4321

1. Using separation of variables, solve the one-dimensional heat equation

$$\frac{\partial u(x,t)}{\partial t} - k\frac{\partial^2 u(x,t)}{\partial x^2} = 0$$

for the temperature u at position x and time t along a thin metal rod that sits between $x = 0$ and $x = a$. The ends of the rod are in contact with an ice water (0° Celsius) reservoir and at time zero, the middle of the rod from $x = a/4$ to $x = 3a/4$ is heated to 100° C.

2. Make plots of the temperature versus distance for a few times or a single three-dimensional plot of (x,t,u) for the problem above.

7305

1. The Green function (heat kernel) for the one-dimensional heat equation is

$$G(x,t) = \frac{1}{\sqrt{4\pi kt}} \exp\left(\frac{-x^2}{4kt}\right)$$

   (a) Is $G(x,t)$ a solution to the heat equation everywhere/everywhen? Show your work.

   (b) Explain in words what this Green function is physically.

   (c) Given a boundary condition at time zero $u(x,0) = f(x)$, write the integral using the Green function that you would use to find $u(x,t)$.

2. (a) Solve the two-dimensional wave equation

$$\frac{\partial^2 \psi(x,y,t)}{\partial t^2} - c^2\left[\frac{\partial^2 \psi(x,y,t)}{\partial x^2} + \frac{\partial^2 \psi(x,y,t)}{\partial y^2}\right] = 0$$

   for the axially symmetric oscillations of a circular drumhead with radius a, where ψ is the displacement of the drumhead from its equilibrium height.

   (b) What are the lowest three frequencies of oscillation?

   (c) Make plots of the drumheads in the first three modes of oscillation.

Bonus: Solve as much of the other class' assignment as you can.