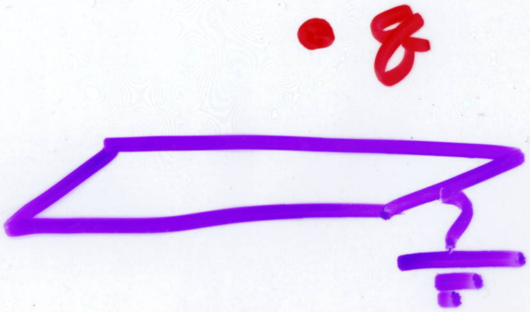
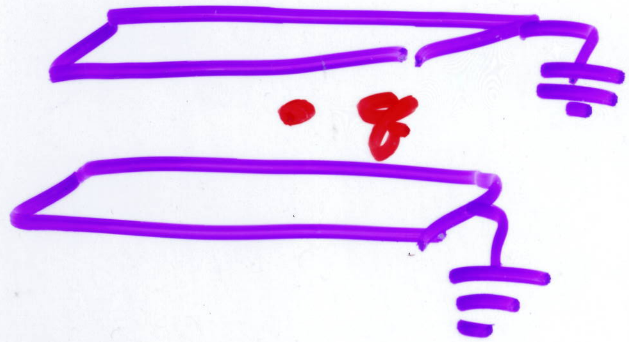
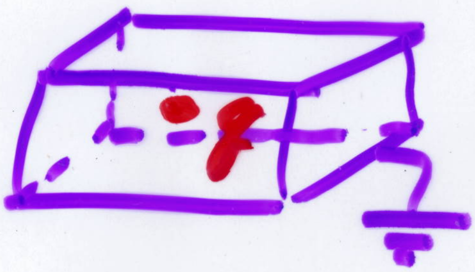
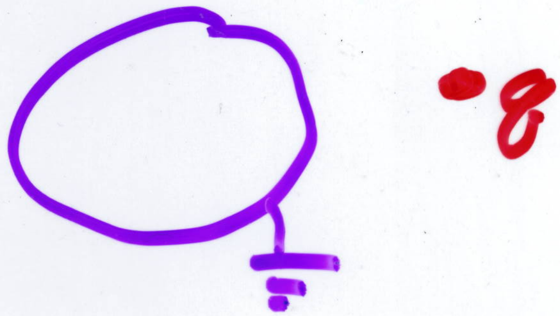
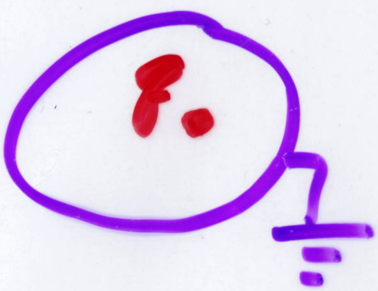
Other Image Geometries









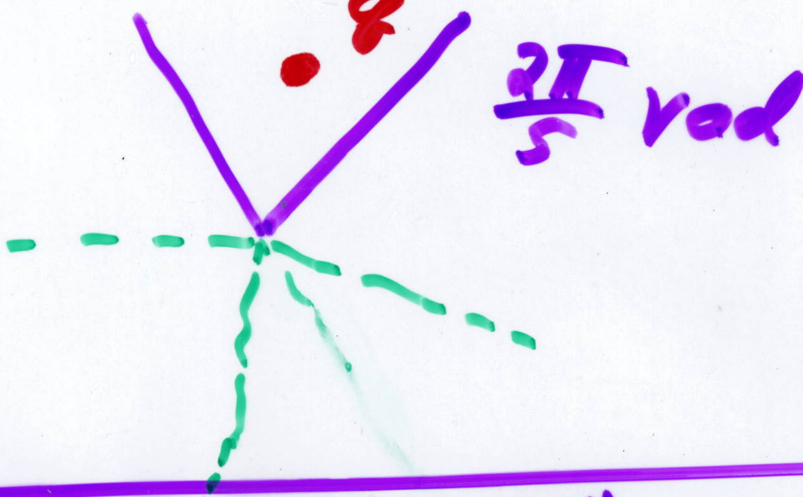


metal

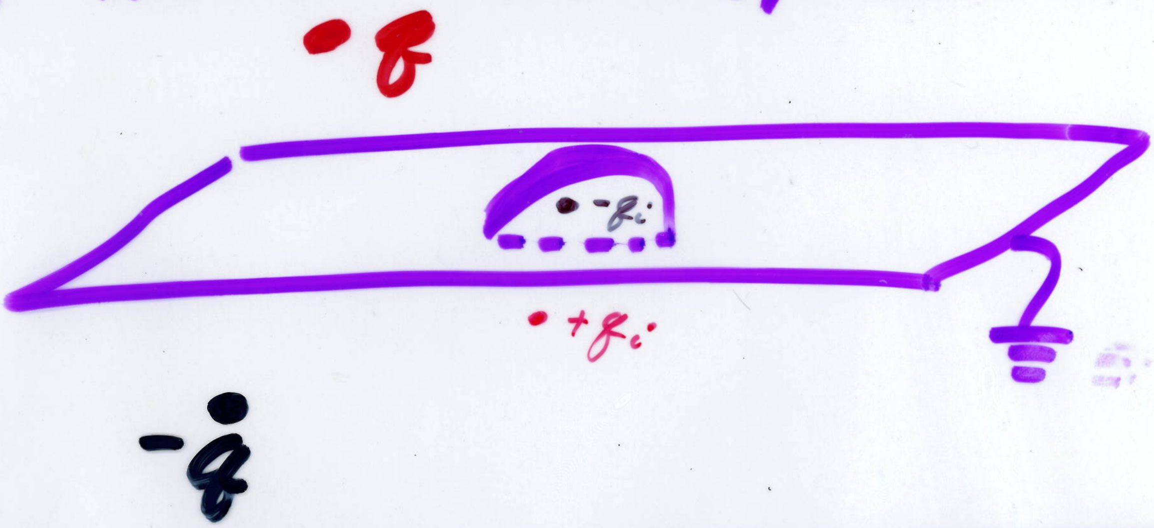
also works for octant

$\frac{\pi}{n}$ rad

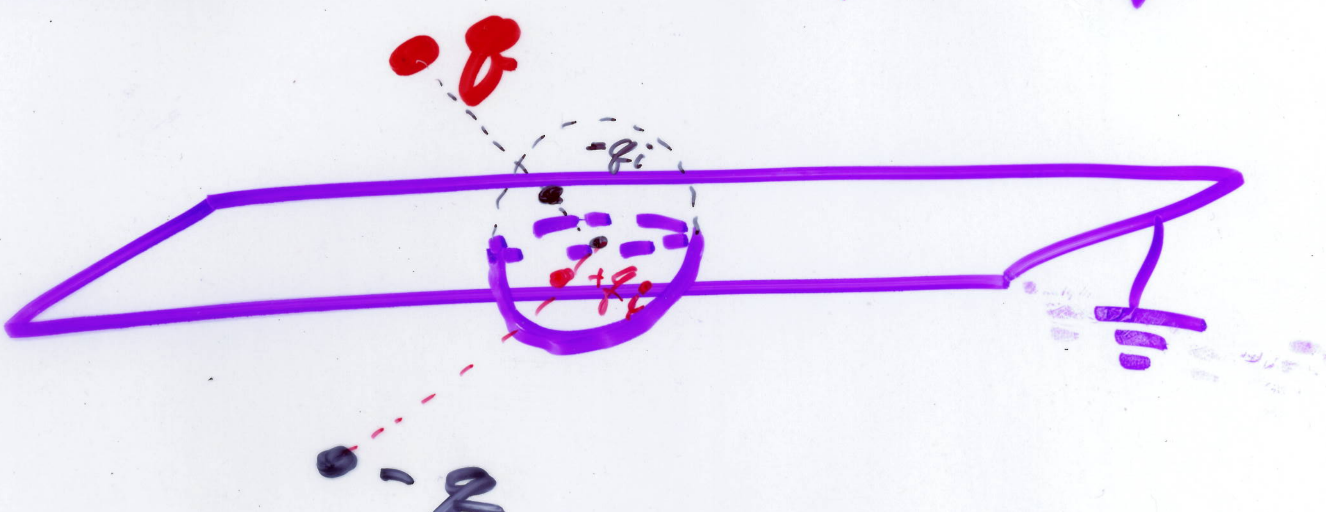
Geometries where Images do NOT work



This works → Hemispherical Boss



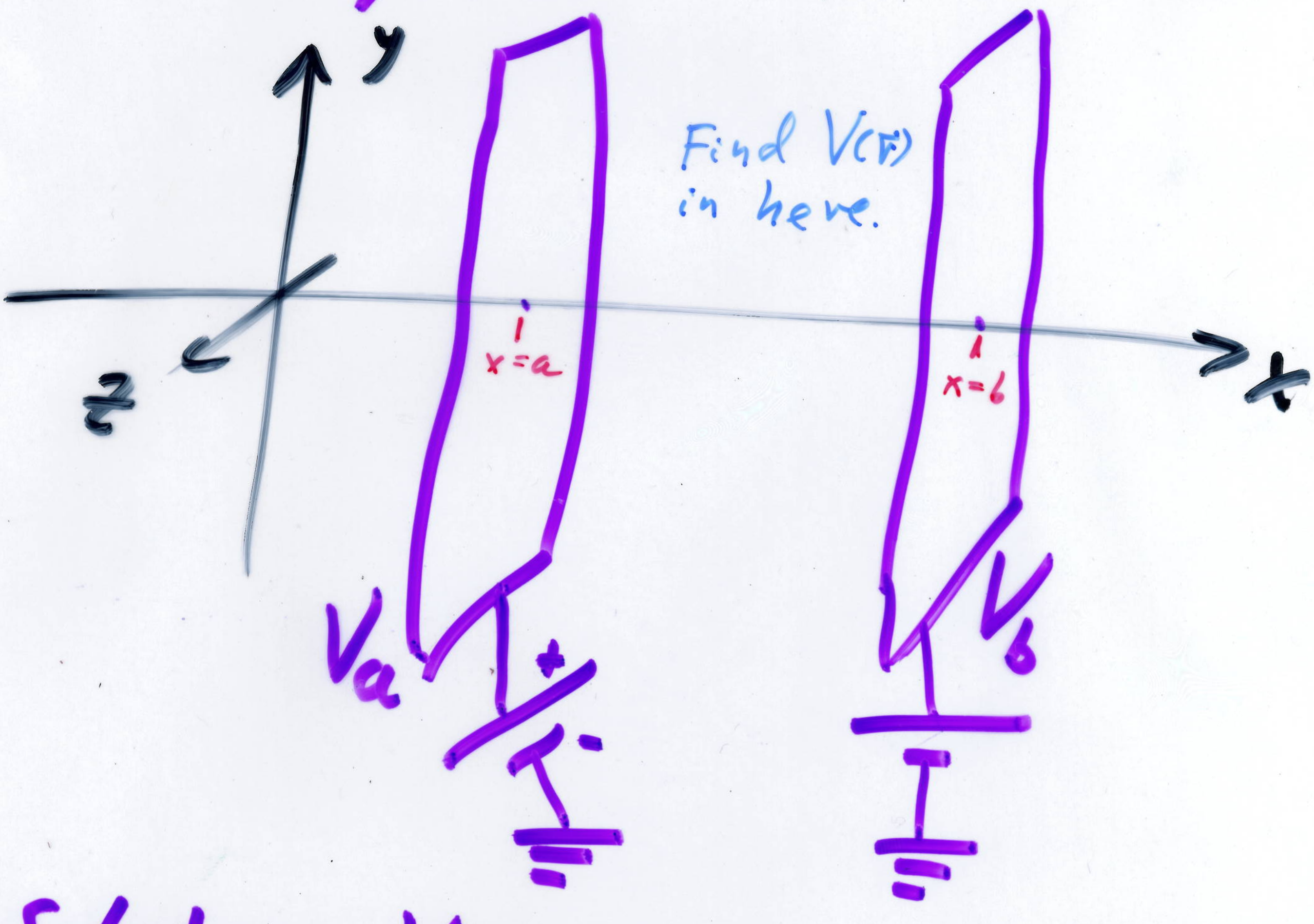
This does not   Hemisph. Dip



Solve Laplace's Eq. $\nabla^2 V(\vec{r}) = 0$ with Separation of Variables:

I. Cartesian Coordinates

Ia) Boundary Conditions depend on only one coordinate, x



Solution $V(\vec{r})$ can only depend on x (not y, not z).

$$V(\vec{r}) = V(x)$$

$$\nabla^2 V(\vec{r}) = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) V(x) = 0$$

$$\frac{d^2}{dx^2} V(x) = 0$$

2nd-order linear ordinary diff. eq.

expect 2 arbitrary constants of integration c_1 and c_2. These will be fixed by the boundary conditions (B.C.)

Solution: $V(x) = c_1 x + c_2$

Solve for c_1 and c_2:

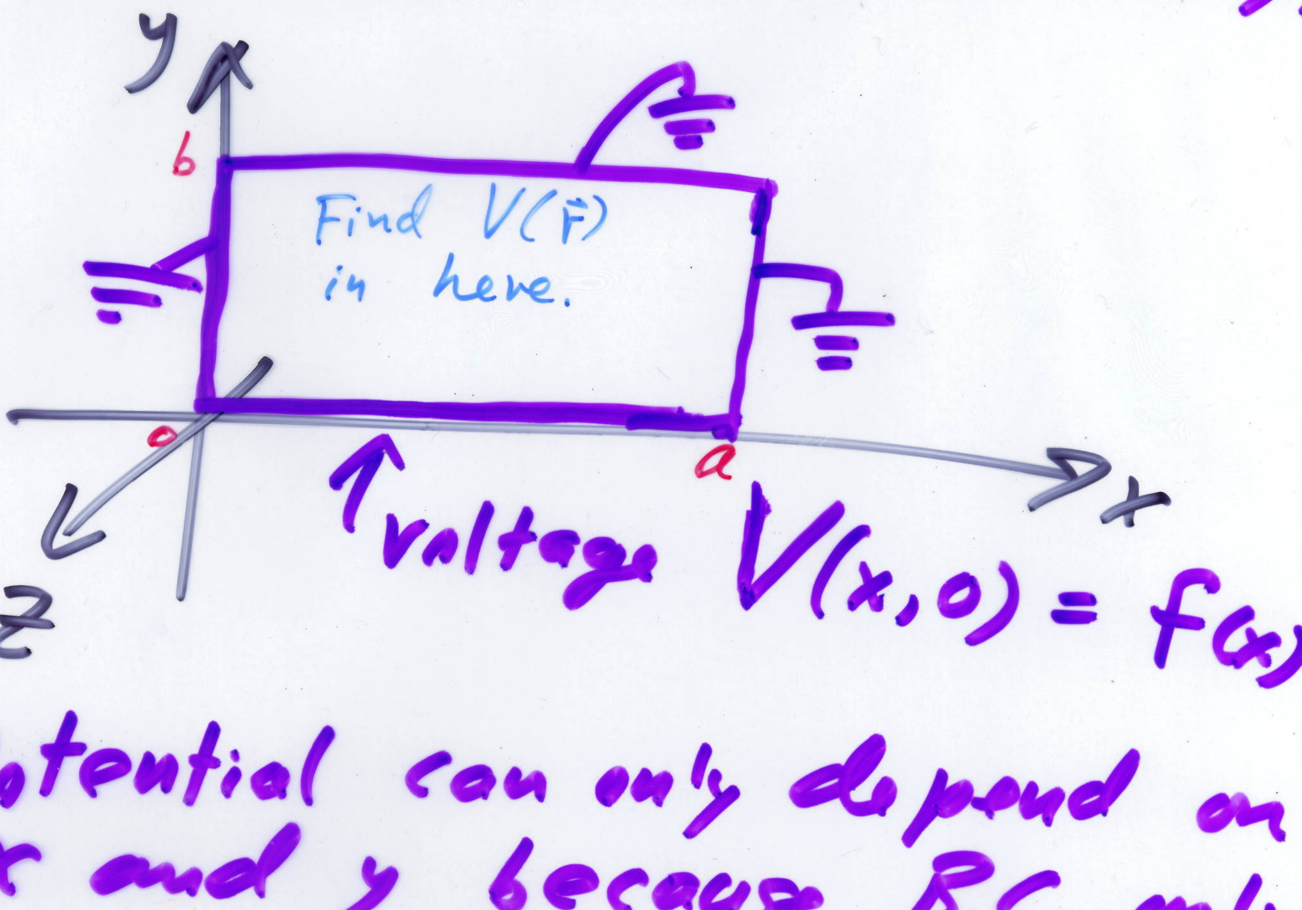
$\underline{x=a}$ $\quad V(a) = c_1 a + c_2 = V_a$

$\underline{x=b}$ $\quad V(b) = c_1 b + c_2 = V_b$

$$c_1 = \frac{V_b - V_a}{b-a} \qquad c_2 = \frac{bV_a - aV_b}{b-a}$$

$$V(x) = \left(\frac{V_b - V_a}{b-a}\right)x + \frac{bV_a - aV_b}{b-a}$$

Ib) Boundary Conditions depend on 2 coordinates (x and y)



Potential can only depend on x and y because B.C. only depend on x and y.

$$V(\vec{r}) = V(x,y)$$

$$\nabla^2 V(\vec{r}) = 0 \Longrightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) V(x,y) = 0$$

Ansatz = Guess: $V(x,y) = C(x)D(y)$

$$\frac{d^2 C(x)}{dx^2} \cdot D(y) + C(x)\frac{d^2 D(y)}{dy^2} = 0$$

Divide both sides by $V(x,y)$

$$\frac{\frac{d^2 C(x)}{dx^2}}{C(x)} + \frac{\frac{d^2 D(y)}{dy^2}}{D(y)} = 0$$

Notation: $$\frac{C''(x)}{C(x)} + \frac{D''(y)}{D(y)} = 0$$

prime (') means derivative with respect to argument.

Separation argument

$$\frac{C''(x)}{C(x)} = \text{constant} = -\alpha^2$$ separation constant

$$\frac{D''(y)}{D(y)} = -\overset{\text{same}}{\text{constant}} = +\alpha^2$$

two 2nd-order linear <u>ordinary</u> D.E.

$$C''(x) + \alpha^2 C(x) = 0$$

$$C(x) = k_1 \sin(\alpha x) + k_2 \cos(\alpha x)$$

or $k_1 e^{+i\alpha x} + k_2 e^{-i\alpha x}$ or $k_1 \sin(\alpha x + k_2)$

or $k_1 \cos(\alpha x + k_2)$

$$D''(y) - \alpha^2 D(y) = 0$$

$$D(y) = k_3 \sinh(\alpha y) + k_4 \cosh(\alpha y)$$

or $k_3 e^{+\alpha y} + k_4 e^{-\alpha y}$

$$V(x,y) = C(x) \cdot D(y)$$

$$= [k_1 \sin(\alpha x) + k_2 \cos(\alpha x)][k_3 \sinh(\alpha y) + k_4 \cdots$$