Separation of Variables

Technique for solving Partial Differential Eqs.
$\Longrightarrow$ several Ordinary Diff Eq.

Eg. Laplace Eq. 2nd, linear, homogeneous PDE

$$\nabla^2 \Phi(\vec{r}) = 0$$ coordinate-free

Cartesian: $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi(x,y,z) = 0$$

Ansatz: $\Phi(x,y,z) = F(x)\,G(y)\,H(z) \leftarrow$ Factorized

not all functions can be factored this way

e.g. $\dfrac{1}{x^2+y^2+z^2}$, $x+y$

Spherical Polar: $\nabla^2 \Phi(r,\theta,\varphi) = 0$

$\Phi(r,\theta,\varphi) = R(r)\,T(\theta)\,F(\varphi) \leftarrow$ factorized

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Inhomogeneous version

$$\nabla^2 \Phi(\vec{r}) = \frac{-\rho(\vec{r})}{\epsilon_0}$$

Poisson's Equation

Schrödinger Eq. 2^{nd} order, linear in Ψ, homogeneous

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V(\vec{r},t)\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)$$

Cartesian: $\Psi(x,y,z,t) = F(x)G(y)H(z)T(t)$

$\Rightarrow$ should get 4 ODE's.

Non-relativistic Spin-0 particles & mass m.

Wave Eq. 2^{nd} order, linear in u, homo.

$$\nabla^2 u(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(\vec{r},t) = 0$$

$$\Box\, u(\vec{r},t) = 0 \quad , \quad \partial^\mu\partial_\mu u(x) = 0$$

D'Alembertian

Helmholtz Eq. 2^{nd} order, linear A, homo.

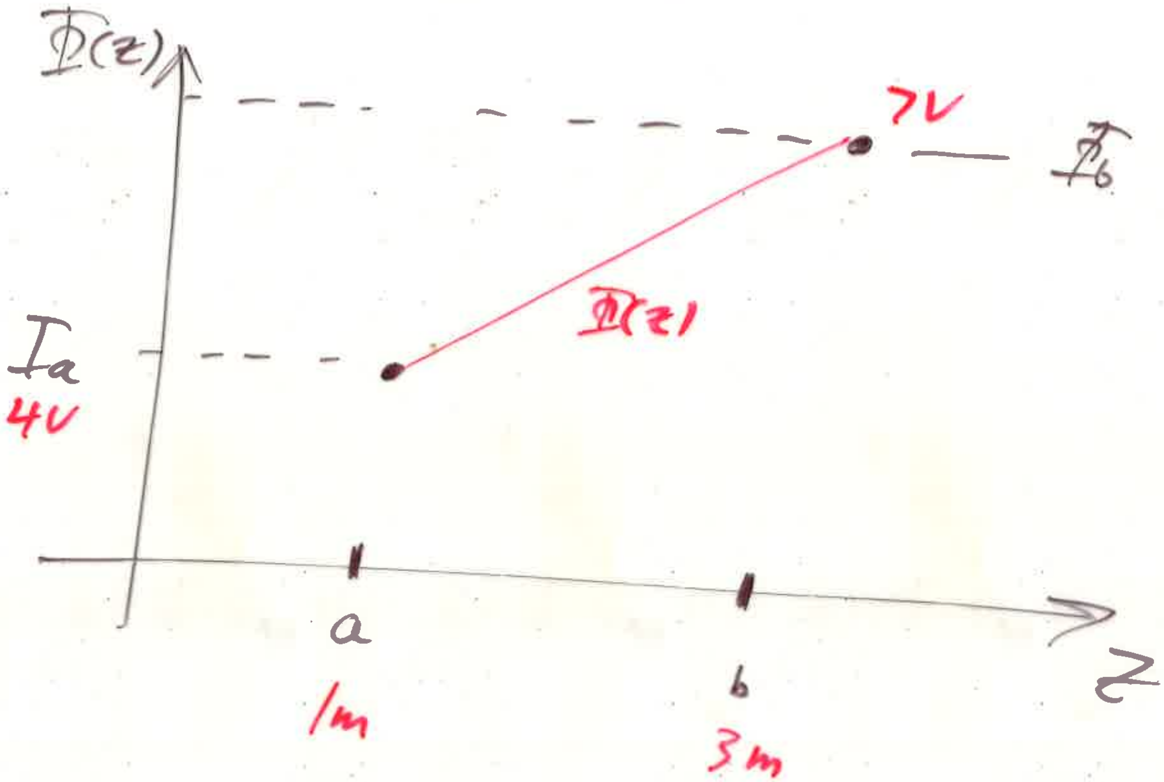
$$\nabla^2 A(\vec{r}) + k^2 A(\vec{r}) = 0$$

Heat Eq. 2^{nd} order, linear in u, homo

$$\frac{\partial}{\partial t}u(\vec{r},t) - k\frac{\partial^2}{\partial x^2}u(\vec{r},t) = 0$$

∇^2

One dimension



$$\Phi(z) = \overset{c_1}{m} z + \overset{c_2}{b}$$

$$= \frac{3}{2} z + \frac{5}{2}$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{3\,V}{2\,m}$$

$$\Phi_a = m z + b$$

$$4V = \frac{3}{2} z + b$$

$$4 = \frac{3}{2}(1) + b$$

Two dimensional Boundary Conditions (BC)