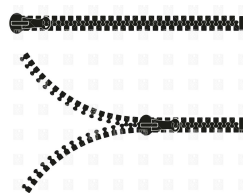

3374

1. Read Schroeder sections 7.1 and 7.2. Did you read all the pages?

2. A system comprises three distinguishable marbles (red, white, and blue) each of mass m which may be found on any step of a staircase whose steps are a distance h apart. The system is known to have energy $3mgh$ with respect to the bottom step (i.e. the bottom step has energy zero).
 - (a) What kind of ensemble is this?
 - (b) List the possible microstates. (A picture would work.)
 - (c) What is the probability that all three marbles are on the same step?
 - (d) What is the average height of the red marble?

3. Solve Schroeder problem 3.34. (See attached.)



4. Consider a horizontal zipper composed of N links which unzips from left to right. A zipped link has energy 0 and an unzipped link has energy ϵ . A link may only unzip if all of the links to its left are unzipped. (This model is used to study DNA molecules.)
 - (a) What is the partition function?
 - (b) What is the average number of unzipped links at temperature T ?

6351

1. A system has energy levels that are equally spaced so that the n th energy level is $E_n = n\epsilon$, where $n = 0, 1, 2, 3, \dots$. The degeneracy g_n of the n th energy level is $n + 1$. Calculate the energy U as a function of temperature. You should be able to evaluate the sum in the partition function exactly.

Bonus: Solve as much of the other class' assignment as you can.

Problem 3.34. Polymers, like rubber, are made of very long molecules, usually tangled up in a configuration that has lots of entropy. As a very crude model of a rubber band, consider a chain of N links, each of length ℓ (see Figure 3.17). Imagine that each link has only two possible states, pointing either left or right. The total length L of the rubber band is the net displacement from the beginning of the first link to the end of the last link.

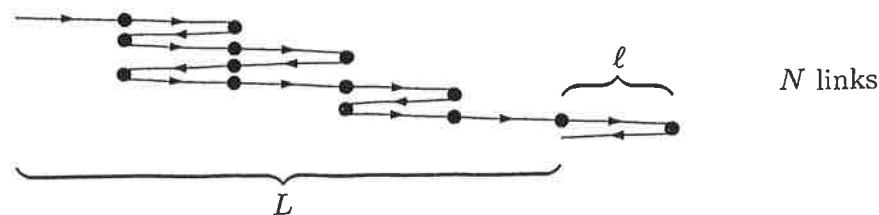


Figure 3.17. A crude model of a rubber band as a chain in which each link can only point left or right.

- (a) Find an expression for the entropy of this system in terms of N and N_R , the number of links pointing to the right.
- (b) Write down a formula for L in terms of N and N_R .
- (c) For a one-dimensional system such as this, the length L is analogous to the volume V of a three-dimensional system. Similarly, the pressure P is replaced by the tension force F . Taking F to be positive when the rubber band is pulling inward, write down and explain the appropriate thermodynamic identity for this system.
- (d) Using the thermodynamic identity, you can now express the tension force F in terms of a partial derivative of the entropy. From this expression, compute the tension in terms of L , T , N , and ℓ .
- (e) Show that when $L \ll N\ell$, the tension force is directly proportional to L (Hooke's law).
- (f) Discuss the dependence of the tension force on temperature. If you increase the temperature of a rubber band, does it tend to expand or contract? Does this behavior make sense?
- (g) Suppose that you hold a relaxed rubber band in both hands and suddenly stretch it. Would you expect its temperature to increase or decrease? Explain. Test your prediction with a real rubber band (preferably a fairly heavy one with lots of stretch), using your lips or forehead as a thermometer. (Hint: The entropy you computed in part (a) is not the total entropy of the rubber band. There is additional entropy associated with the vibrational energy of the molecules; this entropy depends on U but is approximately independent of L .)