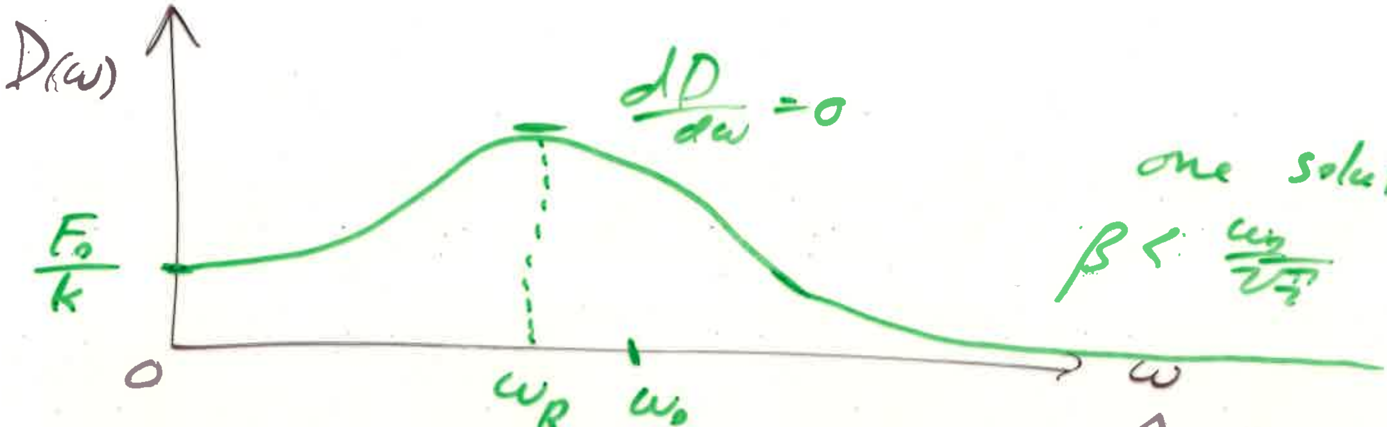
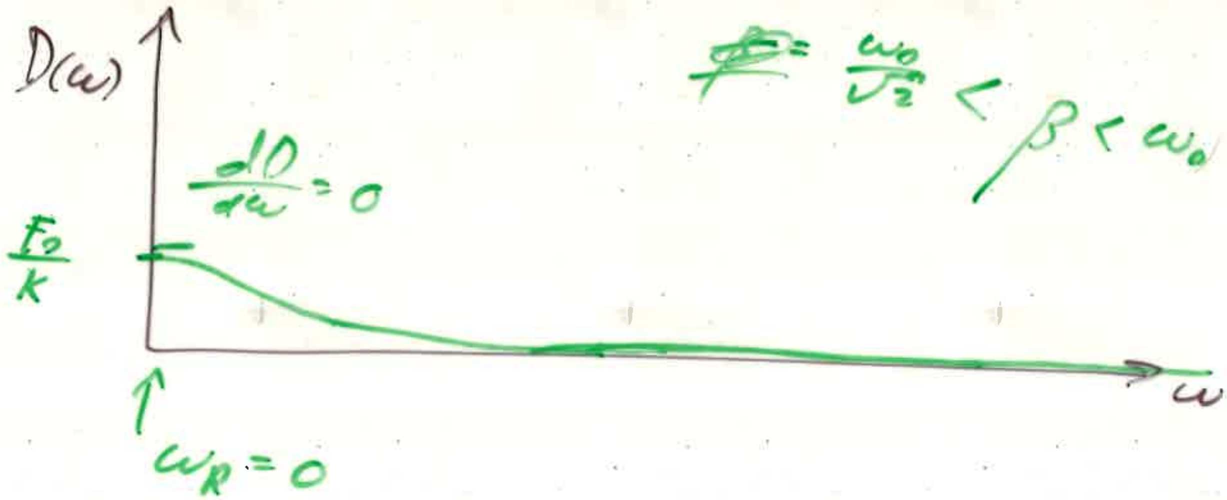


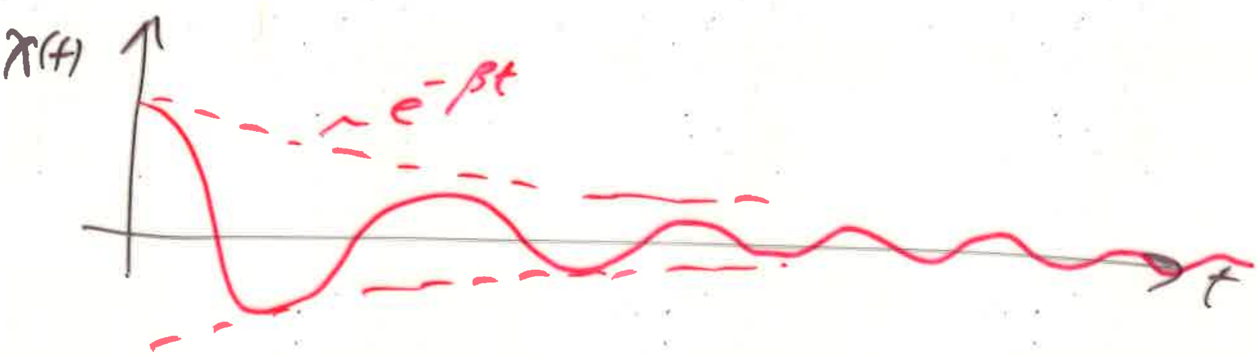
one solution $\beta < \frac{\omega_0}{\sqrt{2}}$

$\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$

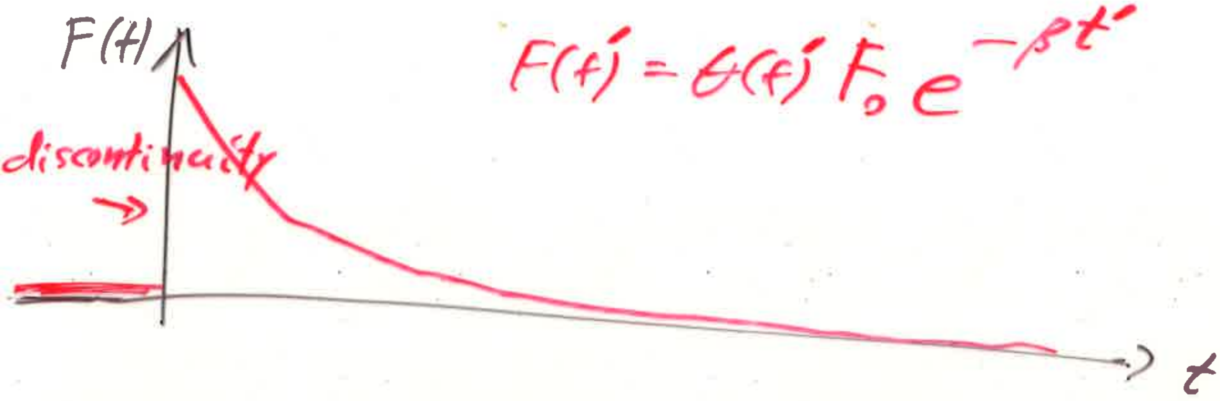
driving frequency



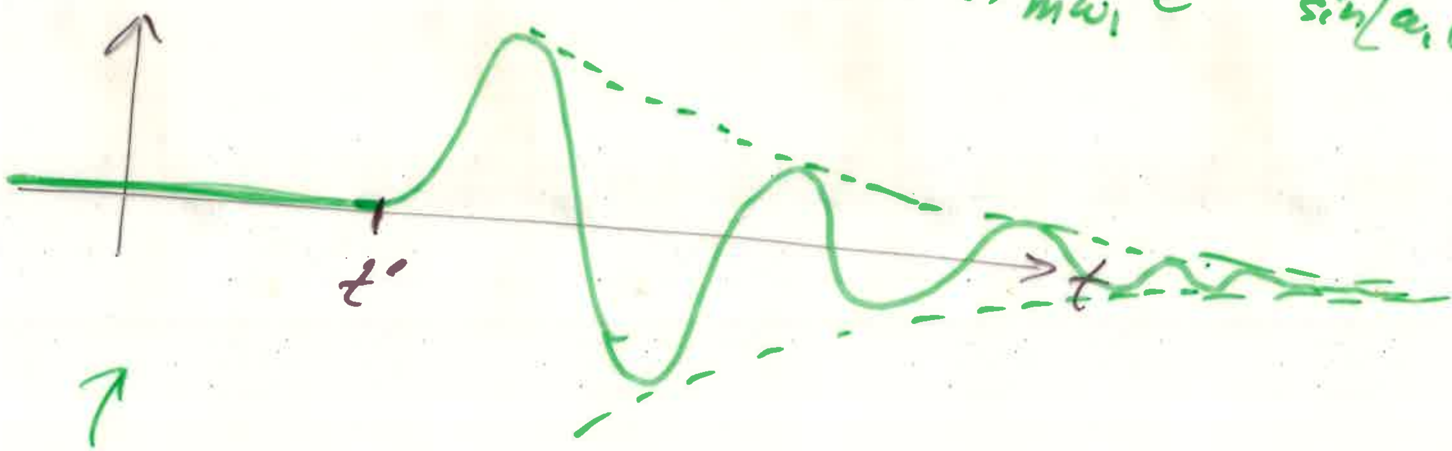
$\cancel{\beta} = \frac{\omega_0}{\sqrt{2}} < \beta < \omega_0$ underdamped



$$F(t') = \theta(t')\, F_0\, e^{-\beta t'}$$



$$G(t,t') = \theta(t-t')\,\frac{1}{m\omega_1}\, e^{-\beta(t-t')} \sin[\omega_1 (t-t')]$$



Green function, depends of the D.E. (eg. SHM) and initial conditions $x(t')=0$, $v(t')=0$

Response

$$x(t) = \int_{t'=-\infty}^{+\infty} F(t')\, G(t,t')\, dt'$$

$$= \int_{t'=-\infty}^{+\infty} \boxed{\theta(t')}\, F_0\, \boxed{e^{-\beta t'}}\, \theta(t-t')\, \frac{1}{m\omega_1}\, e^{-\beta(t-\boxed{t'})} \sin[\omega_1 (t-t')]\, dt'$$

0

$$x(t) = \int_{t'=0}^{\infty} \dots$$

$$x(t) = \frac{F_0}{m\omega_1} e^{-\beta t} \int_{t'=0}^{\infty} \overbrace{\theta(t-t')}^{= \begin{cases} 0 & \text{if } t<t' \\ 1 & \text{if } t>t' \end{cases}} \sin[\omega_1(t-t')]\, dt'$$

$$x(t) = \frac{F_0}{m\omega_1} e^{-\beta t} \int_{t'=0}^{t} \sin[\omega_1(t-t') dt'$$

$$t - t' \equiv u \qquad t'=0 \Leftrightarrow u=t$$

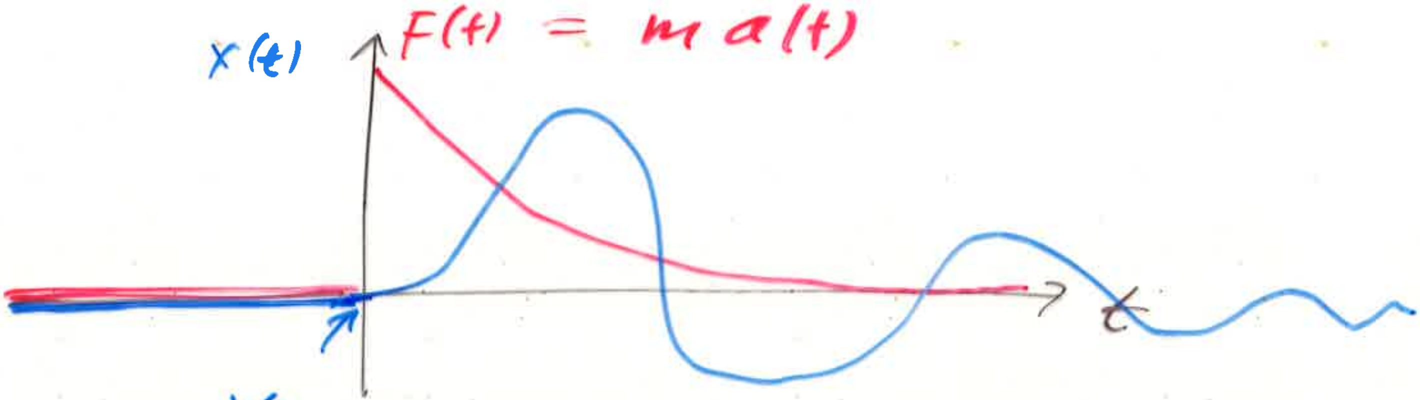
$$-dt' = du \qquad t'=t \Leftrightarrow u=0$$

$$x(t) = \frac{F_0}{m\omega_1} e^{-\beta t} \left[-\int_{u=t}^{0} \sin[\omega_1 u]\, du \right]$$

$$= \frac{F_0}{m\omega_1} e^{-\beta t} \int_{u=0}^{t} \sin(\omega_1 u)\, du$$

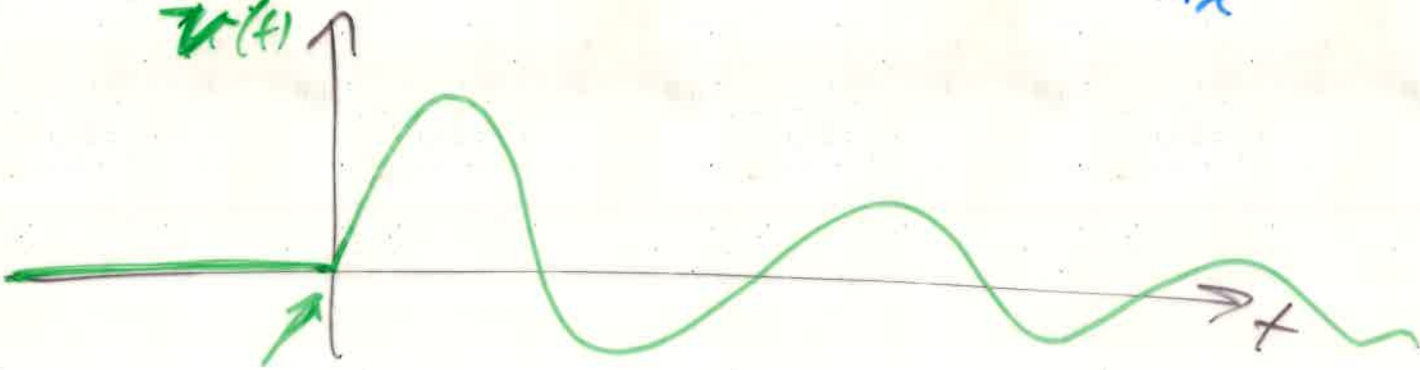
$$= \frac{F_0}{m\omega_1^2} e^{-\beta t} \Big[-\cos(\omega_1 u) \Big]_0^t$$

$$= \frac{F_0}{m\omega_1^2} e^{-\beta t} \left[1 - \cos(\omega_1 t) \right]$$



x(t) is continuous

$x'(t) = v(t)$ is continuous $\Longleftrightarrow$ no kink



kink = change in slope

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(mv)}{dt} = \frac{dm}{dt} v + m \underbrace{\frac{dv}{dt}}_{a}$$

thrust