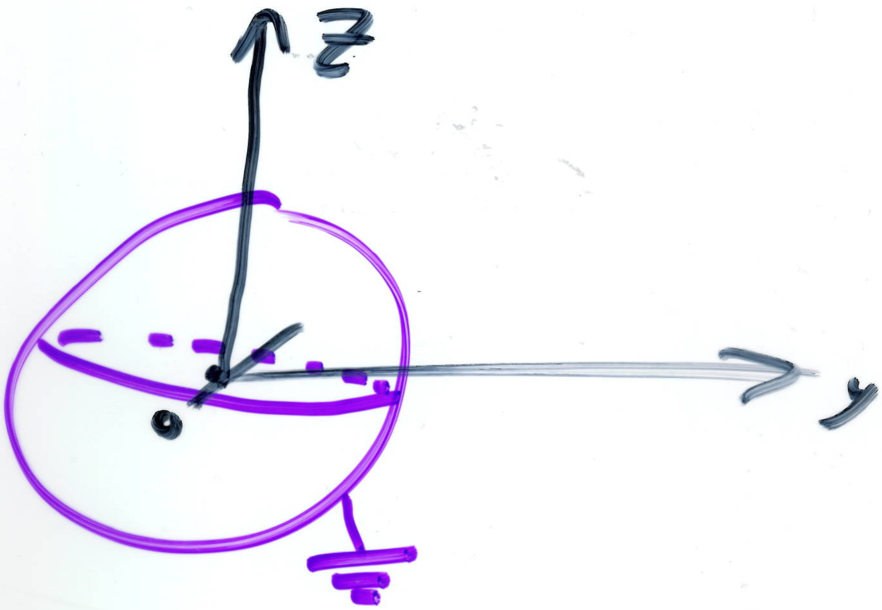
Grounded conducting sphere in a uniform electric field. Find $V(\vec{r})$ everywhere.




Problems with azimuthal symmetry (no $\varphi$ dependence)

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$$

Because the sphere is grounded

$$V(R,\theta) = 0 = \sum_{l=0}^{\infty}\left(A_l R^l + \frac{B_l}{R^{l+1}}\right) P_l(\cos\theta)$$

Boundary Condition at $r=R$.

$$A_\ell R^\ell + \frac{B_\ell}{R^{\ell+1}} = 0 \qquad \forall \ell$$

$$B_\ell = -A_\ell R^{2\ell+1} \qquad \forall \ell$$

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$$V(r,\theta) = \sum_{\ell=0}^{\infty} A_\ell \left[ r^\ell - \frac{R^{2\ell+1}}{r^{\ell+1}} \right] P_\ell(\cos\theta)$$

B.C. at $z = \pm\infty$

$$V\begin{pmatrix}\text{far from} \\ z=0\end{pmatrix} = -E_0 z = -E_0 r\cos\theta$$

$$\boxed{-\vec{\nabla} V = E_0 \hat{k}} \qquad = -E_0 r^1 P_1(\cos\theta)$$

$$\Rightarrow A_1 = -E_0$$

all other $A_\ell$'s $= 0$

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$$V(r,\theta) = A_1 \left[ r^1 - \frac{R^3}{r^2} \right] P_1(\cos\theta)$$

$$= -E_0 \left[ r - \frac{R^3}{r^2} \right] \cos\theta \qquad \text{outside } r > R$$

Inside $r<R$ : $V(r,\theta) = 0$

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Find the surface charge density on the sphere.

Find electric field

$$\vec{E}_{(\vec{r})} = -\vec{\nabla} V(\vec{r})$$

multiply by $\epsilon_0$, dot product with outward normal vector.

$$-\epsilon_0 \frac{\partial V(r,\theta)}{\partial r}\bigg|_{r=R} = \sigma(R,\theta)$$

$$= -\epsilon_0 \frac{\partial}{\partial r}\left(-E_0\left[r - \frac{R^3}{r^2}\right]\cos\theta\right)\bigg|_{r=R}$$

$$= \epsilon_0 E_0 \left[1 + 2\frac{R^3}{r^3}\right]\cos\theta\bigg|_{r=R}$$

$$= \epsilon_0 E_0\, 3 \cos\theta$$

Another problem: Insulating spherical shell with surface charge density $\sigma(R,\theta) = 3\epsilon_0 E_0 \cos\theta$

Find potential everywhere

outside: $r > R$ $\quad V(r,\theta) = E_0 \frac{R^3}{r^2} \cos\theta$ pure dipole potential

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inside:

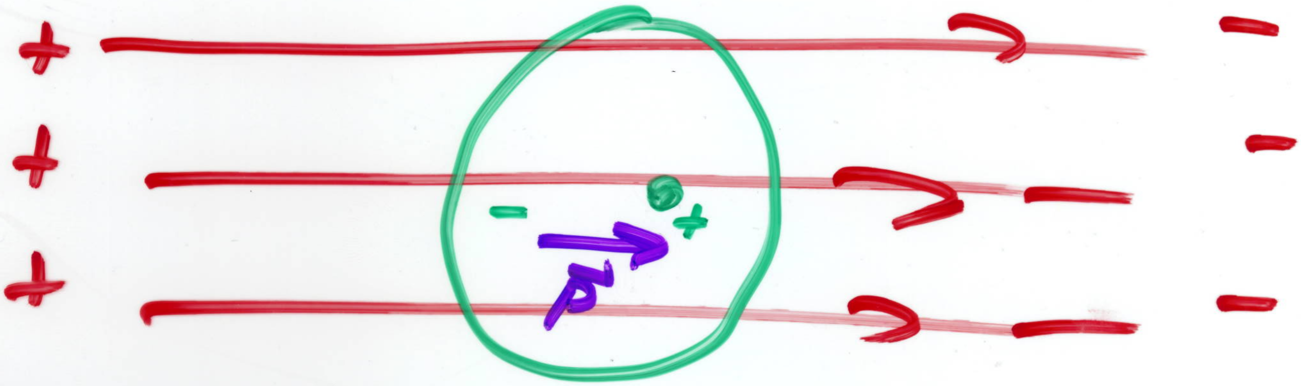
$r < R$ $\quad V(r,\theta) = E_0 r \cos\theta = E_0 z$

electric field

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r}) = -E_0 \hat{z}$$

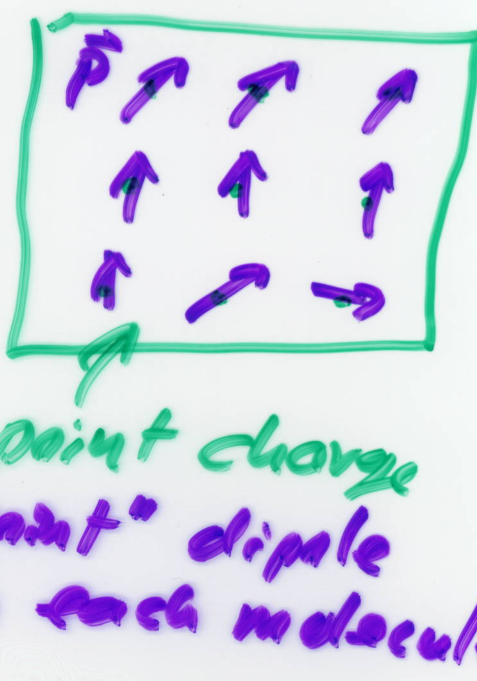
no $\vec{E}$ field

with $\vec{E}$ field

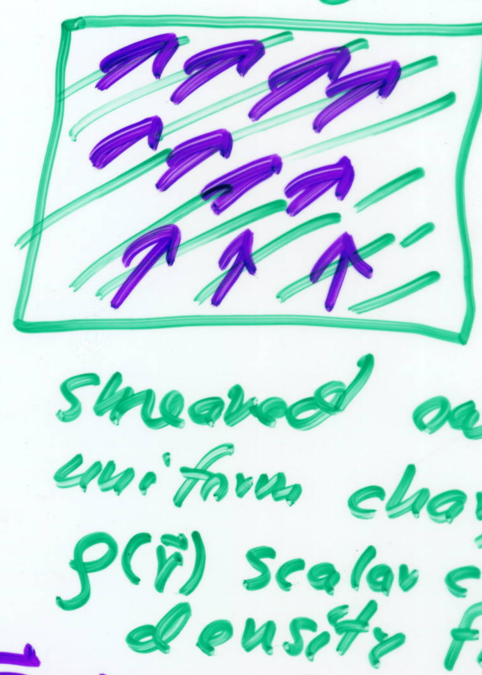



$\vec{p}$ induced dipole moment.

$\vec{p} = \alpha \vec{E}$




gellium



point charge

"point" dipole on each molecule

smeared out uniform charge

$\rho(\vec{r})$ scalar charge density field.

$\vec{P}(\vec{r})$ vector polarization field

$\rho(\vec{r})$ is the monopole (charge) per unit volume.

$\vec{P}(\vec{r})$ is the dipole per unit volume.

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$$P_i = \sum_{j=1}^{3} \alpha_{ij} E_j$$

Mechanics

angular momentum

moment of inertia

angular velocity

$$\vec{L} = I \vec{\omega}$$

$$L_i = \sum_{j=1}^{3} I_{ij} \omega_j$$

moment of inertia tensor