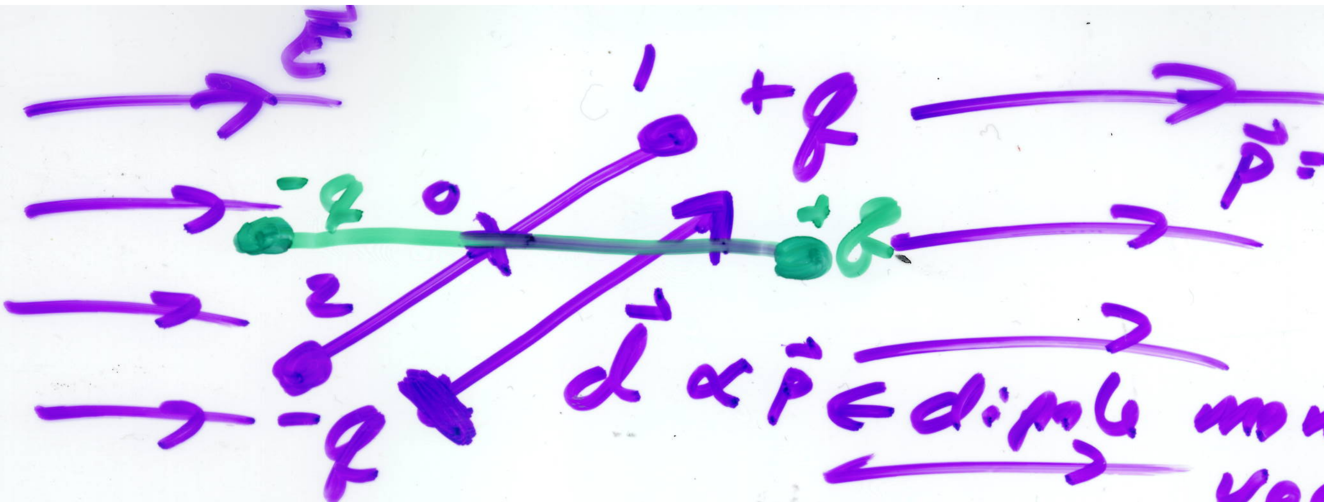


$\vec{p} = q\vec{d}$

$\vec{d} \propto \vec{p} \Leftarrow$ dipole moment vector

Torque $\vec{\tau} = \sum_{i=1}^{2} \vec{r}_i \times \vec{F}_i$

$$\vec{\tau} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$= \frac{\vec{d}}{2} \times q\vec{E} + -\frac{\vec{d}}{2} \times (-q)\vec{E}$$

$$= \vec{d} \times q\vec{E} = \boxed{\vec{p} \times \vec{E}}$$

torque on dipole

Δ is the difference between one side of the dipole and the other.

$$\Delta E_x = \cancel{\vec{E}_0 \cdot 0} + \vec{d} \cdot \vec{\nabla} E_x + \dots$$

$$\Delta E_y = 0 + \vec{d} \cdot \vec{\nabla} E_y$$

$$\Delta E_z = 0 + \vec{d} \cdot \vec{\nabla} E_z$$

$$\Delta \vec{E} = (\vec{d} \cdot \vec{\nabla}) \vec{E}$$

$$\vec{F} = q (\vec{d} \cdot \vec{\nabla}) \vec{E} = (\vec{p} \cdot \vec{\nabla}) \vec{E}$$

$$F_x = \left(p_x \frac{\partial}{\partial x} + p_y \frac{\partial}{\partial y} + p_z \frac{\partial}{\partial z} \right) E_x$$

$$F_y = \text{- - - - - -} \; E_y$$

$$F_z = \text{- - - - - -} \; E_z$$

Potential Energy of an Electric Dipole in an electric field.

$$\Delta U = U_f - U_i = U_2 - U_1 = \int_1^2 \vec{F} \cdot d\vec{s}$$

$$= \int_1^2 \tau \, d\theta = \int_1^2 |\vec{p} \times \vec{E}| \, d\theta$$

$$= \int_1^2 pE \sin\theta \, d\theta = -pE\cos\theta \Big|_{\theta_1}^{\theta_2}$$

$$U_2 = -pE\cos\theta_2$$

$$U_1 = -pE\cos\theta_1$$

$$U = -pE\cos\theta = -\vec{p} \cdot \vec{E}$$

Electric Field of a dipole

$$V(\vec{r}) = \frac{kp\cos\theta}{r^2}$$

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$

$$E_r = -\frac{\partial}{\partial r} V(\vec{r}) = \frac{+2kp\cos\theta}{r^3}$$

radial component of $\vec{E}$ field

$$E_\theta = -\frac{1}{r}\frac{\partial}{\partial \theta} V(\vec{r}) = \frac{kp\sin\theta}{r^2 \cdot r}$$

polar component of $\vec{E}$ field

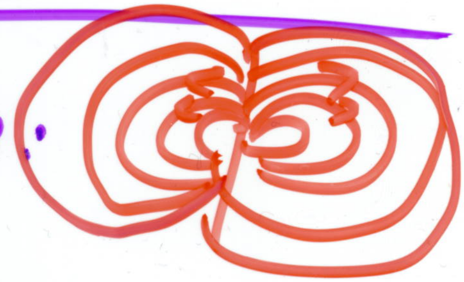
$$E_\varphi = -\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} V(\vec{r}) = 0$$

azimuthal comp. of $\vec{E}$ field

$$\vec{E}(\vec{r}) = E_r\,\hat{r} + E_\theta\,\hat{\theta} + \underline{E_\varphi}\,\hat{\varphi}$$

$$\vec{E}(\vec{r}) = \frac{kp}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

Coordinate-free form.



$$\vec{E}(\vec{r}) = \frac{k}{r^3}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^2} - \vec{p}\right]$$

$\vec{p}$ dipole moment vector

$\vec{r}$ = field point $\quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = r\,\hat{r} + 0\hat{\theta} + 0\hat{\phi}$

$r = |\vec{r}|$

$\vec{p}\cdot\vec{r} = p\,r\cos\theta$

$\vec{p} = p_r\,\hat{r} + p_\theta\,\hat{\theta}$

$p_r = \vec{p}\cdot\hat{r} = p\cos\theta$

$p_\theta = \vec{p}\cdot\hat{\theta} = -p\sin\theta$

$$\vec{E}(\vec{r}) = \frac{kp}{r^3}\left[\frac{3\cancel{r^2}\cos\theta\,\hat{r}}{\cancel{r^2}} - \cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$