Physics 4321/7305 Professor Scalise
Homework Assignment #3 Spring 2023

4321

1. Find the five most important nonzero terms in a complex Fourier expansion of the function $f(x) = \dfrac{x}{\pi}$ on the interval $0 \le x \le 2\pi$, not just the coefficients c_n but also the terms with x dependence. You might want to make a plot to see if your expansion looks like the function. (Your expansion should be real, not complex, of course.)

2. Consider the quantum mechanical wave function $\psi(x) = A \exp\left(-\frac{x^2}{8}\right)$.

   (a) Find A by normalizing $< \psi|\psi >= \int |\psi|^2 dx = 1$. (All integrals are over the entire domain of the variable, $-\infty < x < \infty$ in this case.)

   (b) Graph the normalized function $\psi(x)$.

   (c) Find the Fourier transform $c(k)$ of $\psi(x)$.

   (d) Graph $c(k)$ vs. k.

   (e) What is Parseval's theorem?

   (f) Is $c(k)$ automatically normalized?

   (g) Find $< x >=< \psi|x|\psi >= \int x|\psi|^2 dx$, $< x^2 >$, and the standard deviation or uncertainty $\sigma_x = \Delta x = \sqrt{< x^2 > - < x >^2}$.

   (h) Find $< k >=< c|k|c >$, $< k^2 >$, and σ_k.

   (i) Is the Uncertainty Principle $\sigma_x \sigma_k \ge \frac{1}{2}$ satisfied? (Notice there is no $\hbar$ in this problem. The Uncertainty Principle is a property of *all* waves: sound, water, etc. Physics gives $p = \hbar k$, but there is no momentum p here.)

7305

1. Find the five most important nonzero terms in a complex Fourier expansion of the function

$$f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$$

on the interval $0 \le x \le 2\pi$, not just the coefficients c_n but also the terms with x dependence. You might want to make a plot to see if your expansion looks like the function. (Your expansion should be real, not complex, of course.)

2. Consider the quantum mechanical wave function $\psi(x) = \begin{cases} Ax, & 0 \le x \le \pi \\ 0, & \text{elsewhere} \end{cases}$.

(a) Find A by normalizing $<\psi|\psi> = \int |\psi|^2 dx = 1$.

(b) Graph the function $\psi(x)$.

(c) Find the Fourier transform $c(k)$ of $\psi(x)$.

(d) Graph the real part of $c(k)$ vs. k.

(e) What is Parseval's theorem?

(f) Is $c(k)$ automatically normalized?

(g) Find $< x > = < \psi|x|\psi > = \int x|\psi|^2 dx$, $< x^2 >$, and the standard deviation or uncertainty $\sigma_x = \Delta x = \sqrt{< x^2 > - < x >^2}$.

(h) Find $< k > = < c|k|c >$, $< k^2 >$, and σ_k.

(i) Is the Uncertainty Principle $\sigma_x \sigma_k \ge \frac{1}{2}$ satisfied? (Notice there is no $\hbar$ in this problem. The Uncertainty Principle is a property of *all* waves: sound, water, etc. Physics gives $p = \hbar k$, but there is no momentum p here.)

Bonus: Solve as much of the other class' assignment as you can.