$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0} \qquad \text{Gauss' Law}$$

If $\vec{E}(\vec{r})$ is given, find $\rho(\vec{r})$:

e.g. $\vec{E}(\vec{r}) = \left(Ar + \frac{B}{r}\right)\hat{e}_r + 0\,\hat{e}_\theta + 0\,\hat{e}_\varphi$

$$\epsilon_0\left[\vec{\nabla} \cdot \vec{E}(\vec{r})\right] \Longrightarrow \left[3A + \frac{B}{r^2}\right]\epsilon_0 = \rho(\vec{r})$$

Cartesian Coord's

$$Ar\hat{e}_r = A\vec{r} = A(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)$$

$$\frac{B}{r}\hat{e}_r = \frac{Br\hat{e}_r}{r^2} = \frac{B\vec{r}}{r^2} = \frac{B(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)}{x^2+y^2+z^2}$$

$$\rho(\vec{r}) = \epsilon_0\left[3A + \frac{B}{x^2+y^2+z^2}\right]$$

Divergence Theorem $\Rightarrow$ $\oint\!\!\oint_S \vec{E}\cdot d\vec{A} = \dfrac{q_{\text{enclosed by } S}}{\epsilon_0}$

$\rho(\vec{r}) =$ 

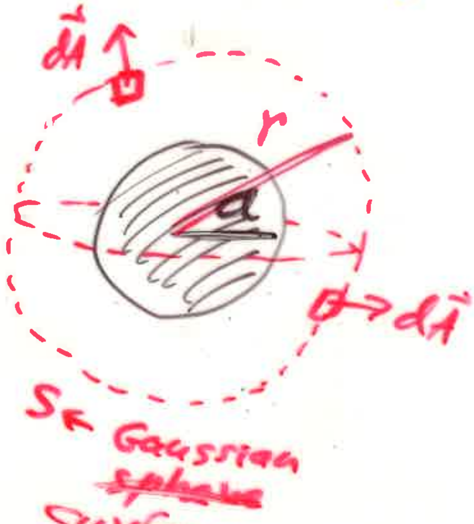
$= \begin{cases} \dfrac{Q}{\frac{4}{3}\pi a^3}, & r < a \\ \emptyset, & r > a \end{cases}$

$$\rho(\vec{r}) = \frac{Q}{\frac{4}{3}\pi a^3} H(a-r)$$

spherical symmetry $\Rightarrow \vec{E}(r,\theta,\varphi) = \vec{E}(r)$

$$= E_r(r)\,\hat{e}_r + \cancelto{0}{E_\theta(r)}\,\hat{e}_\theta + \cancelto{0}{E_\varphi(r)}\,\hat{e}_\varphi = E_r(r)$$

$\underline{r > a}$



$$\oint\!\!\oint_S \underbrace{\vec{E}}_{f(\theta,\varphi)}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$= \frac{Q}{\epsilon_0}$$

$$E_r(r) \oint\!\!\oint_S dA = \text{''}$$

$$= E_r(r)\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

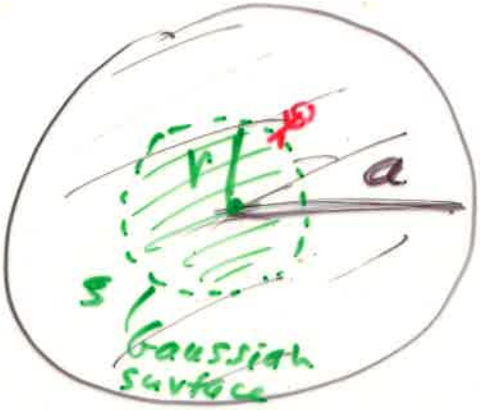
$$E_r(r) = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}$$

$$\vec{E}(\vec{r}) = \left(\frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}\right)\hat{e}_r = \frac{Q}{4\pi\epsilon_0}\frac{\vec{r}}{|\vec{r}|^3}$$

crowd-free

<u>outside $r > a$</u>

$r < a$



$$\oint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$E_r(r)\, 4\pi r^2 = \frac{Q \cancel{\tfrac{4}{3}\pi} r^3}{\cancel{\tfrac{4}{3}\pi} a^3 \,\epsilon_0}$$

$$E_r(r)\, 4\pi \cancel{r^2} = \frac{Q r^3}{\epsilon_0 a^3}$$

$$E_r(r) = \frac{Q}{4\pi\epsilon_0} \frac{r}{a^3}$$

$$\vec{E}(\vec{r}) = \left(\frac{Q}{4\pi\epsilon_0} \frac{r}{a^3}\right) \hat{e}_r = \boxed{\frac{Q\,\vec{r}}{4\pi\epsilon_0\, a^3}}$$

Coordinate-free notation.