Physics 4321/7305 Professor Scalise
Homework Assignment #5 Spring 2020

4321

1. Show that these four ways of writing solutions to the undamped homogeneous simple harmonic oscillator equation, $y''(t) + \omega^2 y(t) = 0$, are equivalent by writing (C and D) in terms of (A and B), then write (E and F) in terms of (A and B), and finally write (G and H) in terms of (A and B).
   $y(t) = A\sin(\omega t) + B\cos(\omega t)$
   $y(t) = C\cos(\omega t + D)$
   $y(t) = E\sin(\omega t + F)$
   $y(t) = G\exp(i\omega t) + H\exp(-i\omega t)$

2. For each of the following ordinary differential equations, state: the function; the variable; whether it is linear or non-linear; and if linear, its order and whether homogeneous or non-homogeneous.

   (a) $4\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2 = 0$

   (b) $4\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2t = 0$

   (c) $4\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$

   (d) $4\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x\sin(t) = 0$

   (e) $\frac{d^3x}{dy^3} = 9x$

   (f) $\frac{d^2y}{dt^2} + 2\beta\frac{dy}{dt} + \omega_0^2 y = \sin(y)$ (β and ω_0 are constants)

   (g) $x^2 + 3\frac{dx}{dt} = 7$

   (h) $\frac{d^4y}{dx^4} + \frac{d^3y}{dx^3} + y^2 = 0$

3. Characterize (order, linear or not, homogeneous or not, etc.) and find by hand the general solution to this differential equation:

$$g''(z) - 16g(z) + z - 1 = 0$$

   then impose the boundary conditions $g(1) = 1$ and $g'(1) = 0$.

7305

1. Evaluate $\int_{x=0}^{\infty} \delta(x^2 + x - 2)\cosh(x)dx$.

2. Find the volume mass density for a thin northern hemispherical shell with uniformly distributed mass m and radius a using generalized functions in spherical polar coordinates.

3. Using Dirac delta functions in the appropriate coordinates, express the following charge distributions as volume charge densities $\rho(\vec{r})$.

   (a) In spherical polar coordinates, a charge Q uniformly distributed over a spherical shell of radius R.

   (b) In cylindrical polar coordinates, a charge per length λ uniformly distributed over a cylindrical surface of radius b.

   (c) In cylindrical polar coordinates, a charge Q spread uniformly over a flat circular disk of negligible thickness and radius R.

   (d) Same as (c), but using spherical polar coordinates.

4. Solve by hand the differential equation, $y(x)y'(x) = 2x^2$.

Bonus: Solve as much of the other class' assignment as you can.