Complete (wiggling) solutions in θ, φ

Spherical coord's ↑ so number of zeros like $\sin, \cos$ J_m

$P_\ell(\cos\theta)$ Legendre functions

(if $\ell \in$ Integers then Legendre polynomials)

$F(\varphi) = A\sin(m\varphi) + B\cos(m\varphi)$

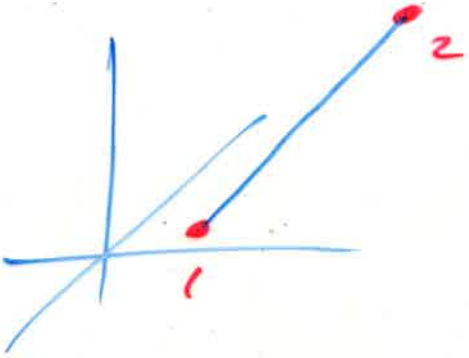
$= e^{\pm i m \varphi}$ $\quad m \in$ Integers

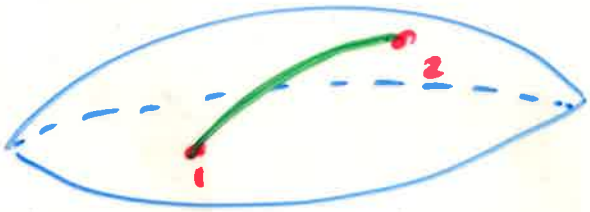
Spherical Harmonics

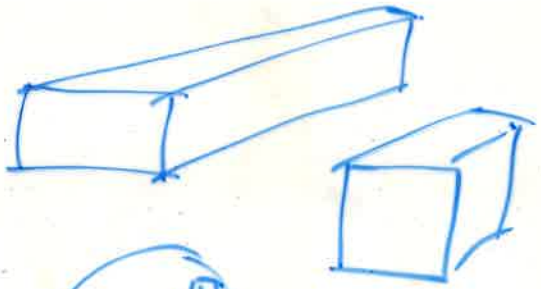
$Y_{\ell m}(\theta, \varphi)$ $\quad Y_{00} = \frac{1}{4\pi}$

Calculus of Variations

e.g.







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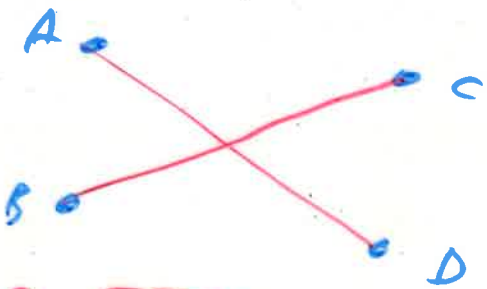
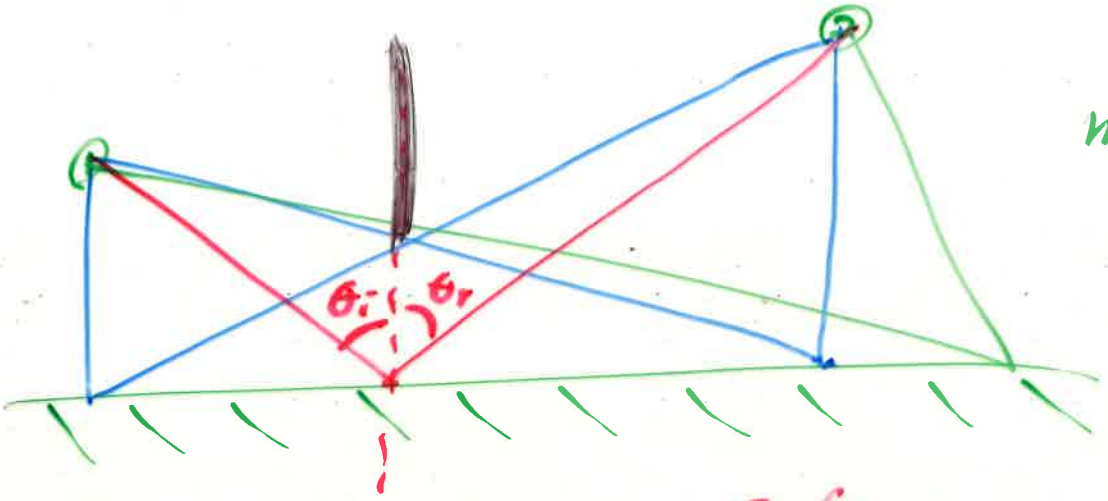
4 cities

$\ell = 2a + 2b$



$2\sqrt{a^2+b^2}$

$2a + b$



Minimize distance & time

Law of specular Reflection

Richard Feynman – Q.E.D. Path Integral formulation of Quantum Mechanics

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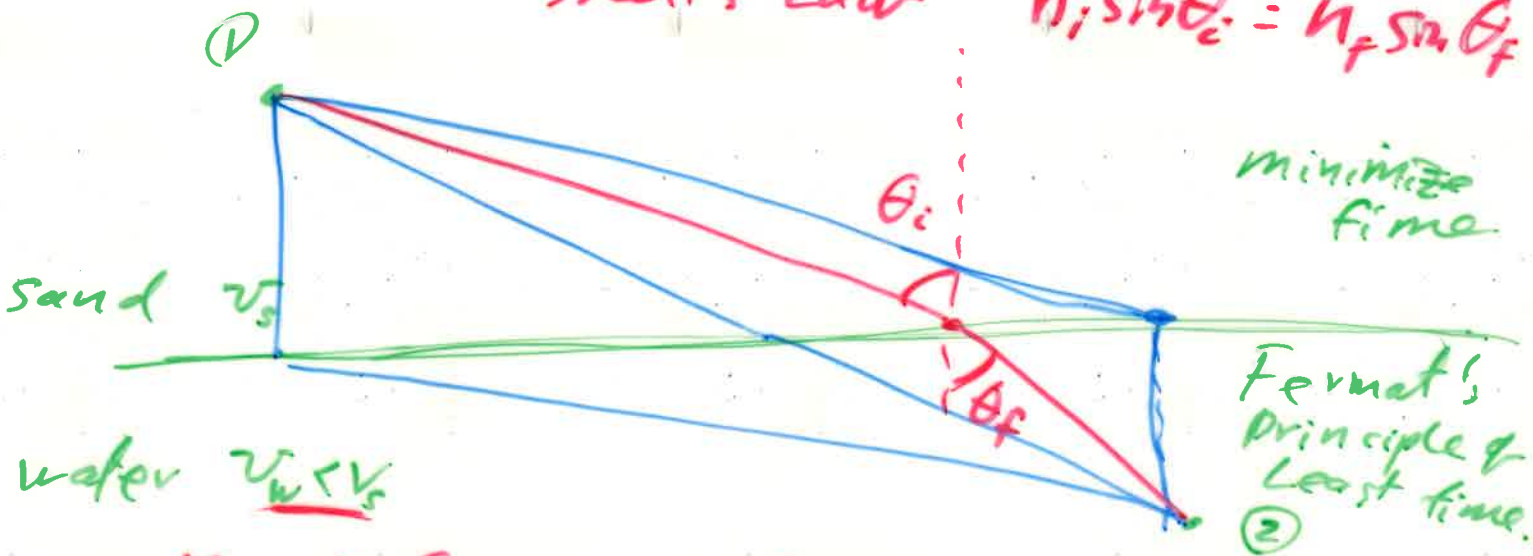
Snell's Law $n_i \sin\theta_i = n_f \sin\theta_f$



minimize time

Fermat's Principle of Least time.

n index of refraction

$v_s = \frac{c}{n_s}$ $\quad v_w = \frac{c}{n_w}$ $\quad n_w > n_s$

$n_{glass} \sim \frac{4}{3}$ $\quad v_{glass} \sim \frac{3}{4} c$

$a(u,v) = u^2 v$ / $b(u,v) = v^3\sqrt{u}$

$G(a,b)$ e.g. $G = a^2 b + \sqrt{b}$

$$\frac{\partial G}{\partial u} = \left(\frac{\partial G}{\partial a}\right)_b \frac{\partial a}{\partial u} + \left(\frac{\partial G}{\partial b}\right)_a \frac{\partial b}{\partial u}$$

$$= (2ab)(2uv) + (a^2 + \tfrac{1}{2}b^{-1/2})\,\frac{1}{2}\frac{v^3}{\sqrt{u}}$$

$$\frac{\partial G}{\partial v} = \left(\frac{\partial G}{\partial a}\right)_b \frac{\partial a}{\partial v} + \left(\frac{\partial G}{\partial b}\right)_a \frac{\partial b}{\partial v}$$