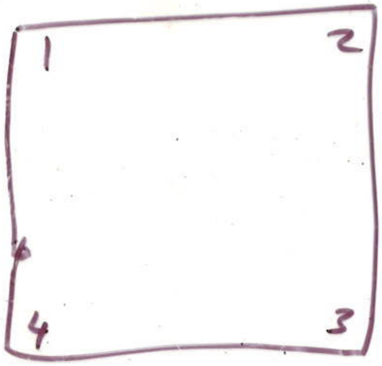

$\{e, r_2\} = C_2$

$r_1 = 90^\circ$ ccw

$r_2 = 180^\circ$ ccw

$r_3 = 270^\circ$ ccw $= 90^\circ$ cw

$C_2 < C_4 < D_4$

C_4

204 → m

1[illegible]3 → n

$\cdot$	e	r_1	r_2	r_3	v	h	m	n
e	e	r_1	r_2	r_3	v	h	m	n
r_1	r_1	r_2	r_3	e	n	m	v	h
r_2	r_2	r_3	e	r_1	h	v	n	m
r_3	r_3	e	r_1	r_2	m	n	h	v
v	v	m	h	n	e	r_2	r_1	r_3
h	h	n	v	m	r_2	e	r_3	r_1
m	m	h	n	v	r_3	r_1	e	r_2
n	n	v	m	h	r_1	r_3	r_2	e

D_4 symmetries of a square (two-sided)

dihedral

$\{e, r_2\}$ $\{e, h\}$, $\{e, v\}$, $\{e, m\}$ $\{e, n\}$

$= C_2$

$C_4 < D_4$ C_4 is a proper subgroup of D_4

$$\left.\begin{array}{l}\{e\} = C_1 \leq D_4 \\ D_4 \leq D_4\end{array}\right\}\text{trivial}$$

ϵ_{ijk}

Permutation groups ("on n letters")

on 3 ~~elements~~ letters 6 group elements.

$$\left.\begin{array}{ll}\{1\,2\,3\} & e \\ \{2\,3\,1\} & R_{120} \\ \{3\,1\,2\} & R_{240}\end{array}\right\}\text{even}$$

$$\left.\begin{array}{ll}\{1\,3\,2\} & F_1 \\ \{3\,2\,1\} & F_2 \\ \{2\,1\,3\} & F_3\end{array}\right\}\text{odd}$$

Symmetric groups S_n — <u>order</u> $n!$ elements

$$S_3 = D_3$$

Alternating group A_n ← even permutations only

$$A_3 = C_3$$

<u>order</u> $\frac{n!}{2}$ elements

$$S_4 \neq D_4$$

order $4! = 24 \neq 8$

$$\{1,2,3,4\} \to \{2,1,3,4\}$$