Physics 4392 — Professor Scalise

Homework Assignment #1

---

1. Simplify these expressions:
   (a) $\int_{x=-\infty}^{+\infty} \delta(x) \sin(x) dx$
   (b) $\int_{x=-\infty}^{+\infty} \delta(x) \sin(t) dx$
   (c) $\int_{x=-\infty}^{+\infty} \delta(x-t) \sin(x) dx$
   (d) $\int_{x=-\infty}^{+\infty} \delta(x-t) \sin(x+t) dx$
   (e) $\int_{x=-\infty}^{+\infty} \delta(3x) \cos(x) dx$
   (f) $\int_{x=14}^{+\infty} \delta(3x) \cos(x) dx$

2. (a) Write the displacement vector $\vec{r}$ in Cartesian coordinates using unit vector notation.
   (b) Calculate the divergence of $\vec{r}$.
   (c) Calculate the curl of $\vec{r}$.
   (d) Calculate the gradient of the magnitude of $\vec{r}$.

3. Consider the vector field $\vec{v}(\vec{r}) = (x^2 + y^2)\hat{e}_x + (x^2 + y^2)\hat{e}_y + z^2\hat{e}_z$.
   (a) Calculate the divergence of $\vec{v}(\vec{r})$
   (b) Calculate the curl of $\vec{v}(\vec{r})$
   (c) Calculate the gradient of the divergence of $\vec{v}(\vec{r})$
   (d) Decompose the vector field $\vec{v}(\vec{r})$ into the sum of two other vector fields, $\vec{a}(\vec{r})$ and $\vec{b}(\vec{r})$, such that $\vec{a}(\vec{r})$ has no divergence (it is solenoidal) and $\vec{b}(\vec{r})$ has no curl (it is irrotational).

4. (a) Show that the divergence of the curl of any vector field is zero.
   (b) Show that the curl of the gradient of any scalar field is zero.