Displacement Vector

c) $\vec{r} = x\,\hat{e}_x + y\,\hat{e}_y + z\,\hat{e}_z$

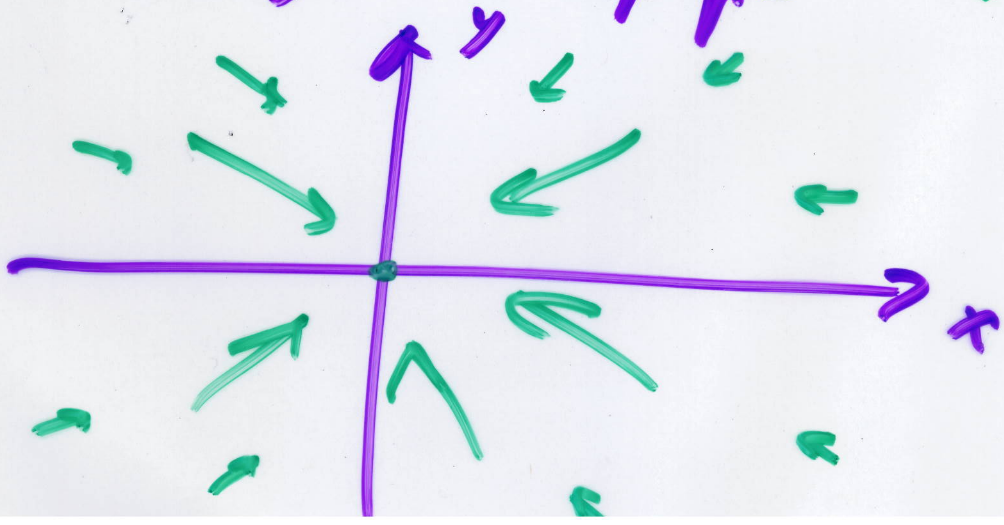
s) $\vec{r} = r\,\hat{e}_r \cancel{+ \theta\,\hat{e}_\theta} \cancel{+ \varphi\,\hat{e}_\varphi}$

$\vec{\nabla}\left(\frac{1}{|\vec{r}|}\right)$

c) $\sum_{i=1}^{3} \hat{e}_i \frac{\partial}{\partial x_i}\left[\frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}\right] = \ldots$

s) $\left[\hat{e}_r \frac{\partial}{\partial r} + \hat{e}_\theta \frac{1}{r}\cancel{\frac{\partial}{\partial \theta}} + \hat{e}_\varphi \frac{1}{r\sin\theta}\cancel{\frac{\partial}{\partial \varphi}}\right]\frac{1}{r}$

$= \hat{e}_r \frac{\partial}{\partial r}\left(\frac{1}{r}\right) = -\hat{e}_r \frac{1}{r^2} \equiv \vec{V}(\vec{r})$



$$\vec{\nabla}\cdot\vec{V}(\vec{r}) = \vec{\nabla}\cdot\left[\frac{-\hat{e}_r}{r^2}\right] = \vec{\nabla}\cdot\vec{\nabla}\left(\frac{1}{r}\right)$$

$$\mathrm{div}\left[\mathrm{grad}\left(\frac{1}{r}\right)\right] = \nabla^2\left(\frac{1}{r}\right)$$

$$\mathrm{laplacian}\left(\frac{1}{r}\right) =$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 V_r\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(-\frac{1}{r^2}\right)\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(-1) = \boxed{0} \quad \text{if } r \neq 0$$

$$= ? \quad \text{if } r = 0$$

Another Way : Divergence Thm

$$\iiint_{Vol} \vec{\nabla}\cdot\vec{V}(r)\, dV = \oiint_S \vec{V}(r)\cdot d\vec{a}$$

$\hat{e}_r\, h_\theta h_\varphi\, d\theta\, d\varphi$

closed surface S

$$= \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\left(\frac{-\hat{e}_r}{R^2}\right)\cdot\left(R^2\sin\theta\, d\theta\, d\varphi\, \hat{e}_r\right)$$

$$= -\int_{\varphi=0}^{2\pi} d\varphi \times \int_{\theta=0}^{\pi}\sin\theta\, d\theta = \boxed{-4\pi}$$

the Monster

$$\nabla^2\left(\frac{1}{r}\right) = \vec{\nabla}\cdot\vec{\nabla}\left(\frac{1}{|\vec{r}|}\right) = \begin{cases} 0 & \text{if } r \neq 0 \\ \infty & \text{at } r = 0 \end{cases}$$

$$\iiint_{\text{vol}} \left[\nabla^2\left(\frac{1}{r}\right)\right] dx\,dy\,dz = -4\pi$$

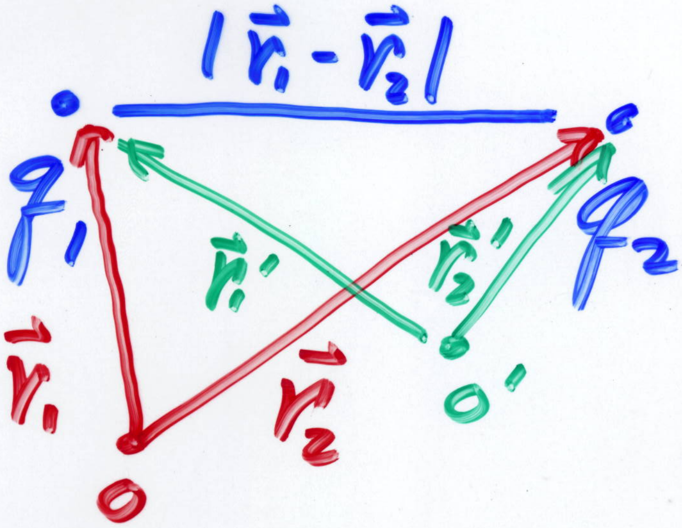
origin included

$$\int_{x}\int_{y}\int_{z}{}_{-\infty}^{+\infty} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) \frac{1}{\sqrt{x^2+y^2+z^2}}\, dx\,dy\,dz$$

c)
$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\,\delta(x)\,\delta(y)\,\delta(z)$$

$$= -4\pi\,\delta^{(3)}(\vec{r})$$

s)
$$= -\frac{1}{r^2 \sin\theta}\,\delta(r)$$



"Point" charges

$$\boxed{k = \frac{1}{4\pi\epsilon_0}} \text{ MKS}$$

$$\vec{F}_{\substack{\text{on } 1 \\ \text{caused by } 2}} = \frac{-k\, q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^2}\,\widehat{r_2 - r_1} =$$

$$k = 8.99 \times 10^{9} \frac{Nm^2}{C^2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} = \text{Permittivity of free space.}$$

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cgs : $k \equiv 1$ Coulombs' law

$$\vec{F}_{\substack{\text{on } 1 \\ \text{by } 2}} = \frac{-k\, q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$= \frac{k\, q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}$$

Force on q at $\vec{r}$ caused by $\{q_i\}$ at $\vec{r}_i$

$$\vec{F}_{\text{on } q} = kq\sum_{i=1}^{N} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

Superposition

$$\vec{F}_{\text{on } q} = q\left[k\sum_{i=1}^{N} \frac{q_i(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}\right]$$

$$\equiv q\,\vec{E}(\vec{r})$$

$\vec{r}$ field point

$\vec{r}_i$ source points