Empirically, many materials are linear dielectrics.

$$\vec{P}(\vec{r}) = \chi_e \epsilon_0 \vec{E}(\vec{r}) + \text{higher order}$$

χ_e = electric susceptibility

$$\vec{D} = (\epsilon_0 E + \vec{P}) = (\epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E})$$

$$= \underbrace{\boxed{(1+\chi_e)\epsilon_0}}_{\epsilon} \vec{E} = \epsilon \vec{E}$$

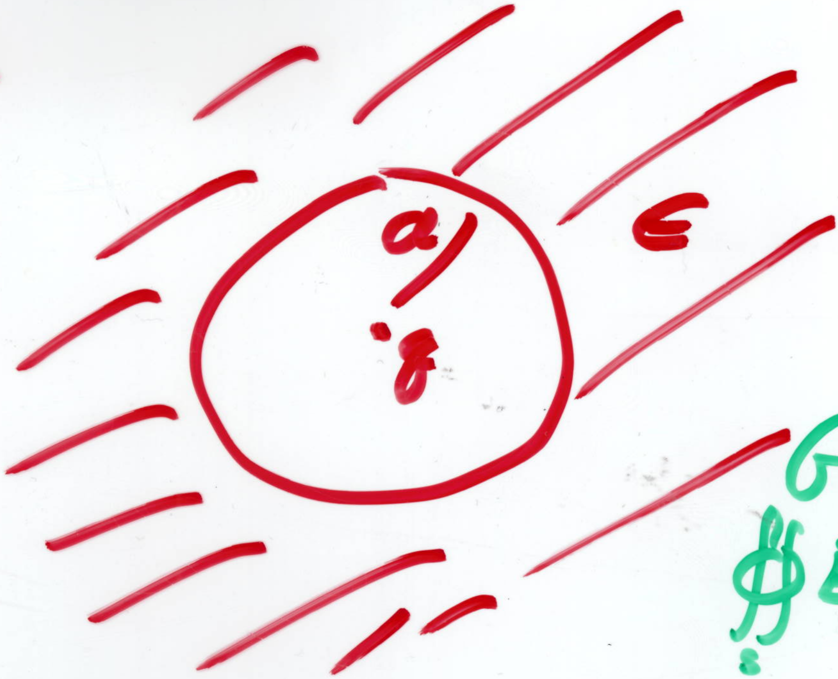
ϵ = permittivity.

$\frac{\epsilon}{\epsilon_0} = \epsilon_r$ ← relative permittivity

$= (1+\chi_e)$ a.k.a. dielectric constant

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$

e.g.



Find $\vec{E}(\vec{r})$ everywhere

Gauss'

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\vec{\nabla}\cdot\vec{D}(\vec{r}) = \rho_{free} = q\,\delta(x)\,\delta(y)\,\delta(z)$$

$$\oint_S \vec{D}\cdot d\vec{a} = Q_{\substack{free\\ enclosed}}$$

$$D(r)\,4\pi r^2 = q_{free}$$

$$\vec{D} = \epsilon\vec{E}$$

$$\vec{D}(r) = \frac{q\,\hat{r}}{4\pi r^2}$$

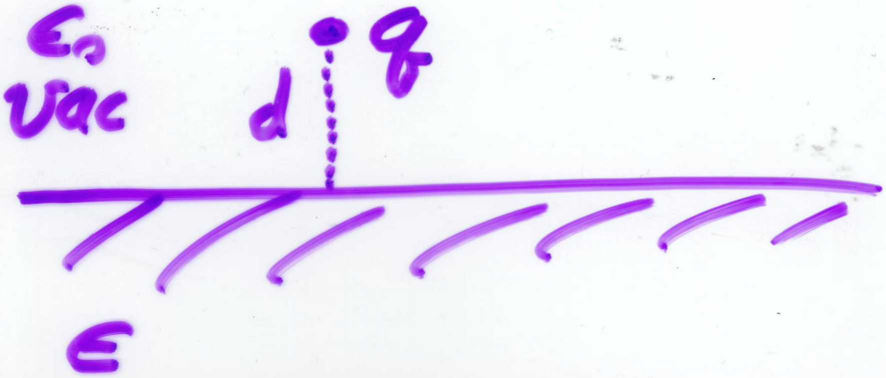
$$\epsilon = \begin{cases} \epsilon_0, & r<a \\ \epsilon, & r>a \end{cases}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon}$$

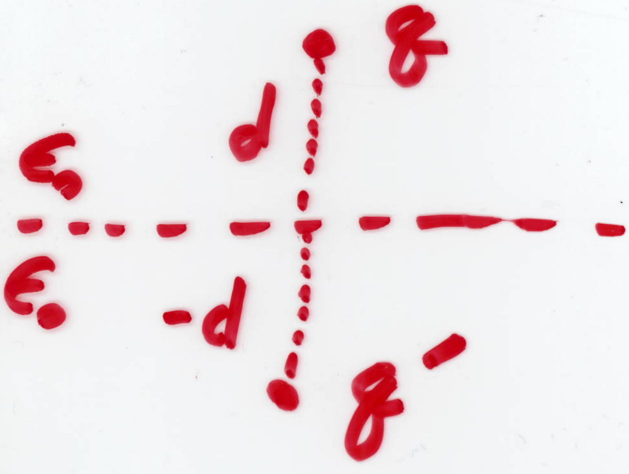
$r<a$: $\vec{E}(\vec{r}) = \frac{\vec{D}}{\epsilon_0} = \frac{q\,\hat{r}}{4\pi\epsilon_0 r^2} = \frac{kq\vec{r}}{r^3}$

$r>a$: $\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{q\,\hat{r}}{4\pi\epsilon\,r^2} = \frac{kq\vec{r}}{\epsilon_r r^3}$

Method of Images for dielectrics.



upper $\frac{1}{2}$ space



lower $\frac{1}{2}$ space

$$\vec{\nabla}\cdot\vec{D} = q_{free} \Longrightarrow \hat{n}\cdot\vec{D}_1 = \hat{n}\cdot\vec{D}_2$$

$$D_{1z} = D_{2z}$$

$$\vec{\nabla}\times E = 0 \Longrightarrow \hat{n}\times\vec{E}_1 = \hat{n}\times\vec{E}_2$$

$$E_{1y} = E_{2y}$$

$$E_{1x} = E_{2x}$$