Practice Midterm

(3) $$\int_{x=-\infty}^{+\infty} \delta(x)\,(5x^3+4x+2)\,dx = \boxed{2}$$

peaked at $x=0$

peaked at $x=0$

$$\int_{x=-\infty}^{+\infty} \underbrace{\delta(7x)}_{\delta(y)}\,(5x^3+4x+2)\,dx \quad \tfrac{1}{7}dy = \boxed{\frac{2}{7}}$$

$y = 7x$ $\quad \frac{5}{7^3}y^3 + \frac{4}{7}y + 2$

integrate by parts

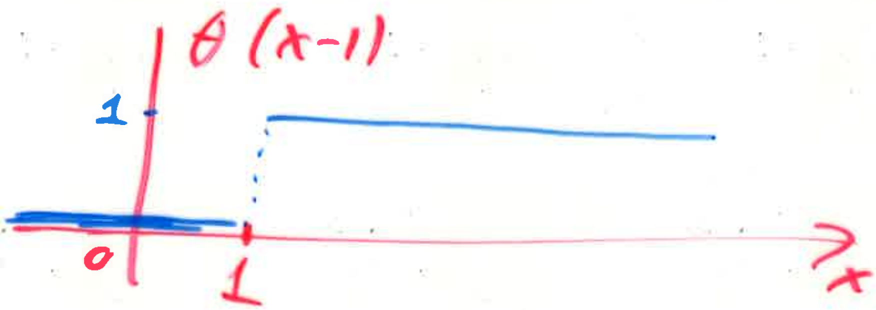
$$\int_{x=-\infty}^{+\infty} \delta'(x)\,(5x^3+4x+2)\,dx = \cancel{\delta(x)(5x^3+4x+2)\Big|_{-\infty}^{+\infty}} \xrightarrow{} 0$$

$$- \int_{x=-\infty}^{\infty} \delta(x)\,(15x^2+4)\,dx = \boxed{-4}$$

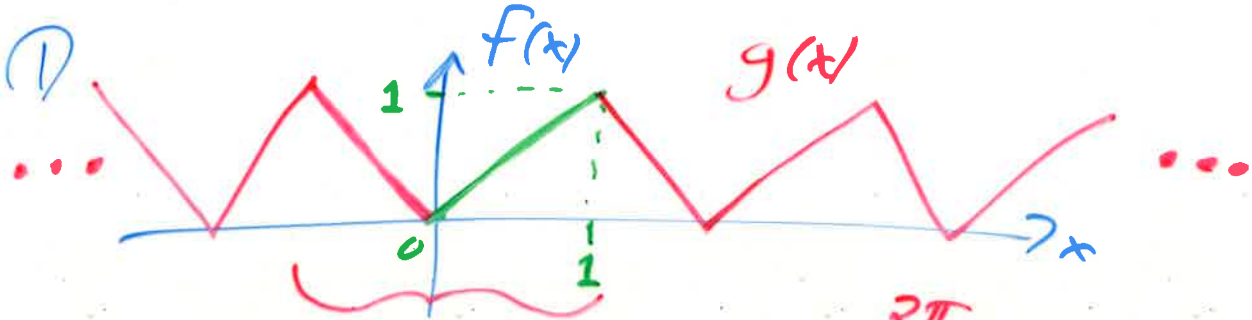
$$\int_{x=-2}^{+2} \theta(x-1)\,(5x^3+4x+2)\,dx = \int_{x=1}^{2} (5x^3+4x+2)\,dx$$

$$= \frac{5}{4}x^4 + 2x^2 + 2x \Big|_{x=1}^{2}$$

$$= \frac{5}{4}16 + 8 + 4 - \left(\frac{5}{4} + 2 + 2\right)$$



1



$T = 2$ $\qquad \omega = \frac{2\pi}{T} = \pi$

continuous $\Rightarrow$ $a_n \propto \frac{1}{n^2}$ = fast convergence.

If discontinuities $\rightarrow$ $a_n \propto \frac{1}{n}$ slow convergence

$$a_n = \langle \cos(n\omega x) | f(x) \rangle$$

$$= \frac{2}{T} \int_{\text{full period}} \cos(n\omega x)\, g(x)\, dx$$

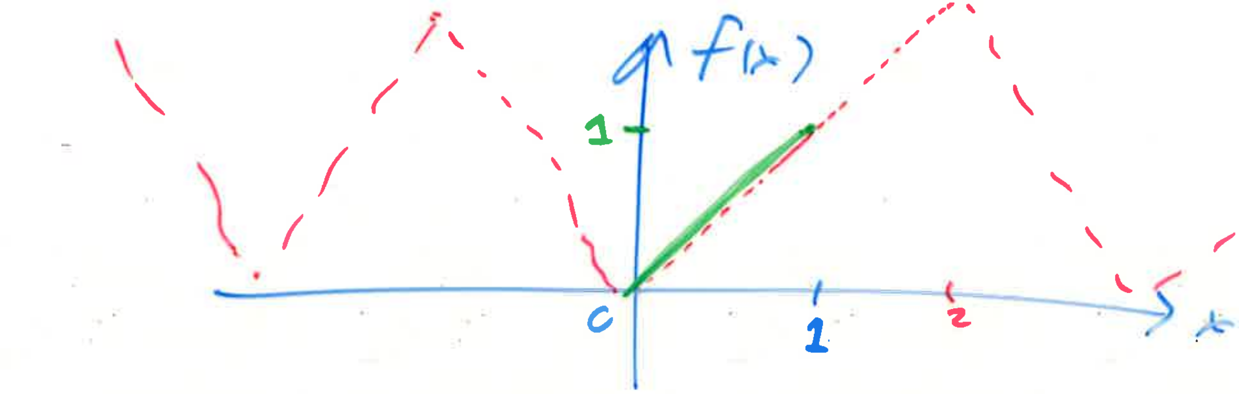
$$= \int_{-1}^{+1} \cos(n\pi x)\, |x|\, dx$$

$$= 2 \int_{x=0}^{1} \cos(n\pi x)\, x\, dx$$

$\frac{a_0}{2}$ is $f_{AVG} = \frac{1}{2}$ (prediction)

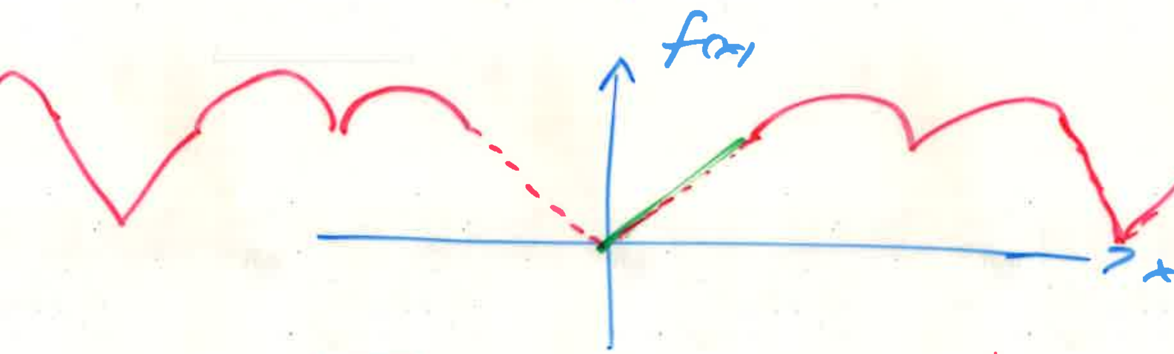
half of a_n's should be zero.

$a_n \propto \frac{1}{n^2}$

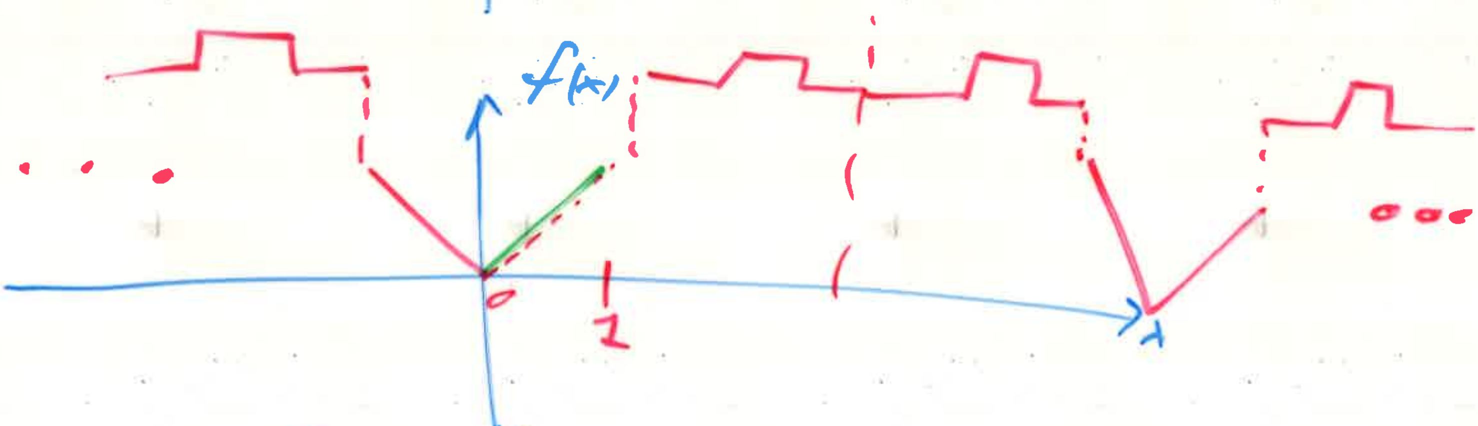


continuous

$T = 4$



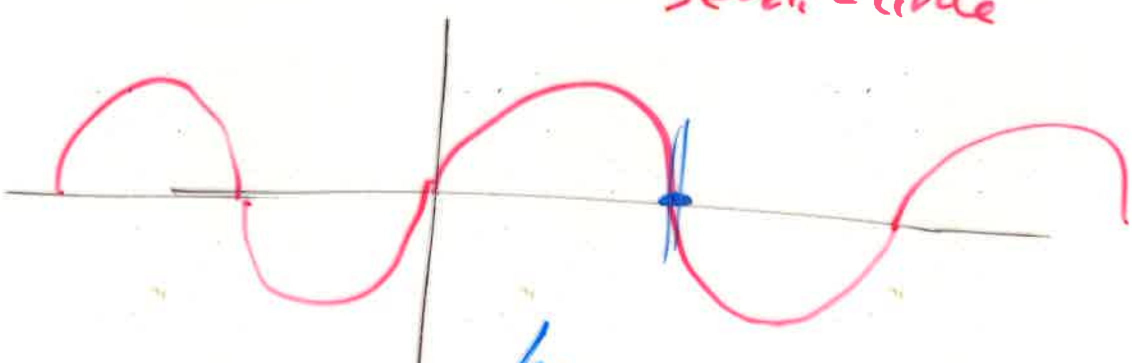
continuous



$a_n \propto \frac{1}{n}$

---

semi-circle



$b_n \propto \frac{1}{n^{3/2}}$

$$t^2 y''(t) + 4t\, y'(t) + 2\, y(t) - t^2 = 0$$

2^{nd} order, linear in y, non-homogeneous, ordinary.

standard form:

$$y''(t) + \frac{4}{t} y'(t) + \frac{2}{t^2} y(t) = +1$$

$$\frac{d^2 y(t)}{dt^2} + \frac{4}{t} \frac{dy(t)}{dt} + \frac{2}{t^2} y(t) = +1$$

Complementary solution

homogeneous:

$$y_c''(t) + \frac{4}{t} y_c'(t) + \frac{2}{t^2} y_c(t) = 0$$

guess $y_c = At^n$

$y_c' = nAt^{n-1}$

$y_c'' = n(n-1)At^{n-2}$

$$n(n-1)At^{n-2} + \frac{4}{t} nAt^{n-1} + \frac{2}{t^2} At^n = 0$$

$$At^{n-2}\left[n(n-1) + 4n + 2\right] = 0$$

$$\Rightarrow \quad n^2 + 3n + 2 = 0$$

Particular sol'n

$y_p(t) = Ct^m$

$$n = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2} \Rightarrow n = -1, -2$$

$$y_c(t) = At^{-1} + Bt^{-2} = \frac{A}{t} + \frac{B}{t^2}$$

$$F(t') = A_0\, \delta(t') \qquad F(t') = B_0\, \delta'(t')$$

$$G(t,t') = \frac{1}{m\omega_1} e^{-\beta(t-t')} \sin[\omega_1 (t-t')]$$

$$x(t) = \int_{t'=-\infty}^{+\infty} F(t')\, G(t,t')\, dt'$$

peaked at $t'=0$

$$= \int_{t'=-\infty}^{+\infty} A_0\, \delta(t') \frac{1}{m\omega_1} e^{-\beta(t-t')} \sin[\omega_1 (t-t')]\, dt'$$

$$= \boxed{\frac{A_0}{m\omega_1} e^{-\beta t} \sin[\omega_1 t] = x(t)}$$

$$x(t) = \int_{t'=-\infty}^{+\infty} \frac{B_0}{m\omega_1}\, \delta'(t')\, e^{-\beta(t-t')} \sin[\omega_1 (t-t')]\, dt'$$

integrate by parts

$$= -\int_{t'=-\infty}^{+\infty} \frac{B_0}{m\omega_1}\, \delta(t') \frac{d}{dt'}\left\{ e^{-\beta(t-t')} \sin[\omega_1 (t-t')] \right\} dt'$$