

Due: Monday 24 October 2008

## 4321

- Imagine that you are a lifeguard trying to reach a drowning swimmer. Choose the  $x$  axis along the linear water-land boundary. You are at position  $(-5, -5)$  m and the swimmer is at  $(+5, +5)$  m. Your speed on land is  $v_L$ ; your speed in water is  $v_W \leq v_L$ . The ratio of speeds is  $\frac{v_L}{v_W} \equiv n \geq 1$ .
  - If you try to reach the swimmer in the least possible time, show that you reproduce Snell's Law.
  - If  $n = 2$ , at what point along the  $x$  axis should you enter the water?
- Given the functional

$$f[y(x), y'(x), z(x), z'(x); x] = yy' \sin(z) + x^2 y^2$$

what are the two Euler equations? Don't try to solve them but carry through all the differentiations.

## 7305

- A steamroller cylinder of radius  $a$  is rolled along a flat horizontal road. At some point the cylinder rolls over and crushes an ant. The steamroller operator feels the bump and stops after a few seconds. The operator observes that the stain on the roller is one meter to the right of and one meter above the stain on the road.
  - What is the radius of the steamroller cylinder?
  - Through what angle has the cylinder rolled after crushing the ant?
  - Are the previous answers unique?
  - Is the stain on the roller on its way up toward the maximum possible height of  $2a$ , or on its way down from the maximum height?
- Consider the surface generated by revolving a curve connecting two fixed points,  $(x_1, y_1)$  and  $(x_2, y_2)$ , about the  $x$  axis. Find the equation of the curve which will minimize the area of the surface of revolution. What is the name of this function  $y(x)$ ?

**Bonus:** Solve as much of the other class' assignment as you can.