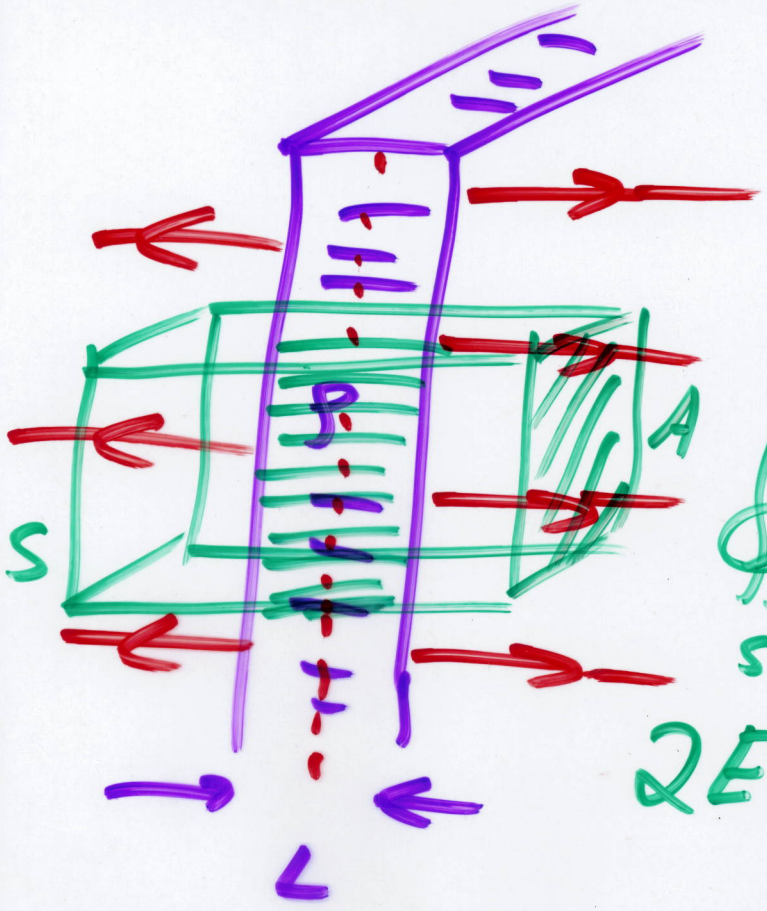


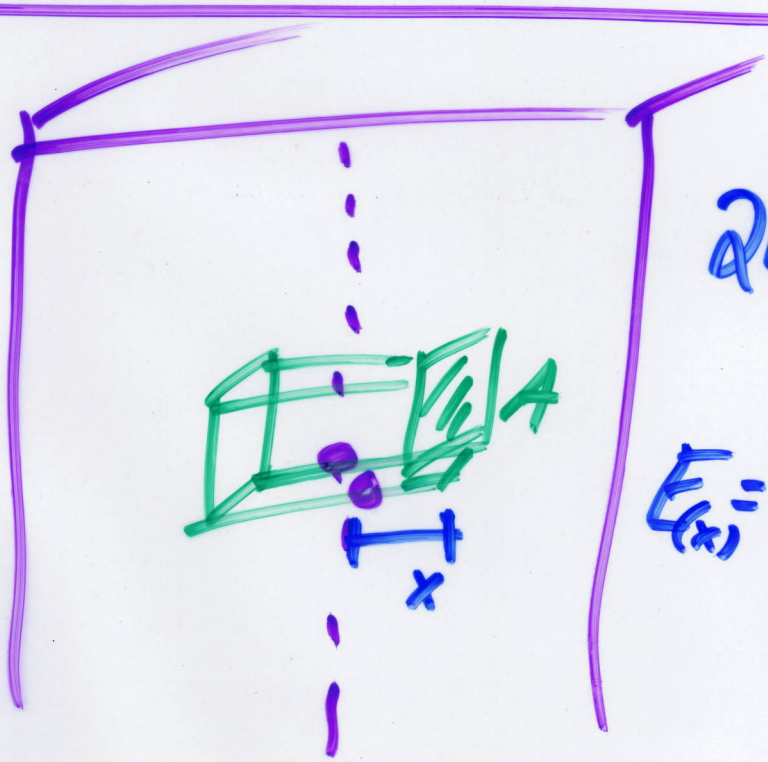
Find $\vec{E}$ everywhere.

Gauss' Law

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

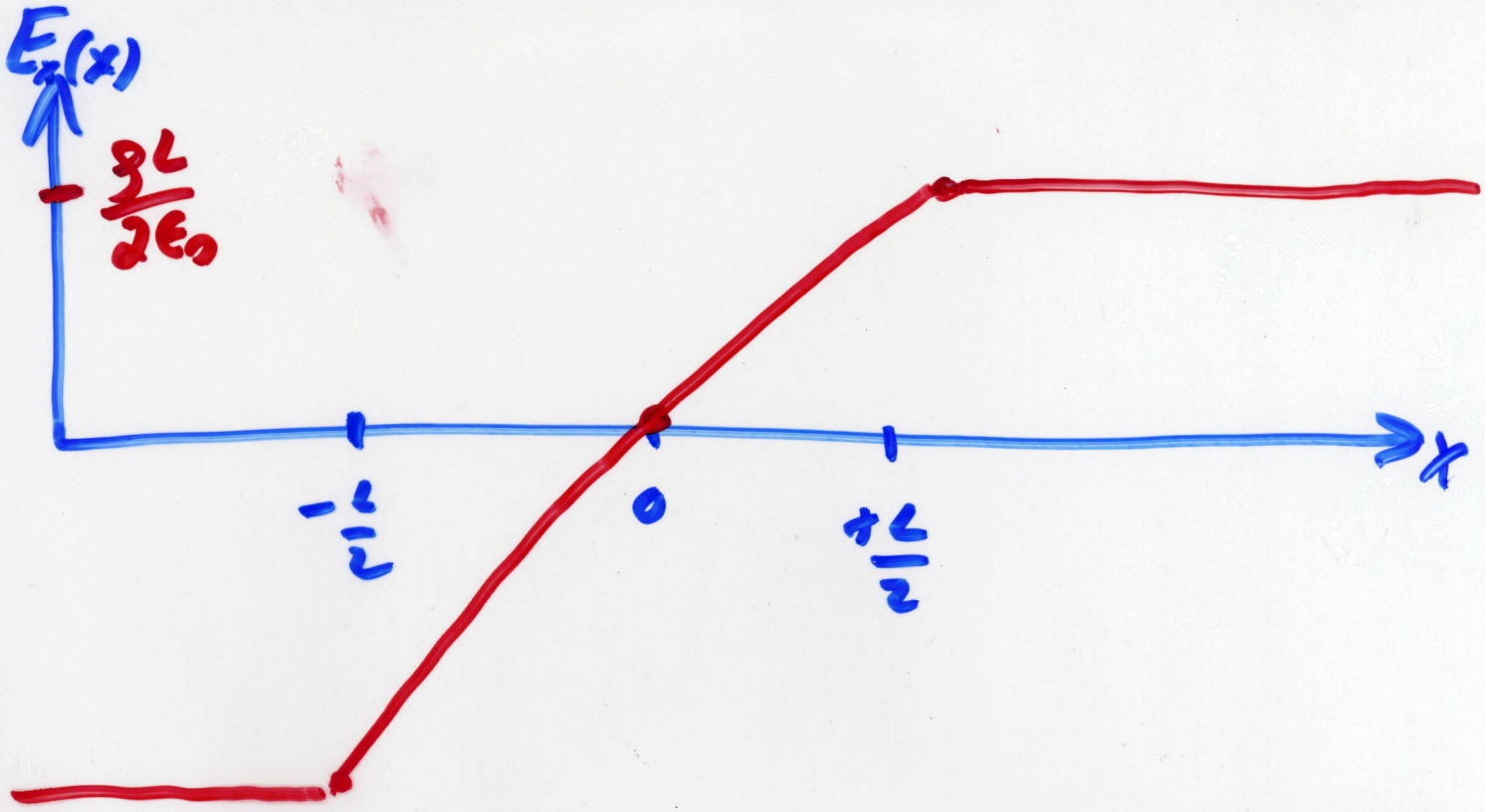
$$2EA = \frac{\rho A L}{\epsilon_0}$$

$$\vec{E} = \frac{\rho L}{2\epsilon_0}$$ away from slab



$$2EA = \frac{\rho A 2x}{\epsilon_0}$$

$$E_{(x)} = \frac{\rho x}{\epsilon_0}$$



$$\vec{E}(\vec{r}) = \iiint k\rho(\vec{r}')\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dV'$$

Divergence

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = \iiint k\rho(\vec{r}')\,\underbrace{\vec{\nabla}\left[\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\right]}_{-4\pi\delta^3(\vec{r}-\vec{r}')}\,dV'$$

$$= \iiint k\,\rho(\vec{r}')(-4\pi)\,\delta^3(\vec{r}-\vec{r}')\,dV'$$

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = -4\pi k\rho(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

differential form of Gauss' law

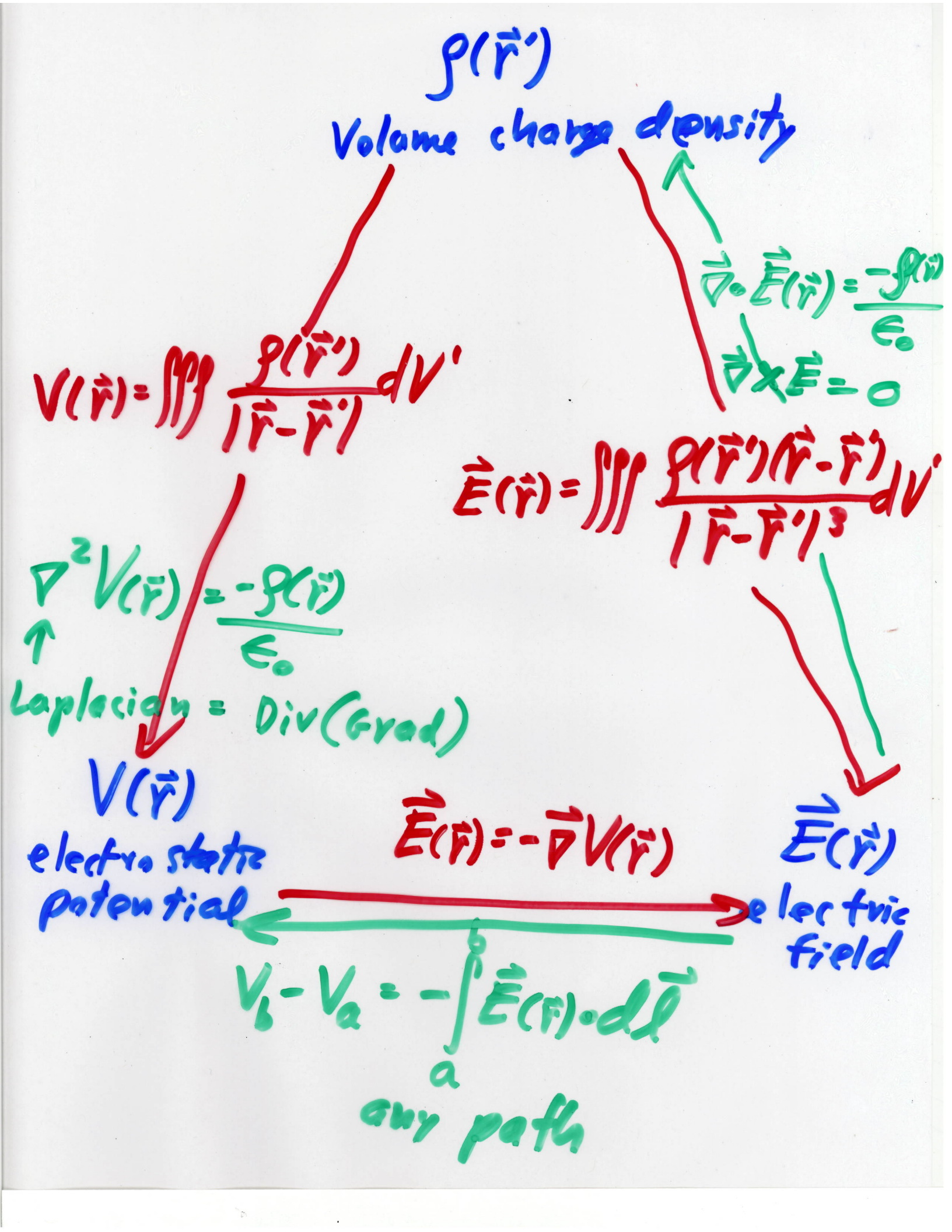
$$\vec{E}(\vec{r}) = \iiint \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,dV'$$

$$\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = -\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}$$

Gradient

$$\vec{E}(\vec{r}) = -\iiint \rho(\vec{r}')\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|}\,dV'$$

$$\vec{E}(\vec{r}) = -\vec{\nabla}\boxed{\iiint \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,dV'} = -\vec{\nabla}\boxed{V(\vec{r})}$$


$\rho(\vec{r}')$
Volume charge density
$V(\vec{r}) = \iiint \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} dV'$
$\vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{-\rho(\vec{r})}{\epsilon_0}$
$\vec{\nabla}\times\vec{E} = 0$
$\vec{E}(\vec{r}) = \iiint \frac{\rho(\vec{r}')(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} dV'$
$\nabla^2 V(\vec{r}) = \frac{-\rho(\vec{r})}{\epsilon_0}$
Laplacian = Div(Grad)
$V(\vec{r})$
electrostatic potential
$\vec{E}(\vec{r}) = -\vec{\nabla}V(\vec{r})$
$\vec{E}(\vec{r})$
electric field
$V_b - V_a = -\int_a^b \vec{E}(\vec{r})\cdot d\vec{l}$
any path

$$\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$ Poisson's Eq. non-homogeneous

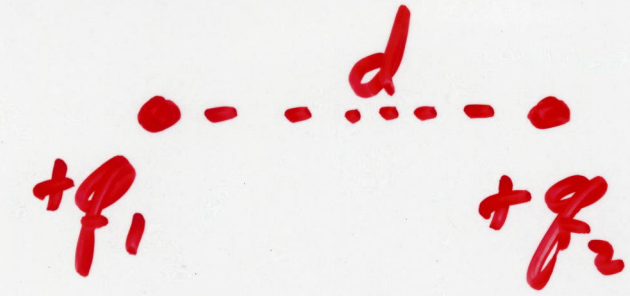
$$\nabla^2 V(\vec{r}) = 0$$ Laplace's Eq. homogeneous

2nd order, linear, partial differential equations

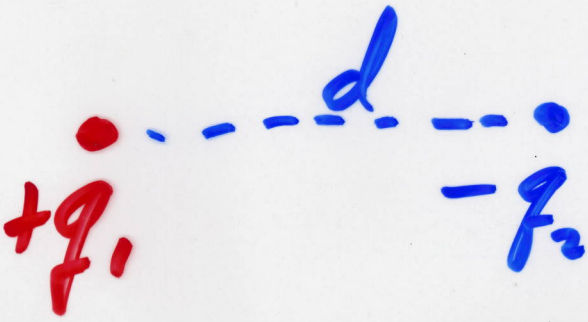
Energy

$$\text{Work I do} = \int \vec{F} \cdot d\vec{l} = \int -\vec{E}_{elec} \cdot d\vec{l}$$

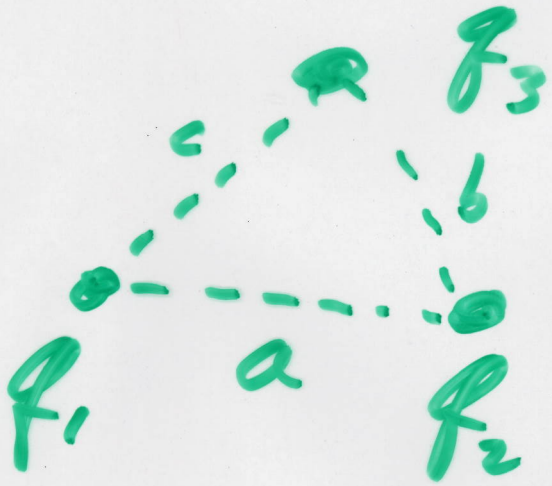
$$= -\int \frac{k q_1 q_2}{r^2} dr = \frac{k q_1 q_2}{r}$$



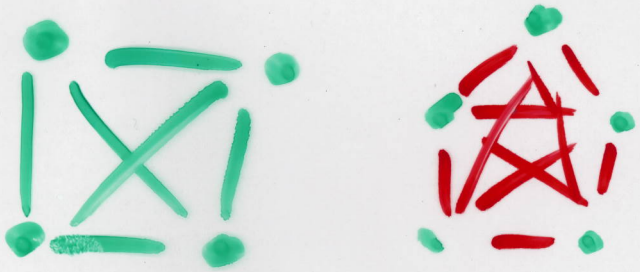
$$U_{12} = \frac{k q_1 q_2}{d}$$



$$U_{12} = -\frac{k q_1 q_2}{d}$$



$$U_{123} = \frac{k q_1 q_2}{a} + \frac{k q_1 q_3}{c} + \frac{k q_2 q_3}{b}$$



# changes	# terms in U
2	1
3	3
4	6
5	10
n	$\frac{n(n-1)}{2}$