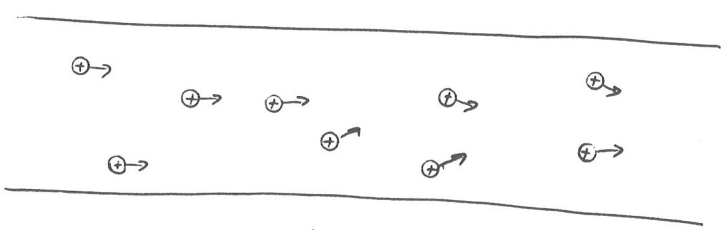
Current + Resistance

Until now, we have been studying Electrostatics (charges at rest).

<u>Current</u>: the rate at which charge moves past a hypothetical plane.

$$i(t) = \frac{dq}{dt}$$



$$q = \int_{t_i}^{t_f} i(t)\, dt$$

MKS Unit

The unit of current is the ampere (A). This is one of the fundamental set

{meter, kilogram, second, ampere}.
L, M, T, current

Steady State

The current is <u>not</u> a function of time — it is constant.

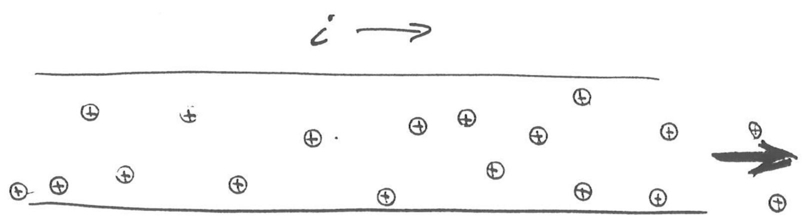
$$i = \text{constant}$$

Under steady state conditions, charge cannot "pile up" in the wire.

Direction of Current

Current (i) is a <u>scalar</u>, but there is an associated <u>direction</u>.

Current is defined by convention to flow in the direction that <u>positive</u> charges would move even if the moving charges are negative!



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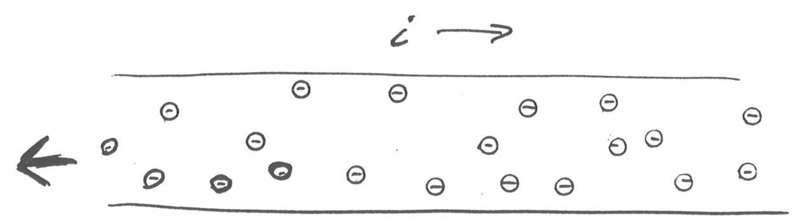
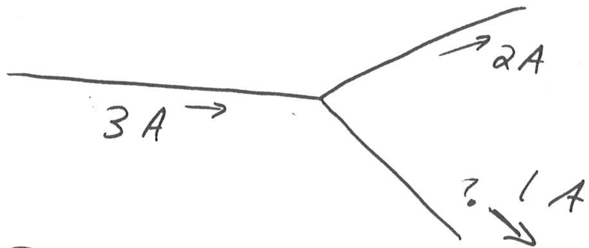
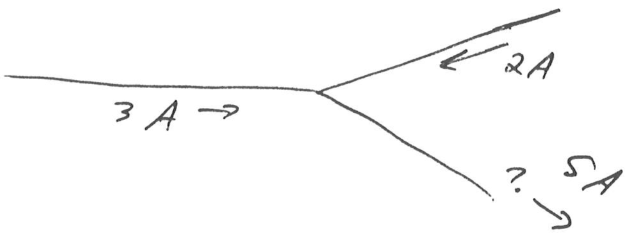


Ex

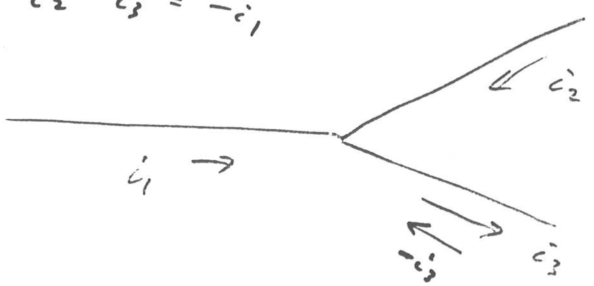


$3A + 2A + (-5A) = 0$ ✓



$i_1 + i_2 = i_3$

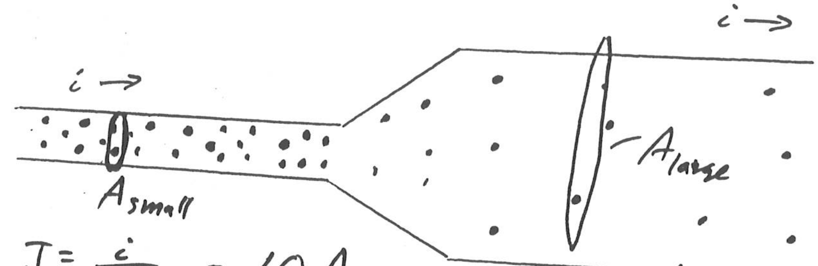
$i_2 - i_3 = -i_1$



$i_1 + i_3 + (-i_2) = 0$

Steady state <u>current conservation</u> is a consequence of <u>charge conservation</u>.

<u>Current Density</u>



$J = \frac{i}{A_{small}} = 10 \frac{A}{m^2}$ $\qquad$ $J = \frac{i}{A_{large}} = 1 \frac{A}{m^2}$

Current Density : $J \equiv \frac{i}{Area}$

(magnitude)

$\vec{J}$ is a <u>vector</u> quantity.

The direction of $\vec{J}$ is the same as that of the electric field $\vec{E}$, regardless of the sign of the charge carriers.

Whoa!

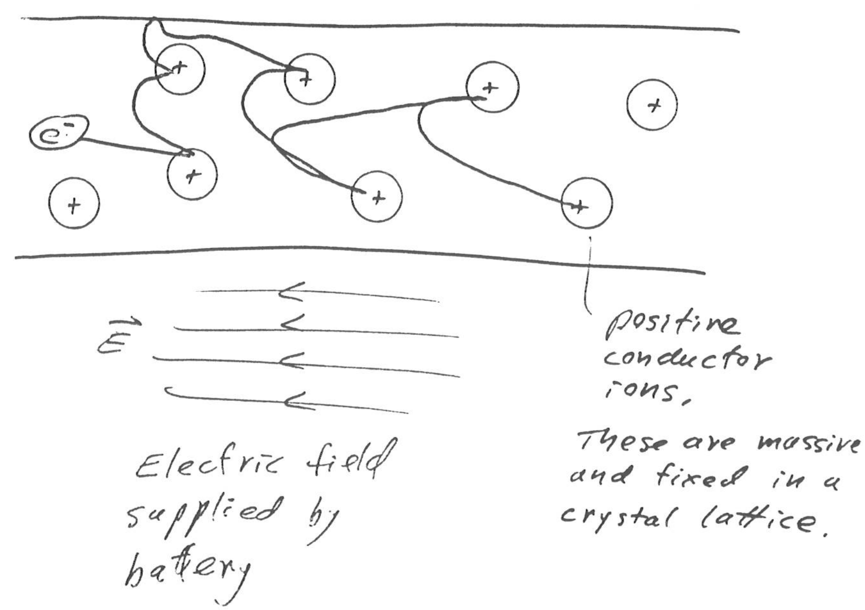
<u>What</u> electric field???

Something must cause the moving charges to move: An <u>electric field</u> in the conductor.

I thought $\vec{E}=0$ inside a conductor. This is true for electro<u>statics</u>. Now we are considering charges in motion: electro<u>dynamics</u>.

Doesn't an electric field cause charges to <u>accelerate</u>, so the current (i) will not be a <u>constant</u> but will increase with time?

An electric field would cause <u>free</u> charges to accelerate. In a conductor (like a wire), the charges accelerate for a <u>very</u> short time (10^{-14} seconds) then collide with atoms in the conductor, scatter, and accelerate again, ...

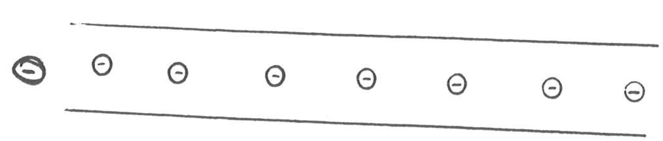


The result of the scattering and acceleration is that electrons move with a constant average velocity called the "drift velocity."

Typically, $|\vec{v}_{drift}| = 10 \frac{cm}{hour}$

A snail could race an electron and win!

So why doesn't it take a week to turn the lights on?



The speed of the "push" is almost the speed of light.

Resistance

It is an experimentally observed fact that for most (not all!) conductors the current is directly proportional to the potential difference across the conductor.

$$V = iR \qquad \text{Ohm's Law}$$

The constant of proportionality is called the resistance.

MKS Unit

$$1 \text{ ohm } (\Omega) \equiv 1 \frac{V}{A} \left(\frac{\text{volt}}{\text{ampere}}\right)$$

1Ω

The Magnetic Field

The magnetic field is **not** produced by "magnetic charges" called magnetic monopoles.

Instead, <u>electric</u> charges in motion, that is, <u>currents</u> produce the magnetic field $\vec{B}$.

Recall: the electric force is

$$\vec{F}_e = q\vec{E}$$

$\vec{F}_e$: force on charge q

$\vec{E}$: field produced by all charges **except** q.

We derived this from Coulomb's Law

$$\vec{F}_{\substack{\text{of } 1\\ \text{on } 2}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(r_{12})^2}\hat{r}_{1\to 2}$$

from experiment

We determine the magnetic force from experiment also:

$$\vec{F}_m = q\vec{v}\times\vec{B}$$

$\vec{F}_m$: force on charge q

$\vec{v}$: velocity of charge q

$\times$: "cross product" or "vector product"

$\vec{B}$: magnetic field produced by all moving charges **except** q

The cross-product is <u>perpendicular</u> to both vectors in the product.

$\vec{F}_m$ is $\perp$ to $\vec{v}$

$\vec{F}_m$ is $\perp$ to $\vec{B}$

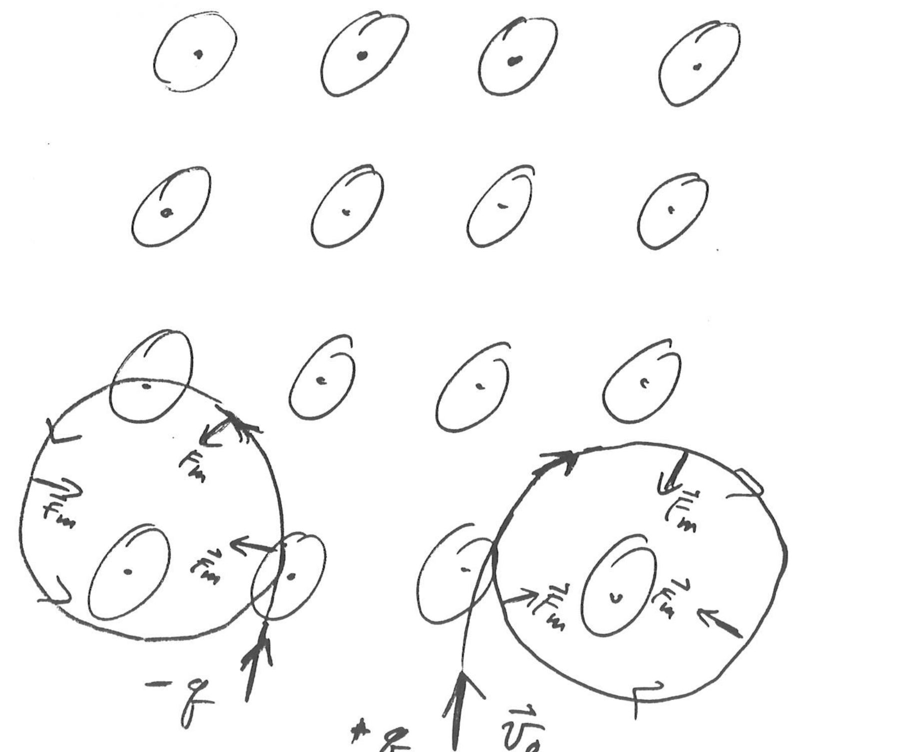
$\vec{v}$ and $\vec{B}$ can be $\perp$ or $\parallel$ or anything in between.

Consequence:

The magnetic force <u>does no work</u>.

<u>Power</u> $P_{inst} = \vec{F} \cdot \vec{v} = \frac{dW}{dt}$

$$W_m = \int_0^T P(t)\, dt = \int_0^T \vec{F}_m \cdot \vec{v} = 0$$

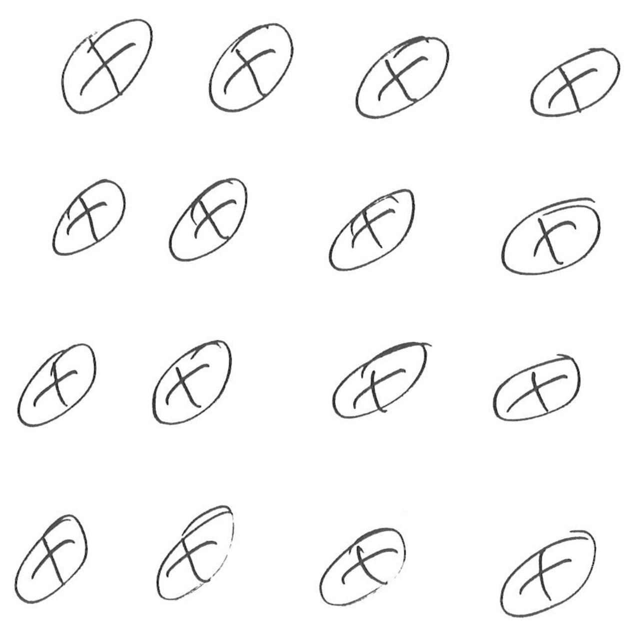


$\vec{B}$ out of page

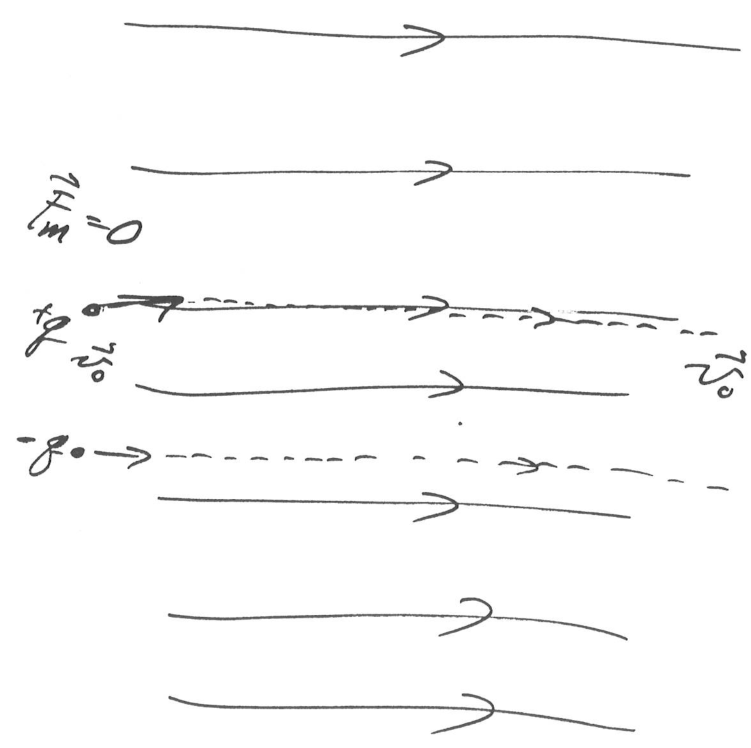
$\vec{F}_m = q\vec{v} \times \vec{B}$

$\vec{F}_m$ centripetal force

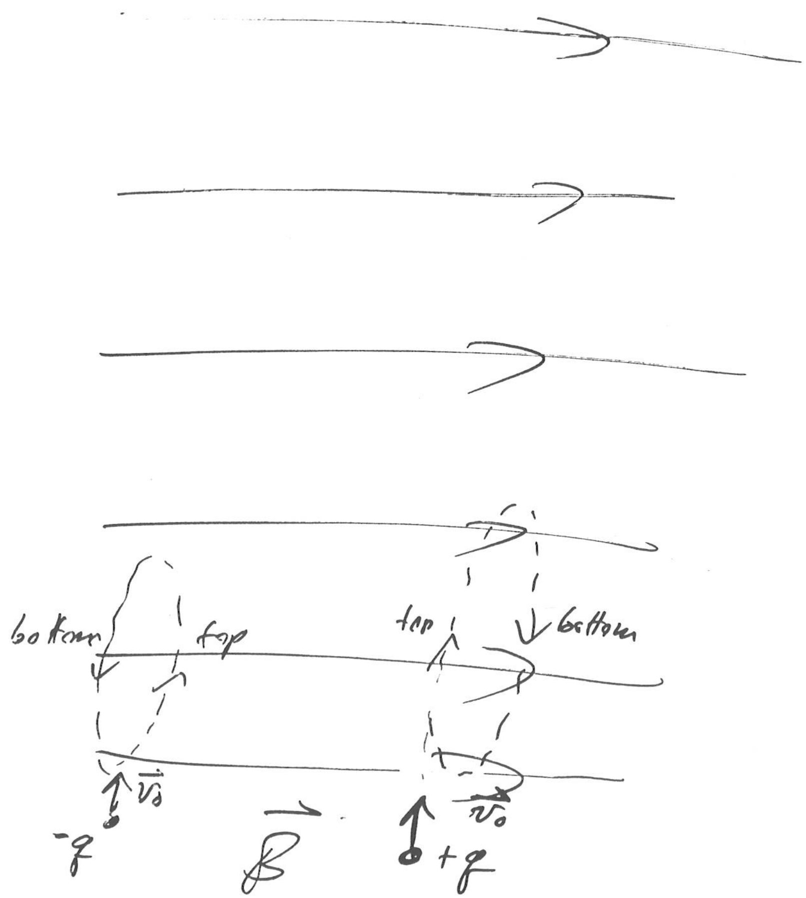
speed = constant = $|\vec{v}_0|$



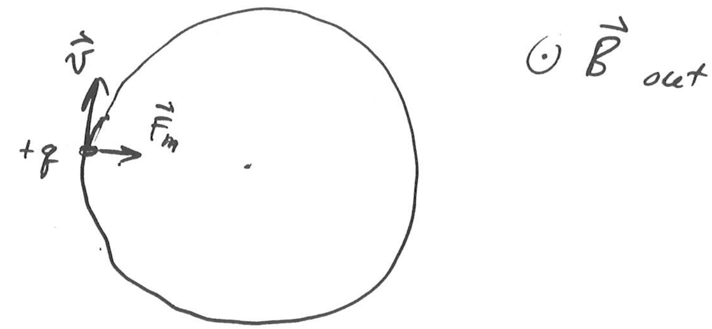
$\vec{B}$ into page



$\vec{v}$ parallel to $\vec{B}$
$\vec{v} \times \vec{B} = 0$



Radius of orbit in a $\vec{B}$ field



$$\sum F_r = m a_r$$

$$F_m = qvB\sin\theta = m\frac{v^2}{R}$$

$$= qvB(1) = \frac{mv^2}{R}$$

$$R = \frac{mv}{qB}$$