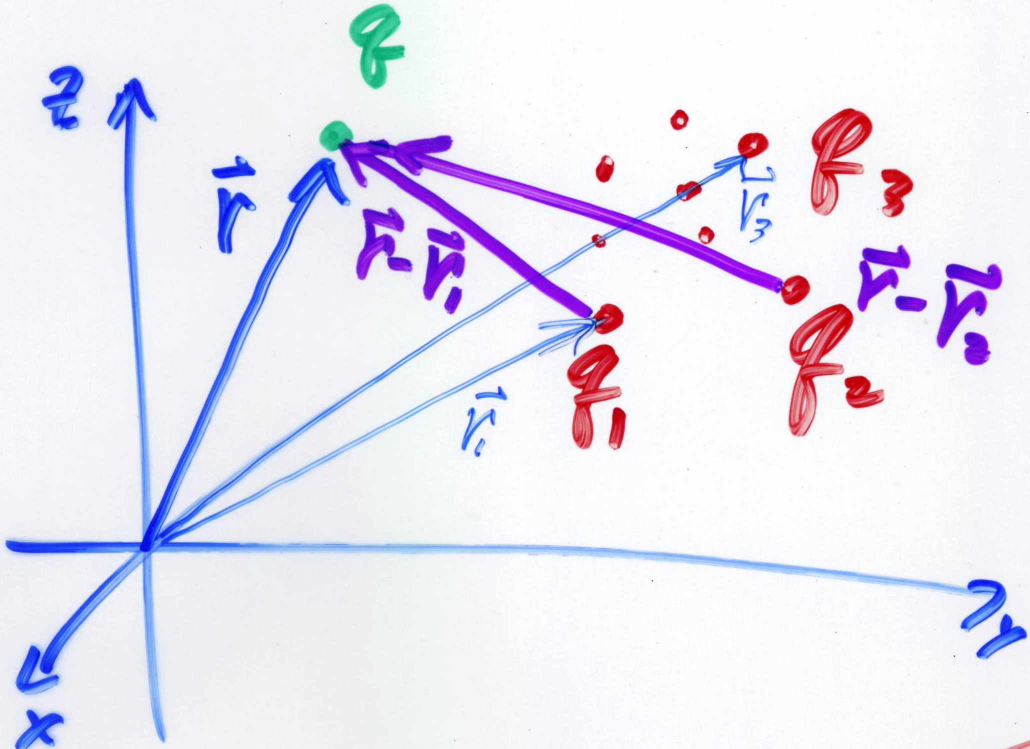
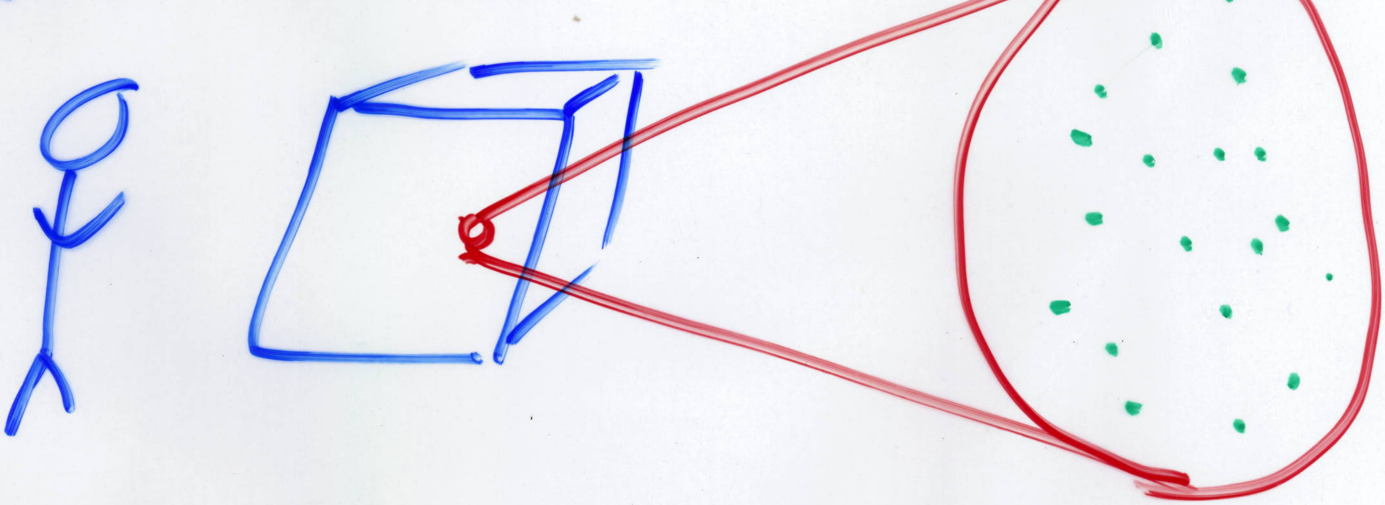
$$\vec{F}_{\text{on } q} = kq \sum_{i=1}^{N} \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

$$\vec{E}(\vec{r}) = K \sum_{i=1}^{N} \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$





$$\vec{E}(\vec{r}) = k \sum_{i}^{N} \frac{q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3} \quad k$$

$$\vec{\mathscr{r}} \equiv \vec{r} - \vec{r}_i \longrightarrow \vec{E}(\vec{r}) = k \sum q_i \frac{\hat{\mathscr{r}}_i}{|\vec{\mathscr{r}}_i|^2}$$

$$\vec{E}(\vec{r}) = k \iiint \frac{dq'(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$dq' = \rho(\vec{r}')\, dV$$

charge density
scalar field

$$\vec{E}(\vec{r}) = k \iiint \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} dV'$$

$\vec{r}$ = field point

$\vec{r}\,'$ = source point

$dx'dy'dz'$ in Cartesian

$r'^2 \sin\theta' dr' d\theta' d\phi'$ in spherical

Volume distribution of charge $\rho(\vec{r}')$

$$\vec{E}(\vec{r}) = k \iiint_{vol} \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \underbrace{dV'}_{dx'dy'dz'}$$

Surface dist. of charge $\sigma(\vec{r}')$

$$\vec{E}(\vec{r}) = k \iint_{S} \frac{\sigma(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \underbrace{da'}_{dx'dy'}$$

Linear dist. of charge $\lambda(\vec{r}')$

$$\vec{E}(\vec{r}) = k \int_{C} \frac{\lambda(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} \underbrace{ds'}_{dx'}$$

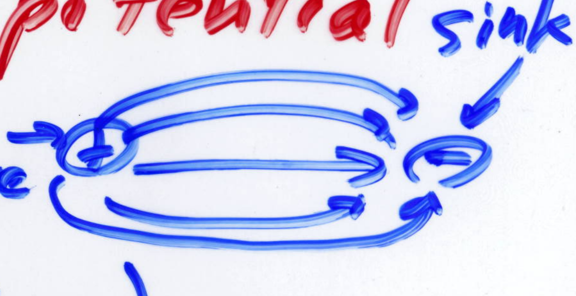
$$-\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}=\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

gradient with respect to unprimed coordinates.

$$\vec{E}(\vec{r})=-\vec{\nabla}\boxed{k\iiint\frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}dV'}$$

$$\vec{E}(\vec{r})=-\vec{\nabla}V(\vec{r})$$

$V(\vec{r})$ = scalar field
electrostatic potential
or voltage.



$$\vec{\nabla}\times\vec{E}(\vec{r})=-\vec{\nabla}\times\left(\vec{\nabla}V(\vec{r})\right)=0$$

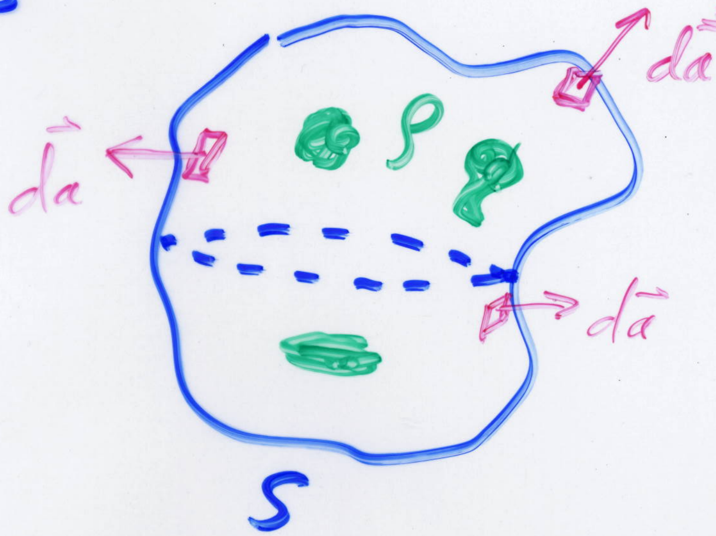
$\Longrightarrow$ Electric field lines cannot close on themselves

Gauss' Law

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{\rho(\vec{r})}{\epsilon_0}$$

differential form

$$\oint\!\!\oint_S \vec{E}(\vec{r})\cdot\vec{da} = \frac{Q_{enclosed}}{\epsilon_0}$$



integral form

Use Gauss' law to find the electric field inside + outside a spherical shell radius R with constant surface charge density σ.

S

$$\oiint_S \vec{E}(\vec{r})\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

gaussian surface

outside $r \geq R$

$\vec{E}$ is parallel to $d\vec{a}$

$$\oiint_S E(\vec{r})\, da = \underline{\underline{\frac{4\pi R^2 \sigma}{\epsilon_0}}}$$

$$E(r) \underbrace{\oiint_S da}_{\text{surface area gaussian sphere}} = \underline{\underline{E(r)\, 4\pi r^2}}$$

$$\boxed{E(r) = \frac{R^2 \sigma}{r^2 \epsilon_0}}$$