<u>From Experiment:</u>

The force due to current i_1 on i_2 in a wire of length ℓ is

$$\vec{F}_{\text{of 1 on 2}} = \left(\frac{\mu_0}{4\pi}\right) \frac{2\ell\, i_1 i_2 \cos\theta}{r_{12}}\, \hat{r}_{21}$$

θ is the angle between i_1 and i_2.

$\hat{r}_{21}$ is a unit vector from i_2 to i_1.

$\theta = 0$
$\cos\theta = 1$

$\theta = 180°$
$\cos\theta = -1$

<u>attractive</u> if i_1 and i_2 are <u>parallel</u>.

<u>repulsive</u> if i_1 and i_2 are <u>antiparallel</u>.

<u>zero</u> if i_1 and i_2 are <u>perpendicular</u>.

The constant

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

is called the <u>permeability of free space</u>.

and "T" stands for "tesla", the MKS unit of magnetic field $\vec{B}$.

$$1\,\mathrm{T} = 1\,\frac{\mathrm{N}}{\mathrm{A \cdot m}}$$

The electric field unit does not have a special name. The MKS unit of $\vec{E}$ is

$$\frac{\mathrm{V}}{\mathrm{m}} = \frac{\mathrm{N}}{\mathrm{C}}$$

$$\boxed{1\,\mathrm{V} = \frac{1\,\mathrm{J}}{\mathrm{C}}}$$

From the last chapter, the force on a charge $\underline{q}$ moving with velocity $\underline{\vec{v}}$ in a magnetic field $\underline{\vec{B}}$ is:

$$\vec{F}_m = q\, \vec{v} \times \vec{B}$$

Can we reconcile this with:

$$\vec{F}_{\text{of 1 on 2}} = \frac{\mu_0}{4\pi} \frac{2\ell\, i_1 i_2 \cos\theta}{r_2}\, \hat{r}_{21} \quad ?$$

- if current i_2 flows for time T, then charge $q_2 = i_2 T$ has passed by.
- if the charge q_2 moves with speed v, then it flows a distance $\ell = vT$.

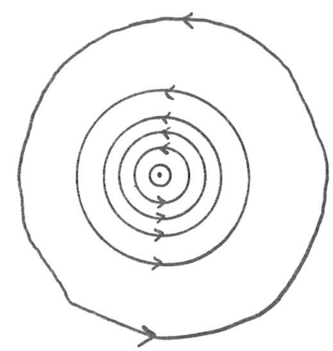
$$\vec{F}_{\text{of 1 on 2}} = \frac{\mu_0}{4\pi} \frac{2(v\cancel{T})\, i_1 \left(\frac{q_2}{\cancel{T}}\right) \cos\theta}{r_2}\, \hat{r}_{21}$$

So the magnetic field due to current i_1 in a <u>straight</u> wire is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2 i_1}{r} \qquad \text{(magnitude)}$$

How about <u>direction</u>?

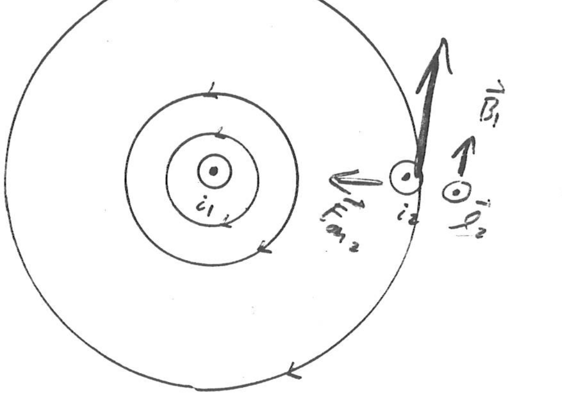
To recover the experimental laws of attraction and repulsion for parallel and antiparallel currents, the $\vec{B}$ field must look like:



- more dense (stronger $\vec{B}$ field) close to the wire
- right hand rule
- lines of $\vec{B}$ never end (no magnetic charges - monopoles)

$\vec{F}_{\text{of 1 on 2}} = i_2 \vec{\ell}_2 \times \vec{B}_1$ $\qquad$ $\vec{F}_{\text{of 1 on 2}} = q_2 \vec{E}_1$

$\vec{F}_{\text{of 2 on 1}} = i_1 \vec{\ell}_1 \times \vec{B}_2$ $\qquad$ $\vec{F}_{\text{of 2 on 1}} = q_1 \vec{E}_2$

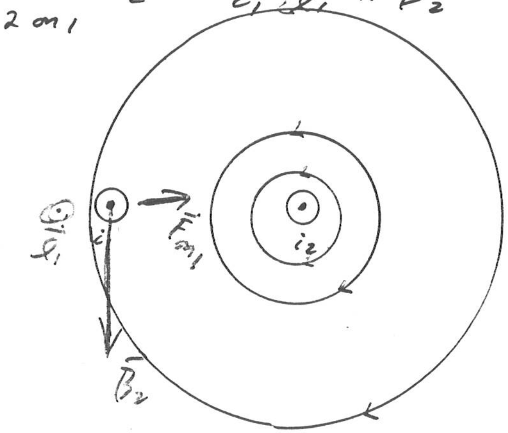


$\vec{\ell}_2$ points along i_2

$\odot$ i_1 $\qquad$ $\otimes$ i_2

$\vec{F}_{\text{of 1 on 2}} = i_2 \vec{\ell}_2 \times \vec{B}_1$ $\qquad$ $\vec{F}_{\text{of 1 on 2}} = q_2 \vec{E}_1$

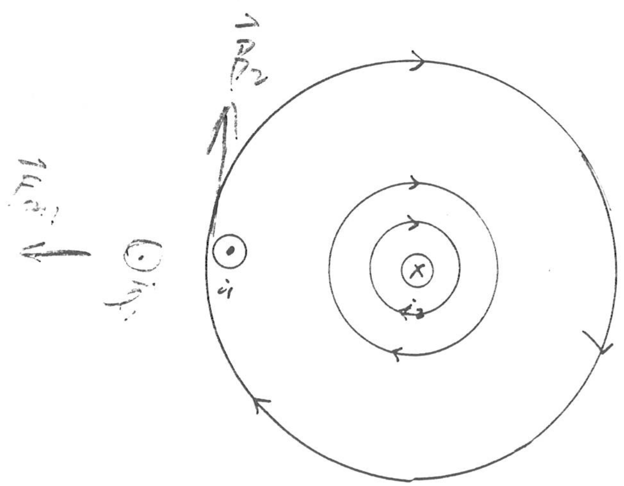
$\vec{F}_{\text{of 2 on 1}} = i_1 \vec{\ell}_1 \times \vec{B}_2$ $\qquad$ $\vec{F}_{\text{of 2 on 1}} = q_1 \vec{E}_2$



$\odot$ i_1 $\qquad$ $\otimes$ i_2

$$\vec{F}_{\text{of 1 on 2}} = i_2\, \vec{\ell}_2 \times \vec{B}_1 \qquad \vec{F}_{\text{of 1 on 2}} = q_2 \vec{E}_1$$

$$\vec{F}_{\text{of 2 on 1}} = i_1\, \vec{\ell}_1 \times \vec{B}_2 \qquad \vec{F}_{\text{of 2 on 1}} = q_1 \vec{E}_2$$



What if the wire is <u>not</u> straight?

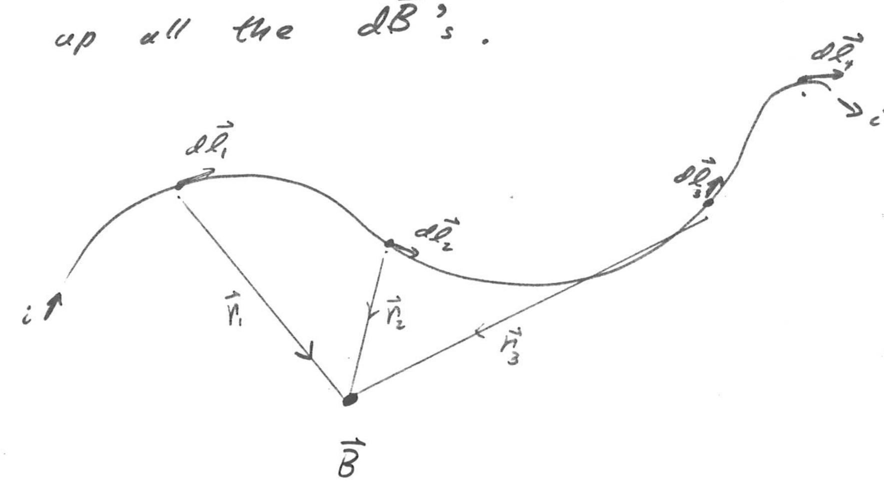
Then the piece of the magnetic field due to an infinitesimal chunk of wire (of length $d\ell$) is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell} \times \vec{r}}{r^3}$$

$\vec{r}$ points <u>from</u> the current element <u>to</u> the field point.

(Biot-Savart Law)

To get the total $\vec{B}$ field, simply integrate along the wire and add up all the $d\vec{B}$'s.



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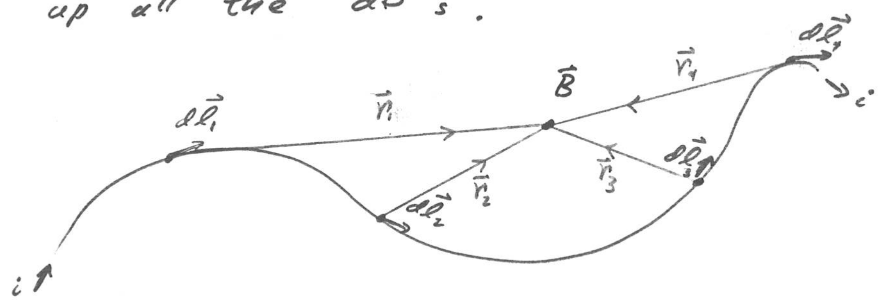
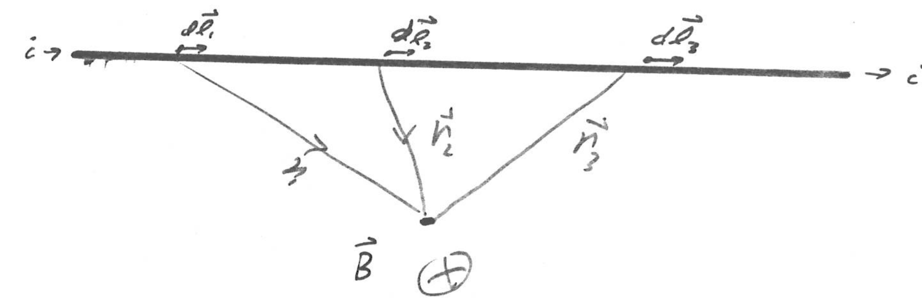


Does this work for a straight wire?

$\vec{B}$ $\odot$

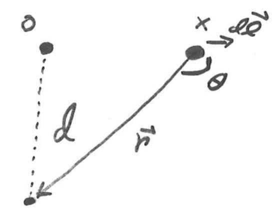


$$\vec{B} = \int_{x=-\infty}^{+\infty} d\vec{B} = \int_{-\infty}^{+\infty} \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell} \times \vec{r}}{r^3} = \sum_{i=1}^{\infty} \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell}_i \times \vec{r}_i}{(r_i)^3}$$

$$B = \frac{\mu_0}{4\pi} \frac{2i}{r}$$

See HRW pages 850-1 for a proof.

$$d\vec{\ell} \times \vec{r} = |d\vec{\ell}|\,|\vec{r}| \sin\theta$$
$$= dx\, r \sin\theta$$



$$|d\vec{\ell}| = dx$$
$$r = \sqrt{d^2+x^2}$$

$$\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell} \times \vec{r}}{r^3}$$

$$|\vec{B}| = \frac{\mu_0 i}{4\pi} \int \frac{dx\, \cancel{r} \sin\theta}{r^{\cancel{3}2}} = \frac{\mu_0 i}{4\pi} \int_{-\infty}^{+\infty} \frac{dx}{d^2+x^2} \frac{d}{r}$$

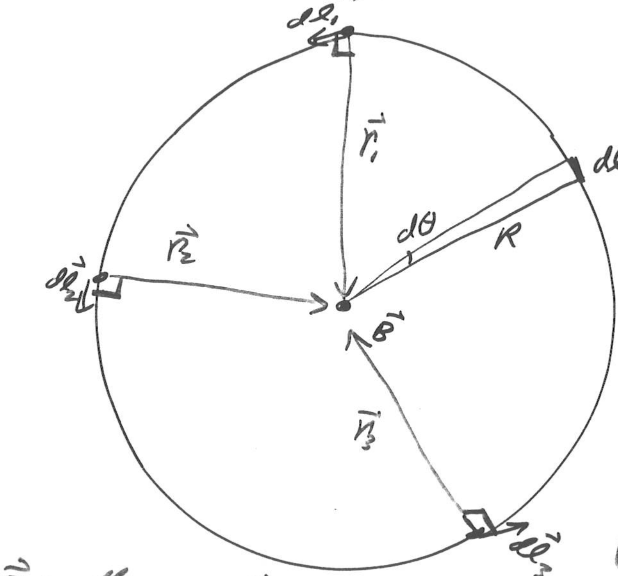
$$= \frac{d\mu_0 i}{4\pi} \int_{-\infty}^{\infty} \frac{dx}{(d^2+x^2)^{3/2}} = \frac{\mu_0}{4\pi} \frac{2i}{d}$$ check this

direction $\boxed{\text{into}}$

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+d^2)^{3/2}} = \frac{x}{d^2\sqrt{x^2+d^2}}\Bigg|_{-\infty}^{\infty}$$

$$\frac{2}{d^2}$$

Ex $\vec{B}$ at the center at a circle of current



$$r_1 = r_2 = r_3 \cdots r_n$$
$$= R$$

$$d\vec{B}_1 = \frac{\mu_0}{4\pi} \frac{i\, d\vec{\ell}_1 \times \vec{r}_1}{r_1^3}$$

$$|d\vec{\ell} \times \vec{r}| = d\ell\, r \cancel{\sin\theta}^{\,1}$$
$$= d\ell\, R$$

$$|\vec{B}| = \int |d\vec{B}| = \int \frac{\mu_0 i}{4\pi} \frac{d\ell\, R}{R^3} = \frac{\mu_0 i}{4\pi R^2} \int d\ell$$

$\int d\ell$ = circumference $= 2\pi R$

$$\int d\ell = \int_0^{2\pi} R\, d\theta = R \int_0^{2\pi} d\theta = 2\pi R$$

$$|\vec{B}| = \left(\frac{\mu_0}{4\pi}\right) \frac{i\, 2\pi}{R} = K_m \frac{2\pi i}{R}$$

direction $\boxed{\text{out}}$

Gauss' Law

infinitesimal vector that points out

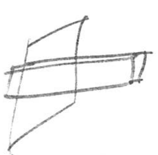
$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

closed surface

evaluated on the closed surface S

spherical Gaussian surface

cylinder



pillbox

Ampere's Law

infinitesimal line element points along the curve C

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 \, i_{enc}$$

closed curve "Amperean Loops"

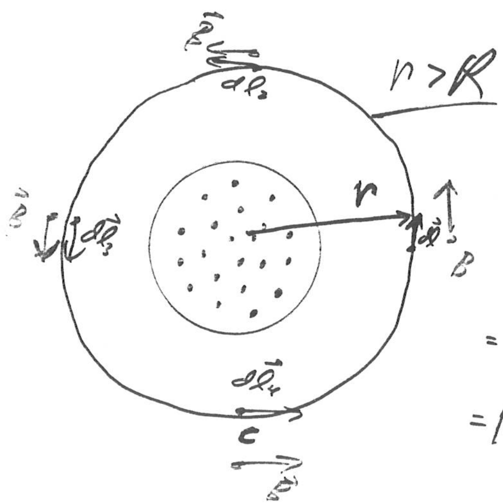
evaluted on closed curve C

current within the closed curve C

Useful in cases of high symmetry

Ex.

A straight wire of radius $\underline{R}$ carries a current $\underline{I}$ distributed uniformly over its cross-sectional area. Find the magnetic field everywhere.



$$\oint_c \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc}$$
$$= \mu_0 I$$
$$= \oint_c |B||d\ell|$$
$$= |B| \oint_c d\ell$$
$$= |B|\, 2\pi r = \mu_0 I$$

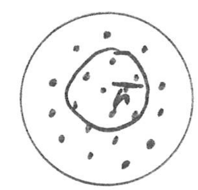
$$\therefore\ |B| = \frac{\mu_0 I}{2\pi r}$$
$$= \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{r}$$

Ex.

A straight wire of radius $\underline{R}$ carries a current $\underline{I}$ distributed uniformly over its cross-sectional area. Find the magnetic field everywhere.

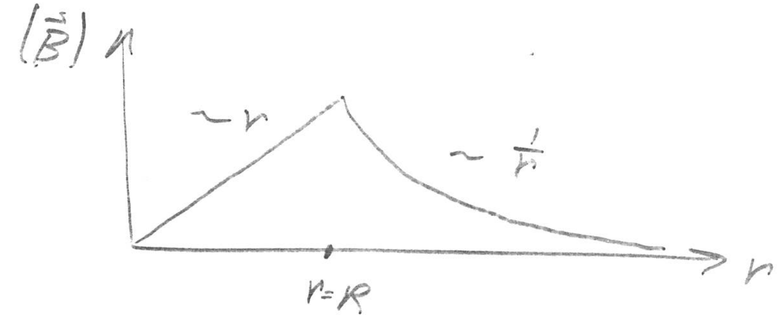
$\underline{r < R}$

$$\oint_c \vec{B}\cdot d\vec{\ell} = \mu_0 i_{enc}$$

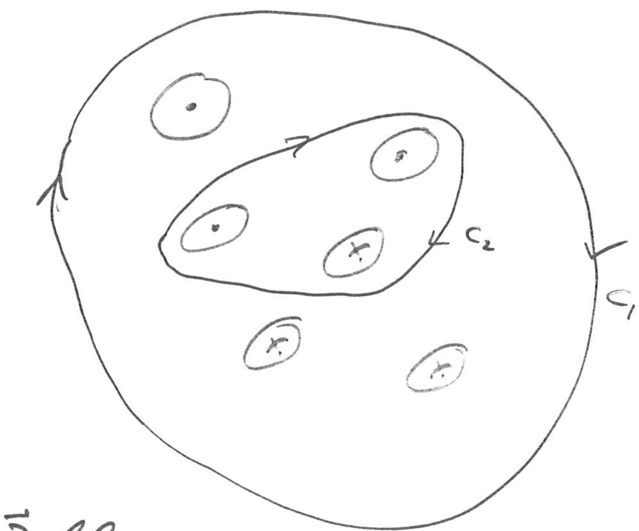


$$|B| \oint_c d\ell = \mu_0\left(\frac{I}{R^2}r^2\right)$$
$$|B|\, 2\pi r = \frac{\mu_0 I r^2}{R^2}$$

$$\therefore\ |B| = \left(\frac{\mu_0}{4\pi}\right)\frac{2Ir}{R^2}$$



Ex



$$\oint_{C_1} \vec{B}\cdot d\ell = 0$$

$$\oint_{C_2} \vec{B}\cdot d\vec{\ell} = \mu_0 I = \mu_0 (I + I - I)$$

If the direction of C_2 is reversed then $i_{enc} = -I = (-I - I + I)$