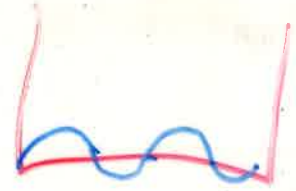
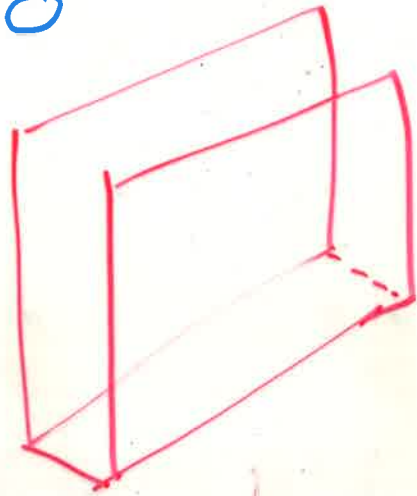
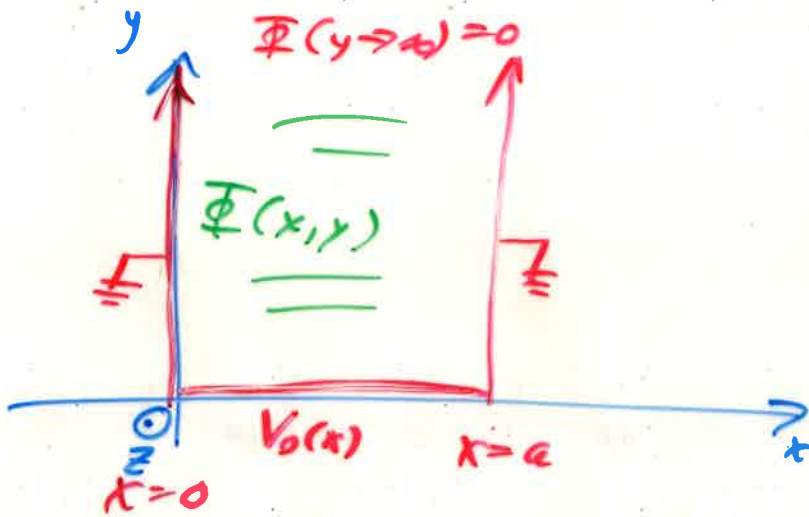


Separation of Variables

B.C. depend on x, y

Laplace Eq. $\nabla^2 \Phi(\vec{r}) = 0$



($y, z = \text{anything}$)

Ausatz: $\Phi(x, y) = F(x) G(y)$

2nd order
linear
PDE

$$\nabla^2 \Phi(x, y) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) [F(x) G(y)] = 0$$

$$= \frac{d^2 F(x)}{dx^2} G(y) + F(x) \frac{d^2 G(y)}{dy^2} = 0$$

$$\frac{\frac{d^2 F(x)}{dx^2}}{F(x)} + \frac{\frac{d^2 G(y)}{dy^2}}{G(y)} = 0$$

$$H(x) + J(y) = 0 \quad \forall x, y$$

$$-\alpha^2 + \alpha^2 = 0$$

Two 2nd-order linear ODEs.

$$\frac{d^2 F(x)}{dx^2} = -\alpha^2 \Rightarrow \frac{d^2 F(x)}{dx^2} + \alpha^2 F(x) = 0$$

$$\frac{d^2 F(x)}{dx^2} + \alpha^2 F(x) = 0$$

$$C_3 e^{i\alpha x} + C_4 e^{-i\alpha x}$$
$$C_5 \sin(\alpha x + C_6)$$
$$C_7 \cos(\alpha x + C_8)$$
$$\cos(\alpha x)$$

$\rightarrow F(x) = C_1 \sin(\alpha x) + C_2 \cos(\alpha x)$
these are complete

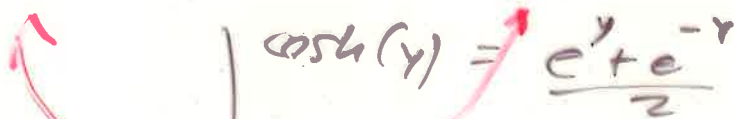
$$\frac{d^2 G(y)}{dy^2} = +\alpha^2 G(y)$$

$$\frac{d^2 G(y)}{dy^2} - \alpha^2 G(y) = 0$$

$$G(y) = d_1 e^{-\alpha y} + d_2 e^{+\alpha y}$$

decay growth

also $d_3 \sinh(\alpha y) + d_4 \cosh(\alpha y)$



$\cosh(y) = \frac{e^y + e^{-y}}{2}$

The graph shows a symmetric, U-shaped curve in the first and second quadrants, passing through (0,1). The x-axis is labeled 'y'.



$\sinh(y) = \frac{e^y - e^{-y}}{2}$

The graph shows an S-shaped curve passing through the origin (0,0), increasing from negative to positive values. The x-axis is labeled 'y'.

not complete

$$F(x) = C_1 \sin(\alpha x) + C_2 \cos(\alpha x)$$

$$\textcircled{1} \text{ B.C. } \Phi(0, y) = 0 \Rightarrow F(0) = 0$$

$$F(0) = C_1 \sin(\alpha \cdot 0) + C_2 \cos(\alpha \cdot 0) = \boxed{C_2 = 0}$$

$$\textcircled{2} \text{ B.C. } \Phi(a, y) = 0 \Rightarrow F(a) = 0$$

$$F(x) = C_1 \sin(\alpha x)$$

$$F(a) = C_1 \sin(\alpha a) = 0 \quad C_1 \neq 0$$

$$\Rightarrow \sin(\alpha a) = 0 \Rightarrow \alpha a = n\pi, \quad n = 1, 2, 3, \dots$$

Separation constant α is quantized

$$\alpha = \frac{n\pi}{a} \quad n = 1, 2, 3, \dots$$

$$G(y) = d_1 e^{-\alpha y} + d_2 e^{+\alpha y}$$
$$= d_1 e^{-\frac{n\pi y}{a}} + d_2 e^{\frac{n\pi y}{a}}$$

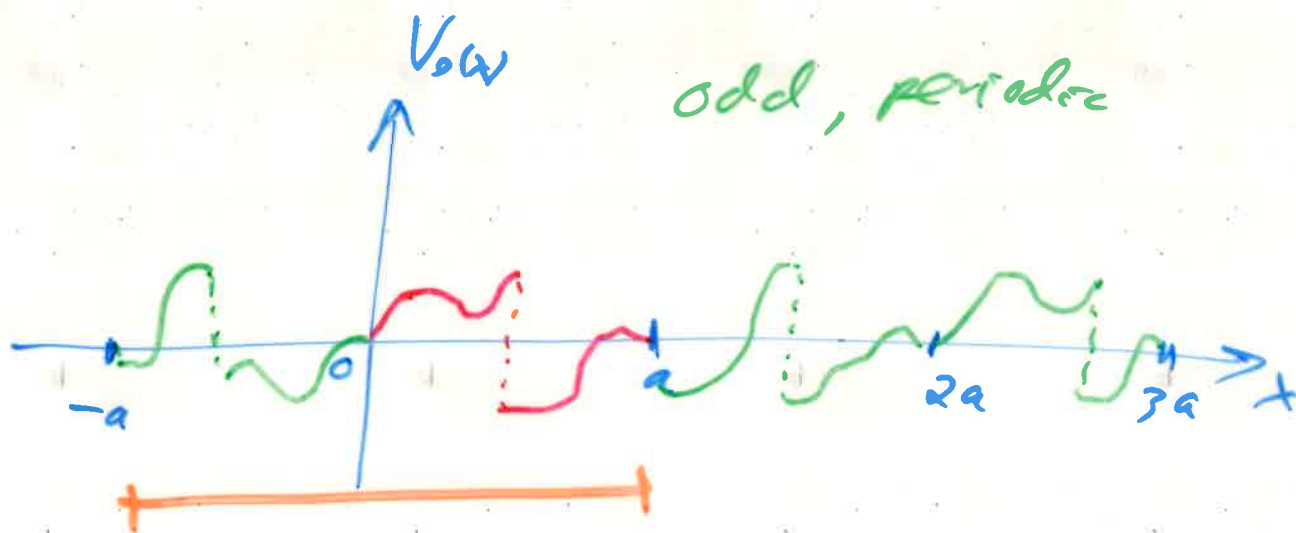
$$\text{B.C. } y \rightarrow a, \quad G(y) \rightarrow 0 \Rightarrow d_2 = 0$$

$$\Phi(x, y) = F(x)G(y) = C_1 \sin\left(\frac{n\pi x}{a}\right) d_1 e^{-\frac{n\pi y}{a}}$$

$$\Phi(x,y) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right) e^{-\frac{n\pi y}{a}}$$

One last Boundary Cond. $y=0$, $\Phi(x,0) = V_0(x)$

$$\Phi(x,0) = V_0(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{a}\right)$$



$$T = \text{period} = 2a$$

→ Use Fourier Analysis to solve for A_n

HW

