4321

1. Consider two-dimensional polar coordinates $r(t)$ and $\phi(t)$.
   (a) Find $\dot{\widehat{e}}_\phi = \frac{d}{dt}\widehat{e}_\phi$ in terms of $\widehat{e}_r, \widehat{e}_\phi, r, \phi, \dot{r}$, and $\dot{\phi}$.
   (b) Find the radial and tangential components of the acceleration.
   (c) Find the radial and tangential components of the jerk (time derivative of the acceleration).
2. Derive the scale functions (h_i's) for cylindrical polar coordinates. Show all the work.
3. Find the scale functions (h_i's) for the transformation from Cartesian (x, y, z) to (u, v, w) coordinates:
   $$x = \tfrac{1}{2}(u^2 - v^2)$$
   $$y = uv$$
   $$z = w$$
4. Give a numerical answer or simplify as much as possible:
   (a) $\vec{\nabla} \cdot \vec{r}$ (divergence of the displacement vector).
   (b) $\vec{\nabla} \times \vec{r}$ (curl of the displacement vector).
   (c) $\vec{\nabla}|\vec{r}|$ (gradient of the length of the displacement vector).
   (d) $\nabla^2|\vec{r}|$ (Laplacian of the length of the displacement vector).
   (e) $\vec{\nabla} \times \hat{\phi}$ (curl of the azimuthal angle unit vector in cylindrical polar coordinates).

7305

1. Consider the vector field $\vec{v}(\vec{r}) = (x^2 + y^2)\hat{e}_x + (x^2 + y^2)\hat{e}_y + z^2\hat{e}_z$. Decompose the vector field $\vec{v}(\vec{r})$ into the sum of two other vector fields, $\vec{a}(\vec{r})$ and $\vec{b}(\vec{r})$, such that $\vec{a}(\vec{r})$ has no divergence (it is solenoidal) and $\vec{b}(\vec{r})$ has no curl (it is irrotational). The answer is not unique. This is the Helmholtz decomposition.
2. What is $\vec{\nabla} \times \hat{\phi}$ (curl of the azimuthal angle unit vector in spherical polar coordinates)?
3. (a) What is $\dfrac{\partial}{\partial x_j}\left(\dfrac{1}{r}\right)$?
   (b) What is $\dfrac{\partial^2}{\partial x_i \partial x_j}\left(\dfrac{1}{r}\right)$?
   (c) Verify that $\nabla^2\left(\dfrac{1}{|\vec{r} - \vec{r}\,'|}\right) = 0$ for $\vec{r} \neq \vec{r}\,'$ by direct calculation in Cartesian coordinates. Use the previous result.

4. The Green function (heat kernel) for the one-dimensional heat equation is

$$G(x,t) = \frac{1}{\sqrt{4\pi kt}} \exp\left(\frac{-x^2}{4kt}\right)$$

(a) Is $G(x,t)$ a solution to the heat equation everywhere/everywhen? Show your work.

(b) Explain in words what this Green function is physically.

(c) Given a boundary condition at time zero $u(x,0) = f(x)$, write the integral using the Green function that you would use to find $u(x,t)$.

Bonus: Solve as much of the other class' assignment as you can.