Hyper areas + hypervolumes

$$\int_{x_1=-\infty}^{\infty} e^{-x_1^2}\,dx_1 \int_{x_2=-\infty}^{\infty} e^{-x_2^2}\,dx_2 \cdots \int_{x_n=-\infty}^{\infty} e^{-x_n^2}\,dx_n = \prod_{k=1}^{n}\int_{x_k=-\infty}^{\infty} e^{-x_k^2}\,dx_k$$

$$= \left(\sqrt{\pi}\right)^n = \underbrace{\int d^{n-1}\Omega}_{\text{solid angle}} \; \underbrace{\int_{r=0}^{\infty} e^{-r^2}\, r^{n-1}\,dr}_{\frac{1}{2}\Gamma\left(\frac{n}{2}\right)}$$

$$r^2 = x_1^2 + x_2^2 + \cdots x_n^2$$

$$\Omega_{n-1} = (n-1)\text{ dimensional solid angle} = \frac{2\pi^{n/2}}{\Gamma\left(\frac{n}{2}\right)} = \frac{2\pi^{n/2}}{\left(\frac{n}{2}-1\right)!}$$

Surface area of an $(n-1)$ dimensional sphere.

Sphere: all points a distance R from a center.

$$S_{n-1} = \Omega_{n-1} R^{n-1} \longrightarrow S_n = \Omega_n R^n = \frac{2\pi^{\frac{n+1}{2}} R^n}{\left(\frac{n-1}{2}\right)!}$$

Ball: all points a distance $\leq R$ from a center.

n	S_n	
0	2	
1	$2\pi R$	
2	$4\pi R^2$	
3	$2\pi^2 R^3$	

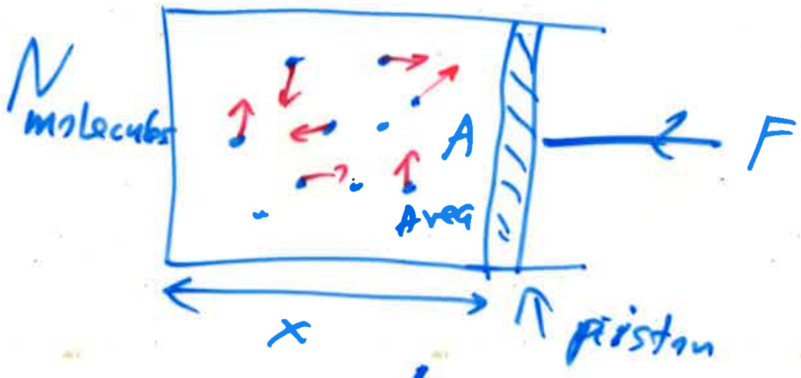
$$V_n = \int_{r=0}^{R} S_{n-1}(r)\, dr = \frac{S_{n-1} R}{n} = \frac{2\pi^{n/2} R^n}{n\left(\frac{n}{2}-1\right)!}$$

n	V_n	
0	1	
1	$2R$	
2	πR^2	
3	$\frac{4}{3}\pi R^3$	
4	$\frac{\pi^2 R^4}{2}$	

Kinetic Theory of Gases

Atoms + Newton's Laws + Maxwell's Eqs.

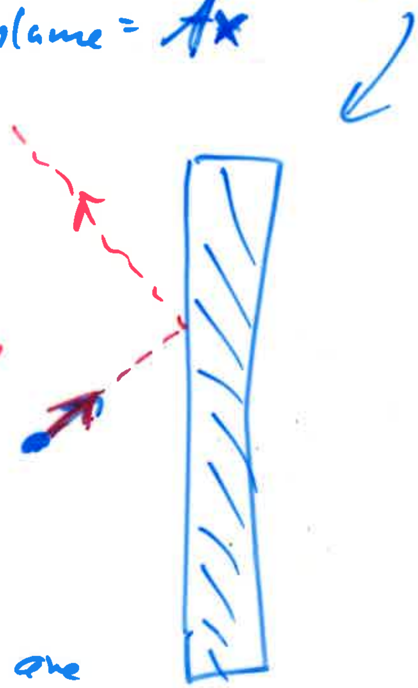
not Quantum Mech.



Volume = Ax

Pressure

$$P = \frac{F}{A}$$



perfect reflection from piston

↑

collisions are elastic

x-component of momentum will be reversed in collisions

$p_y \to p_y \qquad p_z \to p_z$

$p_x \to -p_x$

↳ same kinetic energy before & after collision.

→ same speed before & after.

Total momentum delivered to the piston

$$2mv_x = 2p_x$$

Now watch the box for a time dt. Only atoms within distance $v_x\,dt$ will hit the piston

Number of atoms that hit the piston in time dt

$$d\nu = \frac{N}{V} v_x\,dt\,A$$

Rate at which the piston is hit

$$\frac{d\nu}{dt} = \frac{N}{V} v_x A$$

Force applied to the piston

$$F = \left(\frac{N}{V} v_x A\right)(2m v_x) = \frac{dp}{dt}$$

$$P = \frac{F}{A} = \frac{N}{V} v_x^2\, 2m$$

all v_x's different $\Longrightarrow$ take average

Average Pressure $$P = \frac{Nm}{V}\langle v_x^2\rangle$$

What happened to the 2? Only $\frac{1}{2}$ of the atoms within $v_x\,dt$ were going <u>toward</u> the wall. The other half were moving away.

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle \quad \text{isotropic}$$

$$\langle v_x^2 \rangle = \frac{1}{3} \langle v_x^2 + v_y^2 + v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$\Longrightarrow P = \frac{N}{V} \frac{m}{3} \langle v^2 \rangle = \frac{2}{3} \frac{N}{V} \underbrace{\left\langle \frac{1}{2} m v^2 \right\rangle}$$

average kinetic energy for one molecule.

$$PV = \frac{2}{3} N \langle KE \rangle = \frac{2}{3} U$$

If gas is monatomic
$U = \frac{3}{2} k_B T N$
equipartition theorem

$$PV = N k_B T$$

$$PV = \frac{2}{3} U = (\gamma - 1) U \qquad \gamma = \frac{5}{3} = \frac{C_p}{C_v}$$

Adiabatic $PV^{\gamma} = \text{constant}$.