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1. Read Schroeder chapter 8. Did you read all the pages?
2. How many photons are in one cubic meter at
 - (a) cosmic background temperature
 - (b) room temperature
 - (c) solar surface temperature
 - (d) solar core temperature
 - (e) What is the entropy per photon in natural units (leave k_B in the formula as a symbol)? Does it depend on temperature?
3.
 - (a) How many ways are there to arrange 3 particles in 4 states, all at the same energy, if the particles are
 - i. identical bosons?
 - ii. identical fermions?
 - (b) How many ways are there to arrange 4 identical fermions in 4 states, all at the same energy?
 - (c) How many ways are there to arrange 5 identical fermions in 4 states, all at the same energy?
4.
 - (a) Estimate the moment of inertia I for a CO molecule rotating about an axis through its center of mass and perpendicular to the line joining the atoms. Are the electrons or the nuclei more important?
 - (b) At what temperature T would you expect this rotational degree of freedom to freeze out?
 - (c) Estimate the moment of inertia I for a CO molecule rotating about an axis through its center of mass and parallel to the line joining the atoms. Are the electrons or the nuclei more important?
 - (d) At what temperature T would you expect this rotational degree of freedom to freeze out?
 - (e) Explain why we ignore the latter degree of freedom.
5. Consider a diatomic molecule with non-identical atoms like CO. Define the energy $\epsilon \equiv \frac{\hbar^2}{2I}$. Use a computer to sum the rotational quantum mechanical partition function numerically, keeping terms through $\ell = 7$ (at least). Calculate the average energy, and then get the heat capacity per diatomic molecule in units of k_B . Plot (not sketch) the heat capacity versus x , where $x \equiv k_B T / \epsilon$, for $x = 0$ to 4. Do you see the bump that I drew in lecture?

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1. Schroeder 6.30.

Bonus: Solve as much of the other class' assignment as you can.

Problem 6.28. Use a computer to sum the rotational partition function (equation 6.30) algebraically, keeping terms through $j = 6$. Then calculate the average energy and the heat capacity. Plot the heat capacity for values of kT/ϵ ranging from 0 to 3. Have you kept enough terms in Z to give accurate results within this temperature range?

Problem 6.30. In this problem you will investigate the behavior of ordinary hydrogen, H_2 , at low temperatures. The constant ϵ is 0.0076 eV. As noted in the text, only half of the terms in the rotational partition function, equation 6.30, contribute for any given molecule. More precisely, the set of allowed j values is determined by the *spin* configuration of the two atomic nuclei. There are four independent spin configurations, classified as a single "singlet" state and three "triplet" states. The time required for a molecule to convert between the singlet and triplet configurations is ordinarily quite long, so the properties of the two types of molecules can be studied independently. The singlet molecules are known as **parahydrogen** while the triplet molecules are known as **orthohydrogen**.

- (a) For parahydrogen, only the rotational states with even values of j are allowed.* Use a computer (as in Problem 6.28) to calculate the rotational

*For those who have studied quantum mechanics, here's why: Even- j wavefunctions are symmetric (unchanged) under the operation of replacing \vec{r} with $-\vec{r}$, which is equivalent to interchanging the two nuclei; odd- j wavefunctions are antisymmetric under this operation. The two hydrogen nuclei (protons) are fermions, so their overall wavefunction must be antisymmetric under interchange. The singlet state ($\uparrow\downarrow - \downarrow\uparrow$) is already antisymmetric in

partition function, average energy, and heat capacity of a parahydrogen molecule. Plot the heat capacity as a function of kT/ϵ .*

- (b) For orthohydrogen, only the rotational states with odd values of j are allowed. Repeat part (a) for orthohydrogen.
- (c) At high temperature, where the number of accessible even- j states is essentially the same as the number of accessible odd- j states, a sample of hydrogen gas will ordinarily consist of a mixture of 1/4 parahydrogen and 3/4 orthohydrogen. A mixture with these proportions is called **normal hydrogen**. Suppose that normal hydrogen is cooled to low temperature without allowing the spin configurations of the molecules to change. Plot the rotational heat capacity of this mixture as a function of temperature. At what temperature does the rotational heat capacity fall to half its high-temperature value (i.e., to $k/2$ per molecule)?
- (d) Suppose now that some hydrogen is cooled in the presence of a catalyst that allows the nuclear spins to frequently change alignment. In this case *all* terms in the original partition function are allowed, but the odd- j terms should be counted three times each because of the nuclear spin degeneracy. Calculate the rotational partition function, average energy, and heat capacity of this system, and plot the heat capacity as a function of kT/ϵ .
- (e) A deuterium molecule, D_2 , has nine independent nuclear spin configurations, of which six are "symmetric" and three are "antisymmetric." The rule for nomenclature is that the variety with more independent states gets called "ortho-," while the other gets called "para-." For orthodeuterium only even- j rotational states are allowed, while for paradeuterium only odd- j states are allowed.† Suppose, then, that a sample of D_2 gas, consisting of a normal equilibrium mixture of 2/3 ortho and 1/3 para, is cooled without allowing the nuclear spin configurations to change. Calculate and plot the rotational heat capacity of this system as a function of temperature.‡

spin, so its spatial wavefunction must be symmetric, while the triplet states ($\uparrow\uparrow$, $\downarrow\downarrow$, and $\uparrow\downarrow + \downarrow\uparrow$) are symmetric in spin, so their spatial wavefunctions must be antisymmetric.

*For a molecule such as O_2 with spin-0 nuclei, this graph is the whole story; the only nuclear spin configuration is a singlet and only the even- j states are allowed.

†Deuterium nuclei are bosons, so the overall wavefunction must be symmetric under interchange.

‡For a good discussion of hydrogen at low temperature, with references to experiments, see Gopal (1966).