

# Point Dipole

Limit  $q \rightarrow \infty$ ,  $d \rightarrow 0$

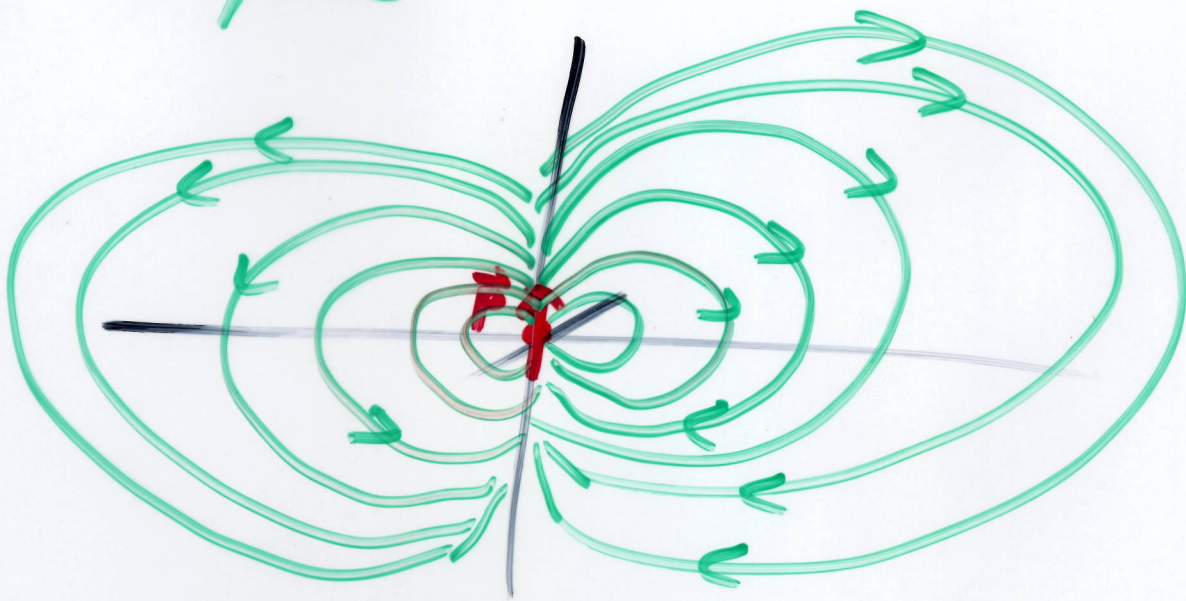
$$qd = \text{constant} = p$$

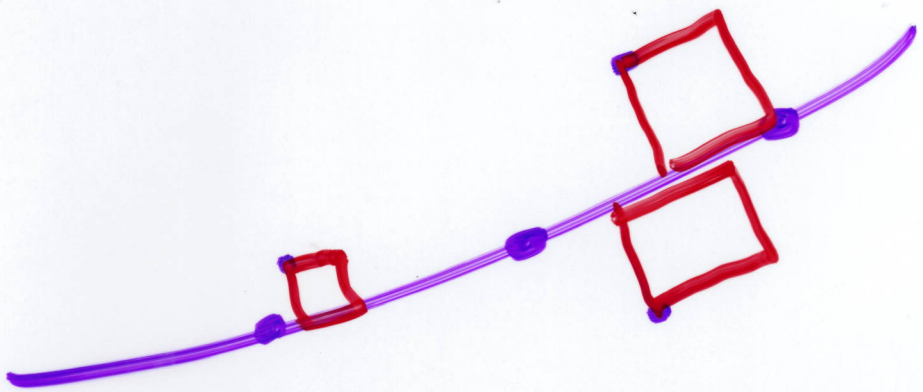
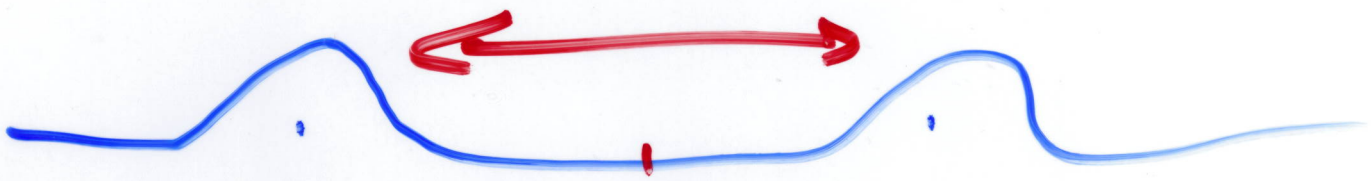
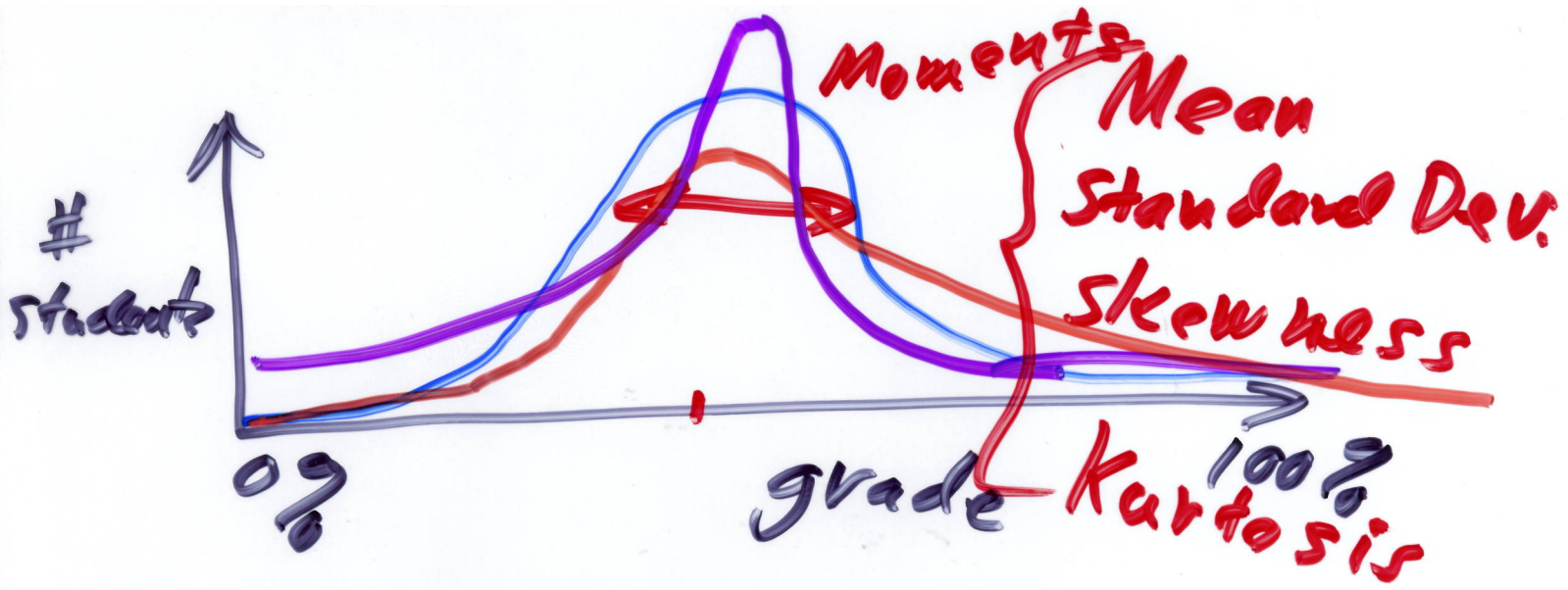
$$V(r) = \frac{k p \cos \theta}{r^2}$$

All higher-order terms go to zero

$$V(\vec{r}) = \frac{k p \cos \theta}{r^2} = \frac{k \vec{p} \cdot \hat{r}}{r^3}$$

Electric field from a point dipole





# Mechanics



Described by volume  
mass density  
 $\rho(\vec{r})$

Total Mass  $M = \iiint \rho(\vec{r}) dV$   
~ Total Charge (scalar)

Center of Mass  $\vec{R}_{\text{cm}} = \frac{\iiint \vec{r} \rho(\vec{r}) dV}{M}$   
Vector ~ Dipole Moment Vector

Moment of inertia ~ Quadrupole  
Moment tensor

$$I = \iiint r^2 \rho(\vec{r}) dV$$



distance from the  
axis of rotation

Not really a scalar  
Rank-2 tensor  $I_{ij} = I_{xx} / I_{yy} / I_{zz}$

Calculating the vector components  
of  $\vec{P}$ :

---

$$P_1 = P_x = \iiint x \rho(\vec{r}) dV$$

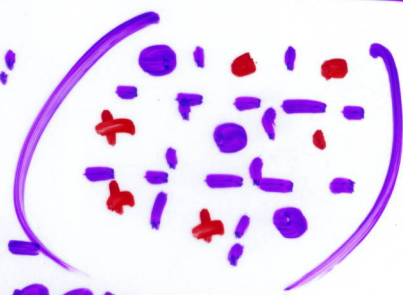
$$P_2 = P_y = \iiint y \rho(\vec{r}) dV$$

$$P_3 = P_z = \iiint z \rho(\vec{r}) dV$$

---

Quadrupole Moments:

Symmetric  $\rightarrow$  6 independent components



$$Q_{11} = Q_{xx} = \iiint x^2 \rho(\vec{r}) dV$$

$$Q_{12} = Q_{xy} = \iiint xy \rho(\vec{r}) dV$$

$\vdots$

$$Q_{33} = Q_{zz} = \iiint z^2 \rho(\vec{r}) dV$$