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**4321**

1. Find the five most important nonzero terms in a complex Fourier expansion of the function  $f(x) = \frac{x}{\pi}$  on the interval  $0 \leq x \leq 2\pi$ , not just the coefficients  $c_n$  but also the terms with  $x$  dependence. You might want to make a plot to see if your expansion looks like the function.
  
2. Consider the quantum mechanical wave function  $\psi(x) = A \exp\left(-\frac{x^2}{8}\right)$ .
  - (a) Find  $A$  by normalizing  $\langle \psi | \psi \rangle = \int |\psi|^2 dx = 1$ . (All integrals are over the entire domain of the variable,  $-\infty < x < \infty$  in this case.)
  - (b) Graph the normalized function  $\psi(x)$ .
  - (c) Find the Fourier transform  $c(k)$  of  $\psi(x)$ .
  - (d) Graph  $c(k)$  vs.  $k$ .
  - (e) What is Parseval's theorem?
  - (f) Is  $c(k)$  automatically normalized?
  - (g) Find  $\langle x \rangle = \langle \psi | x | \psi \rangle = \int x |\psi|^2 dx$ ,  $\langle x^2 \rangle$ , and the standard deviation or uncertainty  $\sigma_x = \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ .
  - (h) Find  $\langle k \rangle = \langle c | k | c \rangle$ ,  $\langle k^2 \rangle$ , and  $\sigma_k$ .
  - (i) Is the Uncertainty Principle  $\sigma_x \sigma_k \geq \frac{1}{2}$  satisfied?

## 7305

1. Find the five most important nonzero terms in a complex Fourier expansion of the function

$$f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 0, & \pi < x \leq 2\pi \end{cases}$$

on the interval  $0 \leq x \leq 2\pi$ , not just the coefficients  $c_n$  but also the terms with  $x$  dependence. You might want to make a plot to see if your expansion looks like the function.

2. Consider the quantum mechanical wave function  $\psi(x) = \begin{cases} Ax, & 0 \leq x \leq \pi \\ 0, & \text{elsewhere} \end{cases}$ .

- (a) Find  $A$  by normalizing  $\langle \psi | \psi \rangle = \int |\psi|^2 dx = 1$ .
- (b) Graph the function  $\psi(x)$ .
- (c) Find the Fourier transform  $c(k)$  of  $\psi(x)$ .
- (d) Graph the real part of  $c(k)$  vs.  $k$ .
- (e) What is Parseval's theorem?
- (f) Is  $c(k)$  automatically normalized?
- (g) Find  $\langle x \rangle = \langle \psi | x | \psi \rangle = \int x |\psi|^2 dx$ ,  $\langle x^2 \rangle$ , and the standard deviation or uncertainty  $\sigma_x = \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ .
- (h) Find  $\langle k \rangle = \langle c | k | c \rangle$ ,  $\langle k^2 \rangle$ , and  $\sigma_k$ .
- (i) Is the Uncertainty Principle  $\sigma_x \sigma_k \geq \frac{1}{2}$  satisfied?

**Bonus:** Solve as much of the other class' assignment as you can.