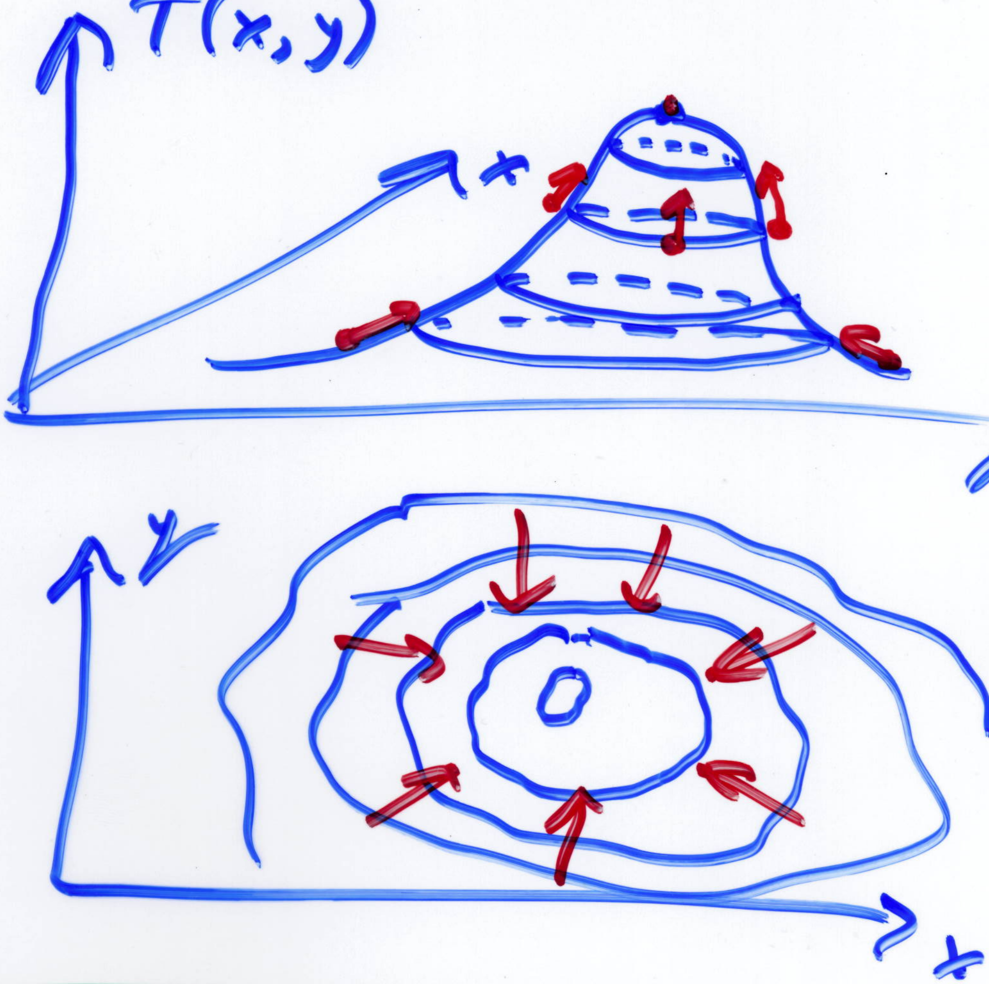


$T(x,y)$
scalar function

$\vec{\nabla} T(x,y) = \vec{V}(x,y)$

Divergence

$\text{div } \vec{V}(\vec{r})$

$$\vec{\nabla} \cdot V(\vec{r}) = S(\vec{r})$$

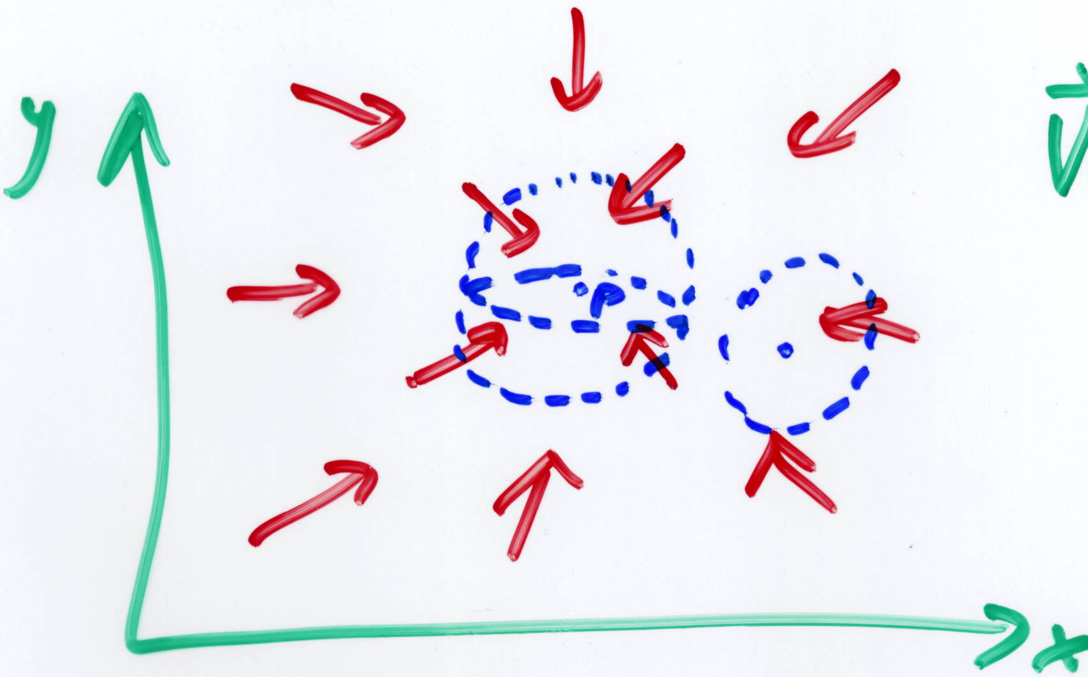
vector field

scalar field

Cartesian

$$\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[\hat{i}V_x(x,y,z) + \hat{j}V_y(\ ) + \hat{k}V_z(\ )\right]$$

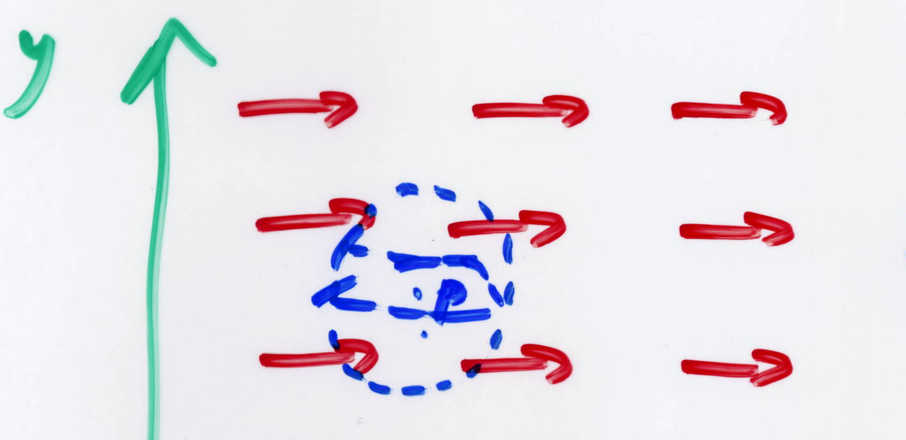
$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = S(x, y, z)$$



$\vec{V}(x,y)$

$P = (x_0, y_0)$ $\qquad \vec{\nabla} \cdot \vec{V}(x,y)\Big|_p < 0$

e.g. $(2, 3)$



$\vec{W}(x,y) = 4\hat{z}$

$\vec{\nabla} \cdot \vec{W}(x,y)\Big|_p = 0$

Curl $\vec{\nabla} \times \vec{V}(\vec{r}) = \vec{W}(\vec{r})$

$(-1)^{r+c}$

Cartesian

$$\vec{\nabla} \times \vec{V}(x,y,z) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x(x,y,z) & V_y(..) & V_z(..) \end{vmatrix}$$

curl $\vec{V}$

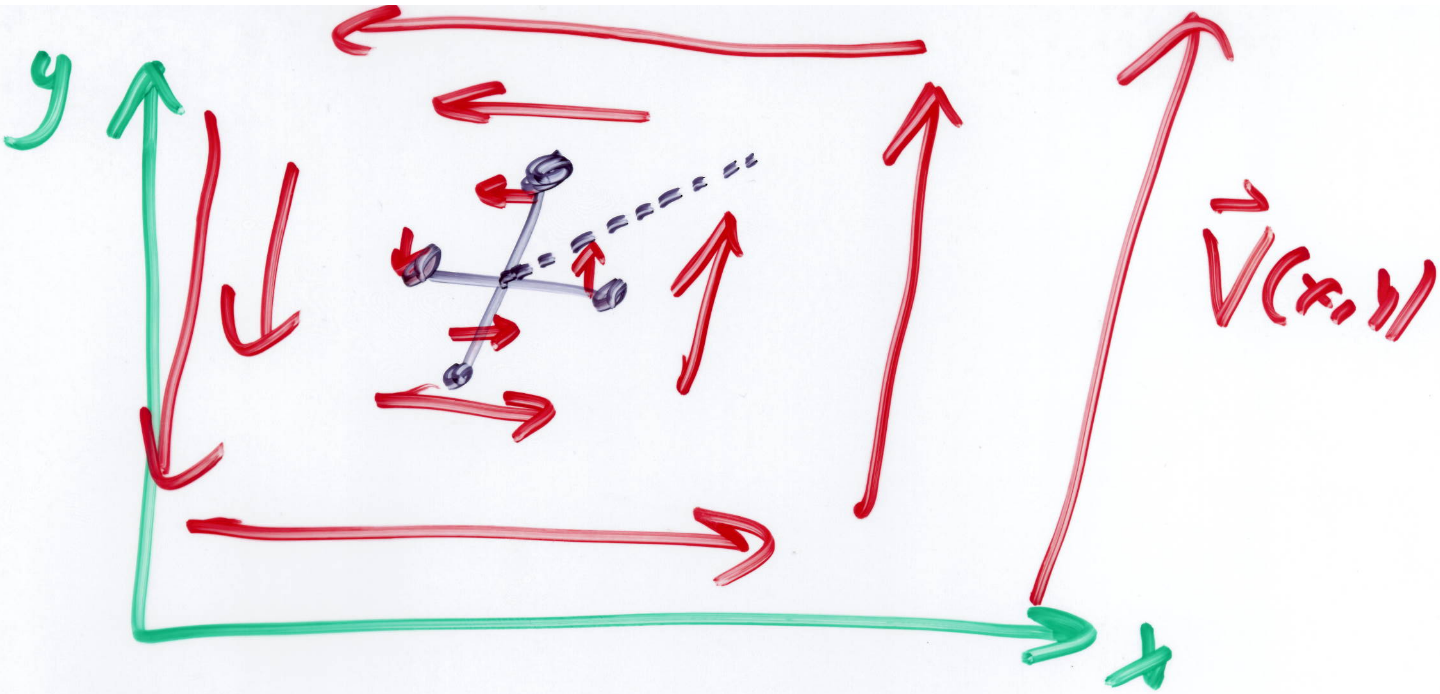
rot $\vec{V}$

$$= \hat{i} \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_y & V_z \end{vmatrix} - \hat{j} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ V_x & V_z \end{vmatrix} + \hat{k} \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ V_x & V_y \end{vmatrix}$$

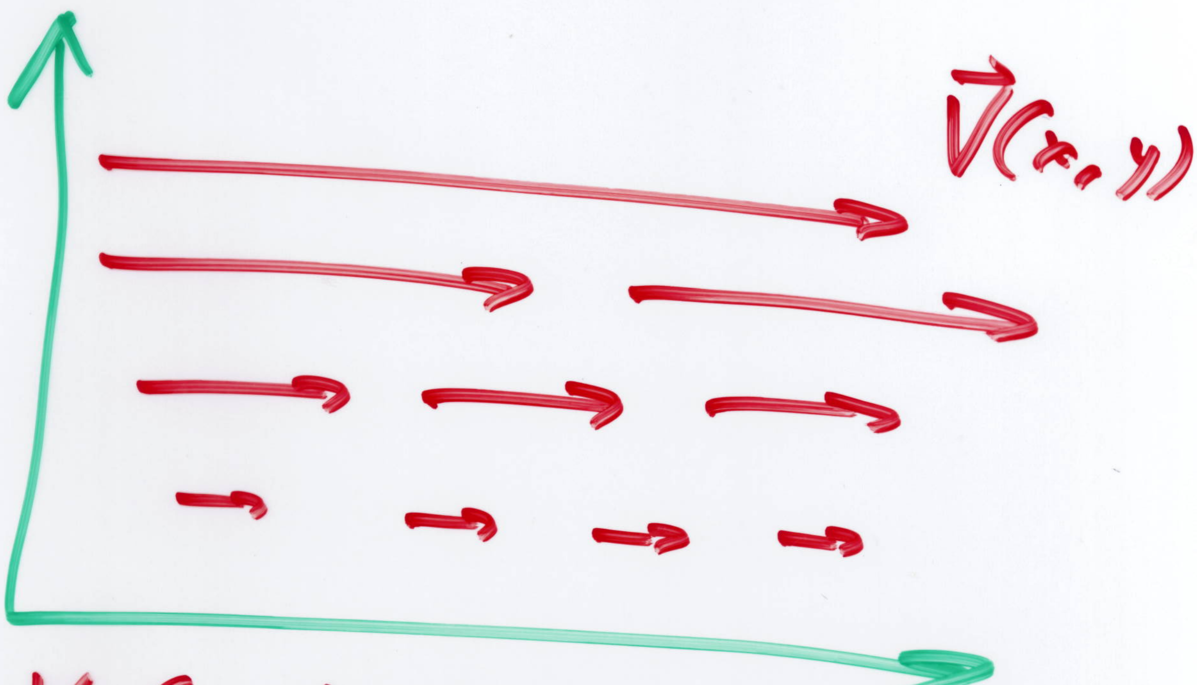
$$= \hat{i} \left[\frac{\partial V_z(x,y,z)}{\partial y} - \frac{\partial V_y(x,y,z)}{\partial z} \right]$$

$$- \hat{j} \left[\qquad \qquad \right]$$

$$+ \hat{k} \left[\qquad \qquad \right]$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$V_y(x,y) = 0$$

$$V_z = 0$$

$$V_x(x,y) = \pi y^2 + 7$$

$$\vec{\nabla} \times \vec{V}(x,y) = \vec{W}(x,y)$$

$$W_x(x,y) = \left[\cancel{\frac{\partial V_z}{\partial y}} - \cancel{\frac{\partial V_y}{\partial z}}\right] = 0$$

$$W_y(x,y) = \left[\cancel{\frac{\partial V_x}{\partial z}} - \cancel{\frac{\partial V_z}{\partial x}}\right] = 0$$

$$W_z(x,y) = \left[\cancel{\frac{\partial V_y}{\partial x}} - \frac{\partial V_x}{\partial y}\right]$$

$$W_z(x,y) = -\frac{\partial}{\partial y}\left[\pi y^2 + 7\right]$$

$$= -2\pi y$$

$$\vec{W}(x,y) = 0\hat{\imath} + 0\hat{\jmath} - 2\pi y\hat{k}$$

2nd derivatives

Laplacian T, $\nabla^2 T = S$

$\vec{\nabla}\cdot\left(\vec{\nabla} T(\vec{r})\right)$	$\cancel{\vec{\nabla}\times\left(\vec{\nabla} T(\vec{r})\right)} \rightarrow 0$
$\vec{\nabla}\left(\vec{\nabla}\cdot\vec{V}(\vec{r})\right)$	
$\cancel{\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{V}(\vec{r})\right)} \rightarrow 0$	$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{V}(\vec{r})\right)$ $= \vec{\nabla}(\vec{\nabla}\cdot\vec{V}) - \nabla^2\vec{V}$

$$\vec{\nabla}\cdot\vec{\nabla} = \left[\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right]\left[\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right]$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

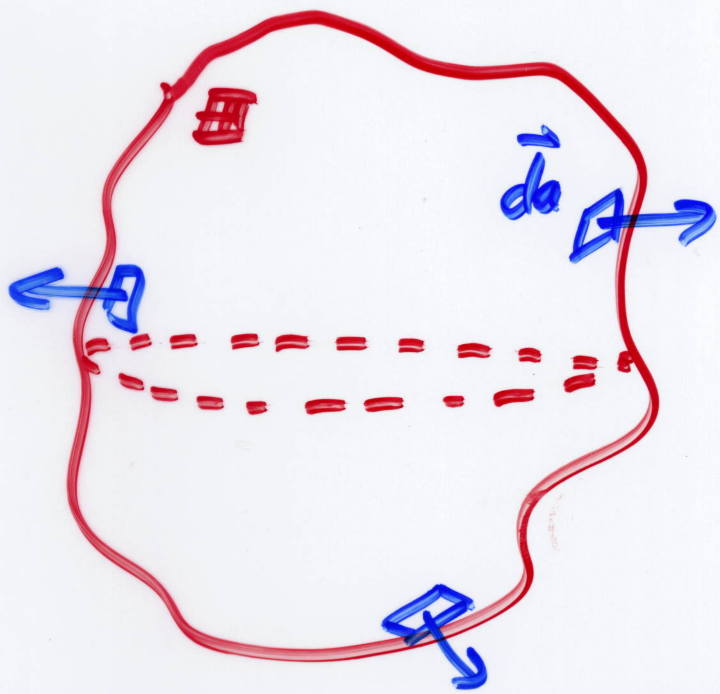
BAC-CAB rule

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

Gauss' Theorem, Green Thm
Divergence Theorem

$$\iiint_V \left[\vec{\nabla}\cdot\vec{V}(\vec{r})\right] dV = \oiint_S \vec{V}(\vec{r})\cdot d\vec{a}$$

S ↑ closed surface

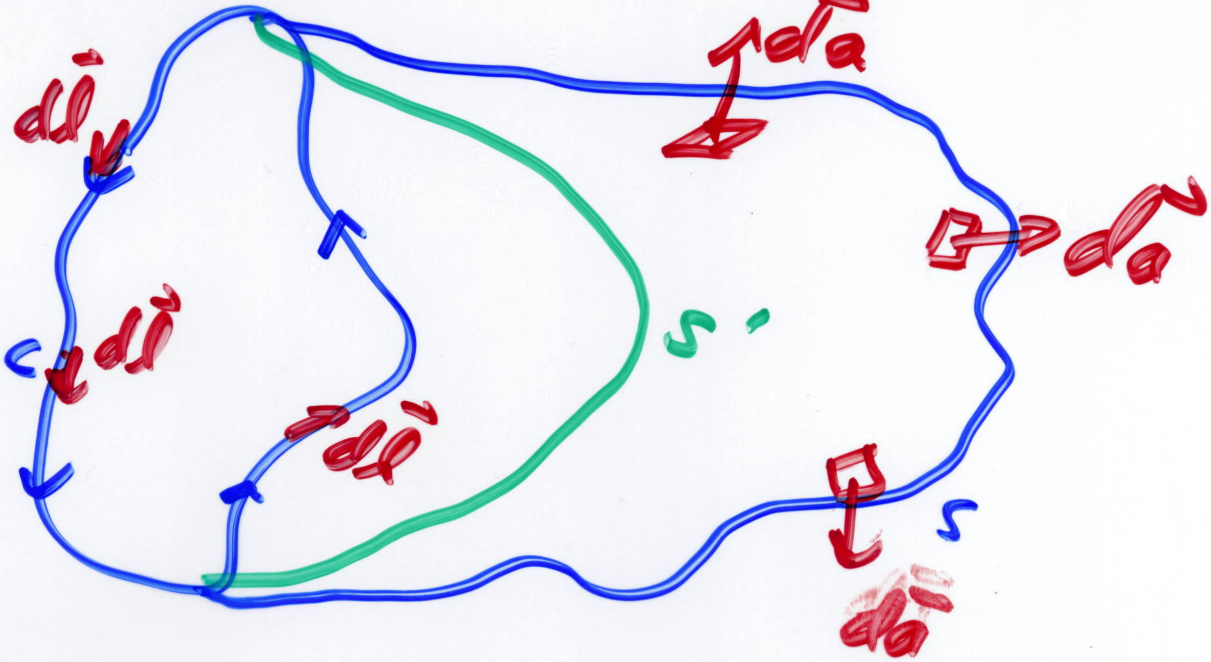


Stokes' Theorem

$$\iint_S \left[\vec{\nabla}\times\vec{V}(\vec{r})\right]\cdot d\vec{a} = \oint_C \vec{V}(\vec{r})\cdot d\vec{l}$$

S open surface

C closed loop


$d\vec{l}$
c
$d\vec{l}$
$d\vec{l}$
$d\vec{a}$
$d\vec{a}$
s'
s
$d\vec{a}$