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1. Read Schroeder chapter 1. Did you read all the pages?
2. Along an adiabat, the pressure and volume of an ideal gas are related by $PV^\gamma =$ constant. Show that
$$\frac{dT}{dP} = F(\gamma)\frac{T}{P}$$
and find the function $F(\gamma)$.
3. See the attached problem with a PV diagram.
4. Schroeder 1.55. See last page of this set.

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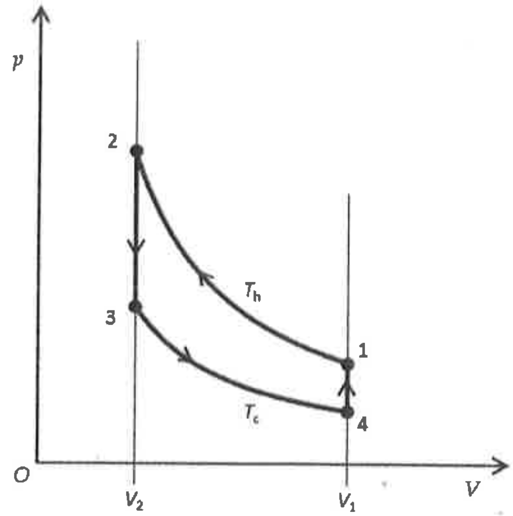
1. (a) Describe and draw all of the quadratic degrees of freedom of a carbon dioxide molecule.
   (b) What is f at high temperature, so that none of the modes are frozen out?
   (c) Treating CO_2 as an ideal gas, what is the molar heat capacity at constant volme in terms of the gas constant R?
   (d) What is the molar heat capacity at constant pressure?
2. Starting from the first law $dU = TdS - PdV$, show that the equation of state for an ideal gas leads to the conclusion that U depends only on T. Proceed as follows: U could depend on T, V, and P, but because of the ideal gas equation of state (a constraint), the internal energy really only depends on two independent variables. The entropy S will also depend on these same two variables. Find the differential dS and use a Maxwell relation.

Bonus: Solve as much of the other class' assignment as you can.

2) The adjacent p-V diagram shows the so-called Stirling cooling cycle (refrigerator). Its working fluid is a monoatomic gas (for instance, helium). Processes 1→2 and 3→4 are isothermal: the fluid is held at constant temperature by thermal baths at temperatures T_h ("hot") and T_c ("cold"), respectively. Processes 2→3 and 4→1 are isochoric: they take place at constant volumes V_2 and V_1, respectively. The heat capacity of the working fluid at constant volume, per mole, is C_V.



Determine the heat absorbed by the fluid in …

a. … the process 1→2
b. … the process 2→3
c. … the process 3→4
d. … the process 4→1

e. Determine the net work done on the fluid per cycle.

f. Is the cycle reversible? Explain.

Problem 1.55. Heat capacities are normally positive, but there is an important class of exceptions: systems of particles held together by gravity, such as stars and star clusters.

(a) Consider a system of just two particles, with identical masses, orbiting in circles about their center of mass. Show that the gravitational potential energy of this system is -2 times the total kinetic energy.

(b) The conclusion of part (a) turns out to be true, at least on average, for *any* system of particles held together by mutual gravitational attraction:

$$\overline{U}_{\text{potential}} = -2\overline{U}_{\text{kinetic}}.$$

Here each $\overline{U}$ refers to the total energy (of that type) for the entire system, averaged over some sufficiently long time period. This result is known as the **virial theorem**. (For a proof, see Carroll and Ostlie (1996), Section 2.4.) Suppose, then, that you add some energy to such a system and then wait for the system to equilibrate. Does the average total kinetic energy increase or decrease? Explain.

(c) A star can be modeled as a gas of particles that interact with each other only gravitationally. According to the equipartition theorem, the average kinetic energy of the particles in such a star should be $\frac{3}{2}kT$, where T is the average temperature. Express the total energy of a star in terms of its average temperature, and calculate the heat capacity. Note the sign.

(d) Use dimensional analysis to argue that a star of mass M and radius R should have a total potential energy of $-GM^2/R$, times some constant of order 1.

(e) Estimate the average temperature of the sun, whose mass is 2×10^{30} kg and whose radius is 7×10^8 m. Assume, for simplicity, that the sun is made entirely of protons and electrons.