Physics 4321/7305 Professor Scalise
Homework Assignment #4 Spring 2024

4321

1. Simplify these expressions:
   (a) $\int_{x=-\infty}^{+\infty} \delta(x)\sin(x)dx$
   (b) $\int_{x=-\infty}^{+\infty} \delta(x)\sin(t)dx$
   (c) $\int_{x=-\infty}^{+\infty} \delta(x-t)\sin(x)dx$
   (d) $\int_{x=-\infty}^{+\infty} \delta(x-t)\sin(x+t)dx$
   (e) $\int_{x=-\infty}^{+\infty} \delta(3x)\cos(x)dx$
   (f) $\int_{x=14}^{+\infty} \delta(3x)\cos(x)dx$
   (g) $\int_{x=-\infty}^{+\infty} \delta'(x)(3x^2+x+7)dx$, where the prime means derivative.
   (h) $\int_{x=-\infty}^{+\infty} \delta''(x)(3x^2+x+7)dx$
   (i) $\int_{x=-\infty}^{+\infty} \delta''(x+2)(3x^2+x+7)dx$

2. Use the Heaviside or step function $\theta(t)$ to define the graph of a square wave pulse of height 4 that begins at t=3 and ends at t=6 using:
   (a) two θ functions additively.
   (b) two θ functions multiplicatively.
   (c) a single θ function.

3. Sketch the graph of $\int_{x'=-\infty}^{x}\left[\int_{x''=-\infty}^{x'} \delta(x'')dx''\right]dx'$ vs. x.

7305

1. If a and b are constants and $\delta^{(m)}$ means the m'th derivative of the delta function, show that $x^n\delta^{(m)}(x) =$
   (a) 0, if $m < n$
   (b) $(-1)^n n!\delta(x)$, if $m = n$
   (c) $\frac{(-1)^n m!}{(m-n)!}\delta^{(m-n)}(x)$, if $m > n$

   Don't forget that these distributions only make physical sense inside an integral and multiplied by a test function $f(x)$ with bounded support.

2. You are most familiar with the Euclidean norm of an n-dimensional vector $\vec{r} = \sum_{i=1}^{n} x_i\hat{e}_i$,

$$|\vec{r}| = \sqrt{\sum_{i=1}^{n}(x_i)^2} = \left(\sum_{i=1}^{n}|x_i|^2\right)^{\frac{1}{2}}$$

   that assigns to every non-zero vector a strictly positive length, but this only one possible norm. In general the p-norm is

$$||\vec{r}||_p = \left(\sum_{i=1}^{n}|x_i|^p\right)^{\frac{1}{p}}$$

(a) Explain why $p = 1$ is called the "taxicab norm".

(b) Explain why $p \to \infty$ is called the "maximum norm".

(c) Draw graphs of "circles" in n=2 dimensions (that is, in the x,y-plane) of unit norm $||\vec{r}||_p = 1$ for

i. $p = 1$

ii. $p = 1.5$

iii. $p = 2$ (This is the only true circle.)

iv. $p = 3$

v. $p = 6$

vi. $p \to \infty$

and describe what happens to the shape as p increases. What happens for $p < 1$?

Bonus: Solve as much of the other class' assignment as you can.