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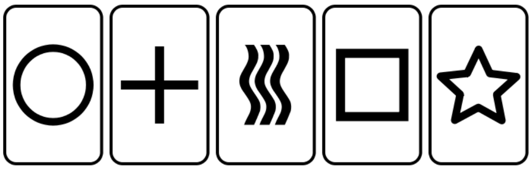
1. Read Schroeder sections 7.1-7.2. Did you read all the pages?

2. (a) What is the energy of the ground state of a hydrogen atom?
   (b) What is the energy of the first excited state of a hydrogen atom?
   (c) What is the degeneracy of the ground state of a hydrogen atom? That is, how many states have the same energy?
   (d) What is the degeneracy of the first excited state of a hydrogen atom?
   (e) What is the relative probability that a hydrogen atom at room temperature will be in its first excited state compared to its ground state?
   (f) At what temperature will there be equal numbers of hydrogen atoms in the ground state and the first excited state?

3. Consider a three-state paramagnet in which an atom with a total spin of 1 $\hbar$ can have a z-projection of spin of $+1$ $\hbar$, 0, or -1 $\hbar$. The corresponding energies are $+\mu B$, 0, and $-\mu B$. There are a total of N atoms. μB is a few electron volts.
   (a) What is the partition function of the system?
   (b) At an absolute temperature of a few nanokelvin, what are the occupancies of the three states?
   (c) At an absolute temperature of a few gigakelvin, what are the occupancies of the three states?
   For the following questions, do not substitute any particular absolute temperature T; leave T as a parameter in the answers.
   (d) What is the Helmholtz free energy of the system as a function of temperature?
   (e) What is the average energy of the system as a function of temperature?
   (f) What is the magnetic moment of the system as a function of temperature?

4. Zener cards are used to conduct experiments for extra-sensory perception (ESP). There are just five different Zener card faces: a circle (one line), a Greek cross (two lines), wavy lines (three lines), a square (four lines), and a star (five lines). There are 25 cards in a deck, five of each design. The backs of all cards in a deck are identical.



A trial consists of randomizing (shuffling) the deck and placing one card face down in front of a test subject. The subject predicts the face of the card. The card is then turned over to reveal the face and the prediction is labeled correct or incorrect. The card is then returned to the deck which is re-randomized before the next trial. An experiment consists of 25 trials.

(a) What is the probability of someone without ESP guessing all 25 trials correctly? (Presumably, someone with ESP should be able to predict all 25 cards correctly with 100% certainty. No one has ever achieved this feat without cheating.)

(b) What is the probability of someone without ESP guessing no trial correctly?

(c) What is the probability of someone without ESP guessing at least one (that is, one or more) trial correctly?

(d) If everyone on Earth were tested for ESP using a Zener card experiment of 25 trials and a histogram were made of the number of cards guessed correctly, what would be the mode of this distribution? That is, what is the most likely number of cards to be guessed correctly?

(e) Every Zener card in the deck is attached to a horizontal wire running along the long axis of the card through the center of mass. If the cards were blank, they would be in equilibrium requiring zero energy to rotate about the wire. But the ink that makes the designs on the faces is heavy and the mass of ink is proportional to the number of lines on the face. It takes energy E_0 to flip the circle (one line), energy $2E_0$ to flip the cross (two lines), etc. If the deck of 25 cards on a wire is placed in thermal contact with a reservoir at absolute temperature T, what is the partition function for the deck?

(f) What is the average number of lines visible (flipped) at absolute temperature T?

(g) How many lines are visible at very high absolute temperature, $k_B T >> E_0$ (not high enough to burn the cards)?

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(a) In this problem, you will relate the difference in heat capacities $C_P - C_V$ to two other experimentally measurable quantities: the isothermal compressibility

$$K_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

and the isobaric thermal expansion coefficient

$$\beta_P = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

i. Regard the entropy as a function of temperature and volume $S(T, V)$, and regard the volume as function of temperature and pressure $V(T, P)$. Write the differentials dS and dV and find an expression for $\left(\frac{\partial S}{\partial T}\right)_P$.

ii. Using the result above and a Maxwell relation, express $C_P - C_V$ in terms of T, $\left(\frac{\partial V}{\partial T}\right)_P$, and $\left(\frac{\partial P}{\partial T}\right)_V$.

iii. Set the differential $dV = 0$ to obtain an expression for $\left(\frac{\partial P}{\partial T}\right)_V$.

iv. What is $C_P - C_V$ in terms of T, V, K_T, and β_P?

Bonus: Solve as much of the other class' assignment as you can.