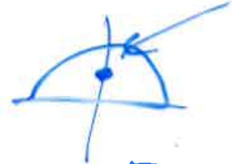
$$\frac{1}{(z^2+1)^2} = \frac{1}{(z-i)^2(z+i)^2}$$



$z=+i$

Expand around $z=+i$

$$= \frac{1}{(z-i)^2} * \text{"Taylor" expansion of } \frac{1}{(z+i)^2} \text{ around } z=+i$$

$$f(x) = f(a) + f'(x)\Big|_{x=a}(x-a) + \frac{1}{2!} f''(x)\Big|_{x=a}(x-a)^2$$

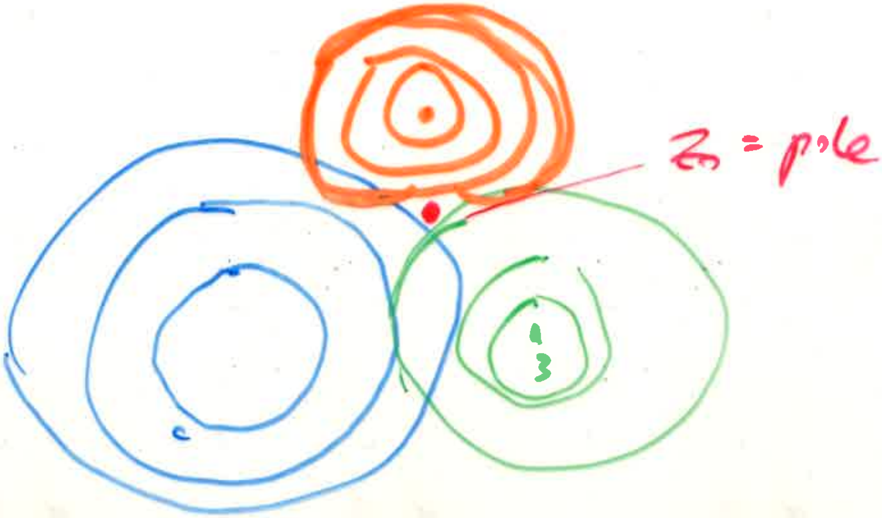
$$f(z) = f(z_0) + \frac{df}{dz}\Big|_{z=z_0}(z-z_0) + \cdots$$

$$\frac{1}{(z+i)^2}\Big|_{z=i} = -\frac{1}{4}, \quad \frac{-i}{4}(z-i),$$

$$\frac{d}{dz}\frac{1}{(z+i)^2} = \frac{d}{dz}(z+i)^{-2} = -2(z+i)^{-3} = \frac{-2}{(z+i)^3}\Big|_{z=i}$$

$$= \frac{-2}{(2i)^3} = -\frac{2}{8(-i)} = \frac{1}{4i} = -\frac{i}{4}$$

$$\frac{1}{(z^2+1)^2} = \frac{1}{(z-i)^2}\left[-\frac{1}{4} - \frac{i}{4}(z-i) + \cdots\right] \to \text{Res} = -\frac{i}{4}$$



① Expand around zero

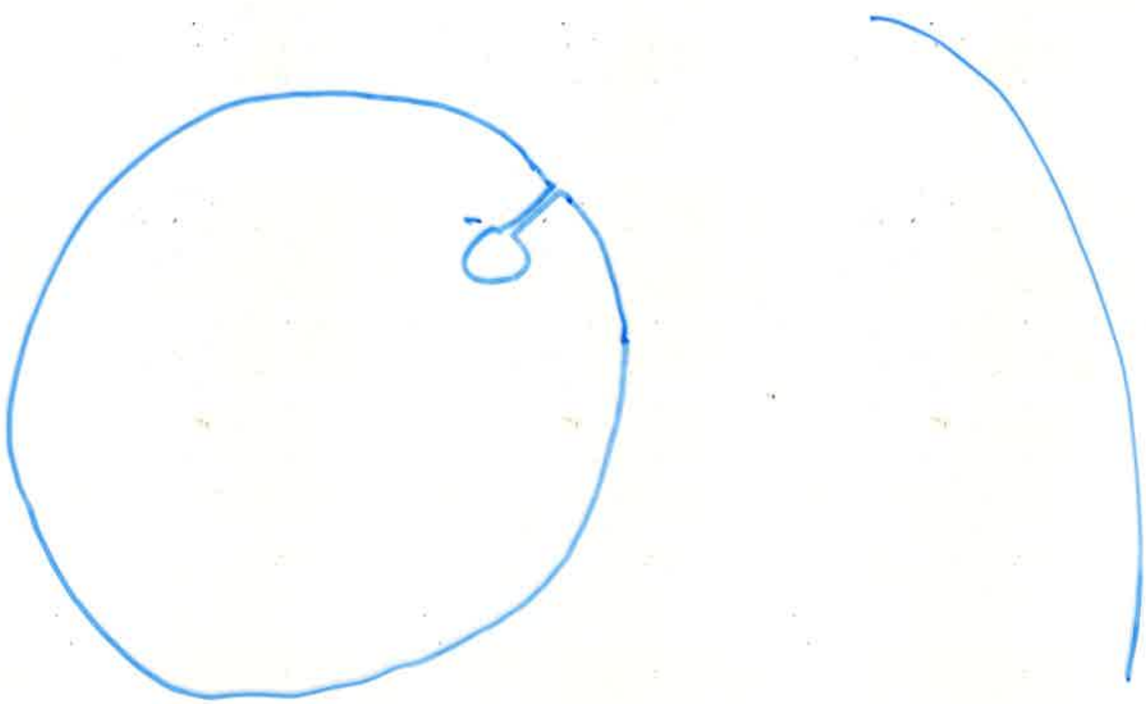
Expand around $z = 3$

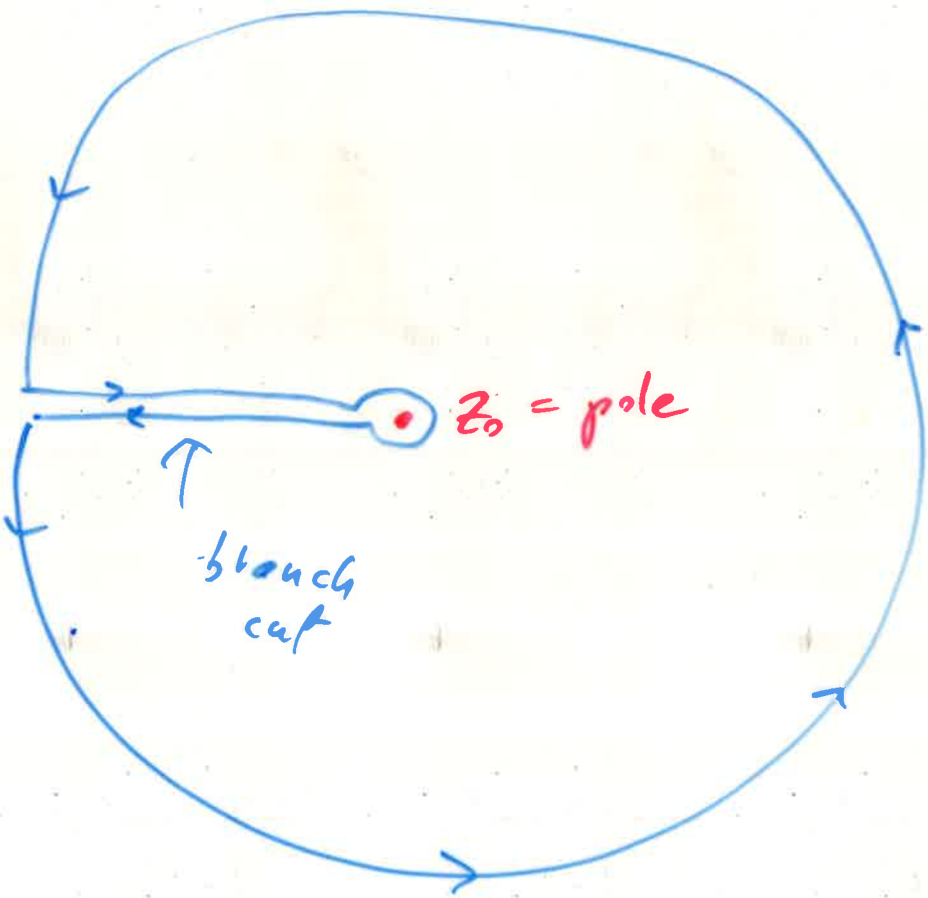
Expand around $z = 2 + 2i$

$$\cdots \frac{a_{-3}}{z^3} + \frac{a_2}{z^2} + \frac{a_{-1}}{z} + a_0 + a_1 z + a_2 z^2 \cdots$$

$$\frac{b_{-3}}{(z-3)^3} + \frac{b_2}{(z-3)^2} \;---$$

OR




z_0 = pole
branch cut