4321

1. Consider the motion of a particle of mass m that starts from rest in a constant gravitational field. If an air drag force force proportional to the square of the speed (i.e., cv^2) is encountered,
   (a) write and characterize the differential equation describing the motion.
   (b) find the terminal speed v_{term}..
   (c) find the distance that the particle falls when its final speed is $v_f < v_{term}$.

2. A mass m on a frictionless horizontal surface is held between two horizontal springs of force constant k and rest length ℓ in such a way that at equilibrium neither spring is stretched nor compressed. The mass is then displaced perpendicular to the line of the springs. Find (but do not solve) the differential equation describing the motion and show that it is intrinsically nonlinear. That is, even for small displacements of the mass, there is no linear term in the Taylor expansion of the force.

7305

1. Starting with the non-linear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - D|\psi|^2\psi$$

show that there is a "soliton solution"

$$\psi(x,t) = Ae^{-i\Omega t}e^{imvx/\hbar}\text{sech}\left(\frac{x-vt}{\Delta}\right)$$

where v is the wave-packet velocity, $\hbar\Omega$ is the soliton energy, D is the potential, and Δ is the soliton width.

   (a) Determine A from normalization: $\int |\psi|^2 = 1$.

   (b) Find Ω and Δ in terms of m, D, v, and constants.

2. (a) Evaluate $\displaystyle\int_{x=-\infty}^{+\infty} \delta'(x/3)\,(x^2+3x+2)\,\sin(x)dx$

   (b) Evaluate $\displaystyle\int_{x=0}^{4} \delta(3)\,\theta(x-1)dx$

   (c) Find a distribution that is equivalent to $\sin(ax)\,\delta'(x)$ but that involves neither sin nor δ'

   (d) What is the volume charge density $\rho(\vec{r})$ for an infinitesimally thin rod of total charge Q bent into a semicircle of radius a? Use cylindrical polar coordinates.

   (e) What is the volume mass density $\rho(\vec{r})$ for an infinitesimally thin icecream cone of total mass M, apex angle β, and slant length ℓ? Use spherical polar coordinates.

   (f) Using generalized functions, what is the volume mass density $\rho(\vec{r})$ for a line of mass m that is parallel to the z-axis, passes through the point $(x=2, y=3, z=4)$ and extends from $z=3$ to $z=7$?

   (g) Using generalized functions, what is the volume mass density $\rho(\vec{r})$ for a sheet of mass m that is perpendicular to the z-axis, passes through the point $(x=2, y=3, z=4)$ and extends over the range $-5 < x < +5$ and $-5 < y < +5$?

Bonus: Solve as much of the other class' assignment as you can.