Physics 4321/7305 Professor Scalise
Homework Assignment #1 Spring 2024

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1. Find the real number coefficients b_i for the set of basis vectors $\hat{u}_i$ from the lecture notes when you decompose the vector $\vec{F} = (4, -4, 4)$.

2. (a) Construct a 3-dimensional orthonormal basis $\{\hat{v}_1, \hat{v}_2, \hat{v}_3\}$ using $(1, 0, 1)$ as one of the directions.

   (b) Expand the vector $\vec{F} = (4, -4, 4)$ in your basis.

   (c) Verify explicitly that your orthonormal basis is complete by showing that $\sum_{n=1}^{3} \hat{v}_n \hat{v}_n$ is the 3 by 3 identity matrix.

3. For $\vec{A} = (1, 2, 3)$ and $\vec{B} = (-1, 0, 2)$, find

   (a) the inner product of $\vec{A}$ and $\vec{B}$. What kind of object is this?

   (b) the cross product of $\vec{A}$ and $\vec{B}$. What kind of object is this?

   (c) the outer product of $\vec{A}$ and $\vec{B}$. What kind of object is this?

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1. The error

$$E_N(c_1, \ldots, c_N) \equiv \int_a^b d\xi \; |f(\xi) - f_N(\xi)|^2 = \int_a^b d\xi \; [f(\xi) - f_N(\xi)]^* [f(\xi) - f_N(\xi)]$$

   made in the approximation

$$f_N(\xi) = \sum_{n=1}^{N} c_n u_n(\xi)$$

   to the function $f(\xi)$ is minimized if the expansion coefficients are chosen as

$$c_n = \int_a^b d\xi \; u_n^*(\xi) f(\xi) \quad .$$

   Prove this assertion. Hint: Write $c_n = a_n + ib_n$ where a_n and b_n are real constants, and $\int_a^b d\xi \; u_n^*(\xi) f(\xi) = A_n + iB_n$ where A_n and B_n are real constants. Then require that

$$\frac{\partial E_N}{\partial a_k} = 0 = \frac{\partial E_N}{\partial b_k} \quad .$$

2. (a) Find $\vec{A}_\parallel$, the piece of $\vec{A}$ parallel to $\vec{B}$, in terms of $\vec{A}$ and $\vec{B}$.

   (b) Find $\vec{A}_\perp$, the piece of $\vec{A}$ perpendicular to $\vec{B}$, in terms of $\vec{A}$ and $\vec{B}$.

   (c) What are the two answers above for $\vec{A} = (1, 2, 3)$ and $\vec{B} = (-1, 0, 2)$?

Bonus: Solve as much of the other class' assignment as you can.