4321

Two dimensional motion with air drag (think golf ball trajectory) is not integrable, but purely horizontal motion and purely vertical motion are separately integrable.

1. A railroad boxcar of mass m on a level frictionless track has initial speed v_0 at the origin. There is an aerodynamic drag force $-cv\vec{v}$ acting horizontally. Find
   (a) $v(t)$
   (b) $x(t)$
   (c) What is the furthest distance the boxcar travels?
   (d) $v(x)$
2. Consider the motion of a particle of mass m that starts from rest in a constant gravitational field. If an air drag force force proportional to the square of the speed (i.e., cv^2, same as problem 1) is encountered,
   (a) write and characterize the differential equation describing the motion.
   (b) find the terminal speed v_{term}..
   (c) find the distance that the particle falls when its final speed is v_f, where $v_f < v_{term}$.
3. Numerically solve the time-independent Schrödinger equation for the one-dimensional quantum harmonic oscillator potential like we did in lecture but for the first excited state (which is odd, so choose the appropriate boundary conditions). Use the first-order forward Euler method with a stepsize $h = 0.01$ and find the dimensionless energy eigenvalue ϵ to at least eight significant figures. Use any programming language that you wish. Plot the wavefunction, $f(u)$ which is a proxy for $\psi(x)$.

7305

1. A mass m attached to a spring of force constant k is constrained to move along a frictionless (no damping) one-dimensional horizontal track. For $t < 0$, the mass is at rest and the spring is unstressed. At $t = 0$, the free end of the spring suddenly acquires a speed v_0 along the track which remains constant thereafter. What is the displacement $x(t)$ of the mass?
2. A mass m on a frictionless horizontal surface is held between two horizontal springs of force constant k and rest length ℓ in such a way that at equilibrium neither spring is stretched nor compressed. The mass is then displaced perpendicular to the line of the springs. Find (but do not solve) the differential equation describing the motion and show that it is *intrinsically* nonlinear. That is, even for small displacements of the mass, there is no linear term in the Taylor expansion of the force.

3. Starting with the non-linear Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - D|\psi|^2\psi$$

show that there is a "soliton solution"

$$\psi(x,t) = Ae^{-i\Omega t}e^{imvx/\hbar}\operatorname{sech}\left(\frac{x-vt}{\Delta}\right)$$

where v is the wave-packet velocity, $\hbar\Omega$ is the soliton energy, D is the potential, and Δ is the soliton width.

(a) Determine A from normalization.

(b) Find Ω and Δ in terms of m, D, v, and constants.

Bonus: Solve as much of the other class' assignment as you can.