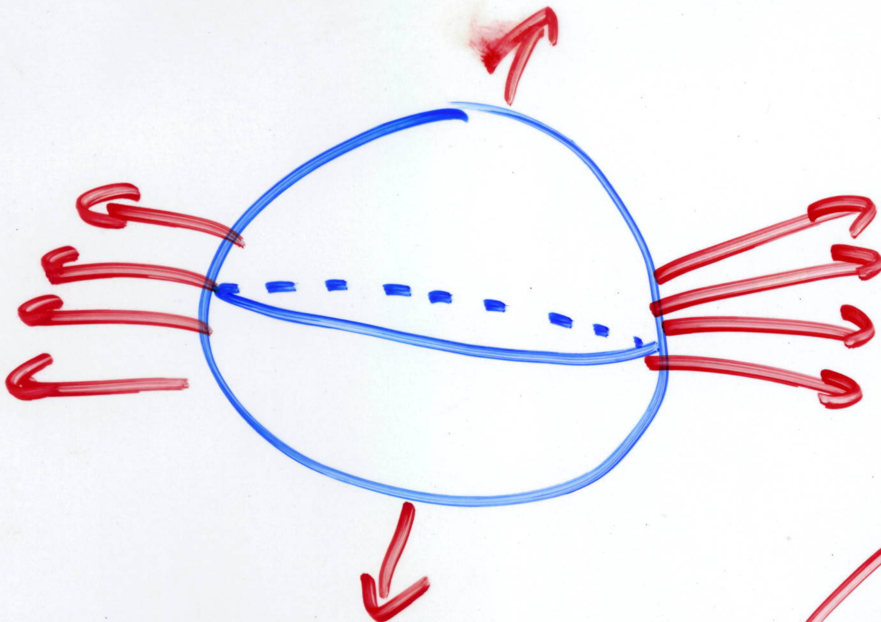
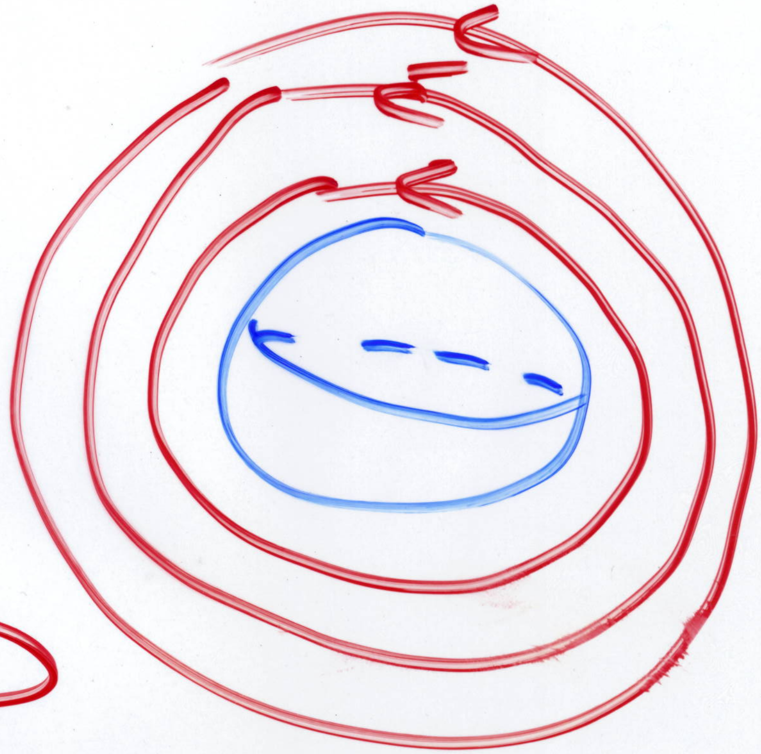
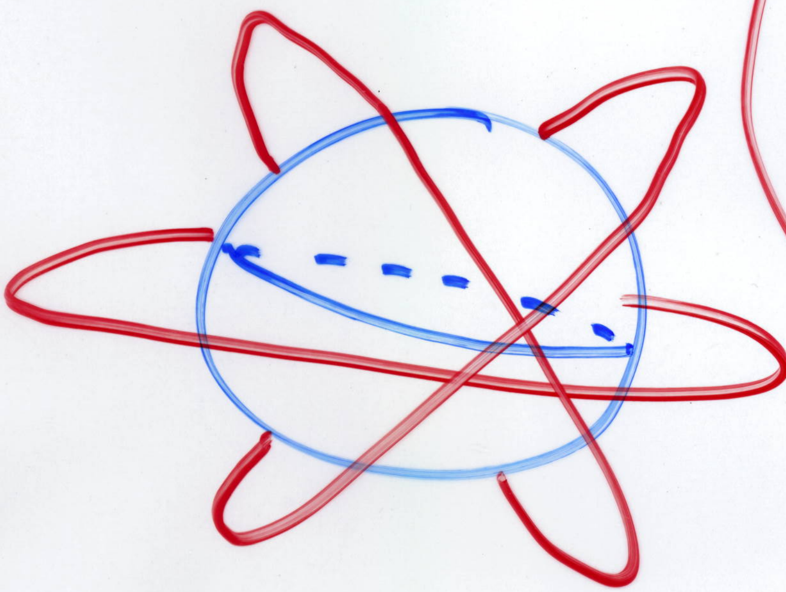
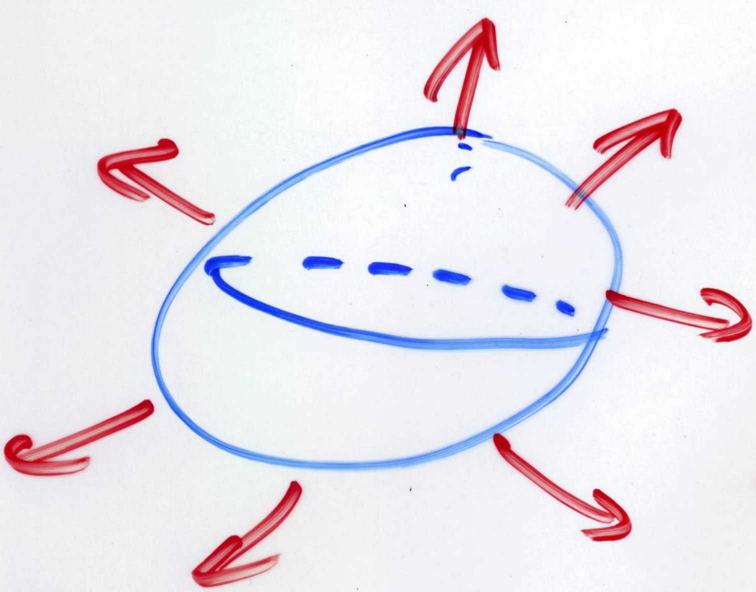
$$\vec{\nabla} \times \vec{E}(\vec{r}) = 0$$



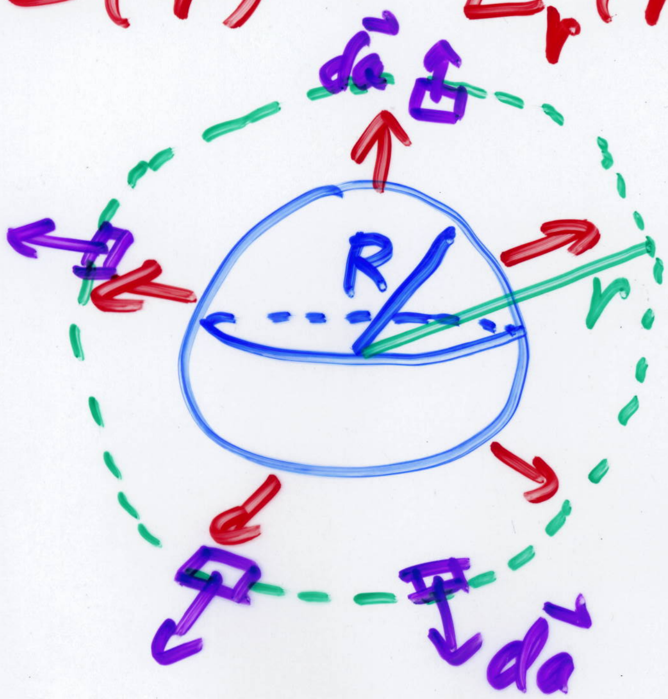






$$\vec{E}(\vec{r}) = E_r(\vec{r})\,\hat{e}_r(\vec{r}) + 0\,\hat{e}_\theta + 0\,\hat{e}_\varphi$$

$$\vec{E}(\vec{r}) = E_r(r)\,\hat{e}_r(\theta,\varphi)$$



$$\oiint_S \vec{E}(\vec{r})\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

Gaussian surface S chosen so that

$\vec{E}(\vec{r})$ is parallel to $d\vec{a}$

$$\vec{E}(\vec{r})\cdot d\vec{a} = |\vec{E}(\vec{r})|\,|d\vec{a}|\cos 0^\circ$$

$$= E(\vec{r})\,da \rightarrow E(r)\,da$$

(r, θ, φ) just r

$$da = r^2 \sin\theta \, d\theta \, d\varphi$$

$$\oiint_S \vec{E}(\vec{r}) \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$= \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} \underline{E(r)\, r^2} \, \underline{\sin\theta} \, \underline{d\theta} \, d\varphi$$

$$= E(r)\, r^2 \underbrace{\int_{\theta=0}^{\pi} \sin\theta \, d\theta}_{2} \underbrace{\int_{\varphi=0}^{2\pi} d\varphi}_{2\pi}$$

$$-\cos\theta \Big|_0^{\pi} = -\cos\pi + \cos 0$$

$$-(-1) + 1 = 2$$

$$= 4\pi E(r)\, r^2 = \frac{4\pi R^2 \sigma}{\epsilon_0}$$

$$\Rightarrow \boxed{E(r) = \frac{R^2 \sigma}{r^2 \epsilon_0} \quad (r \geq R)}$$

Inside $r < R$

Gauss' Law

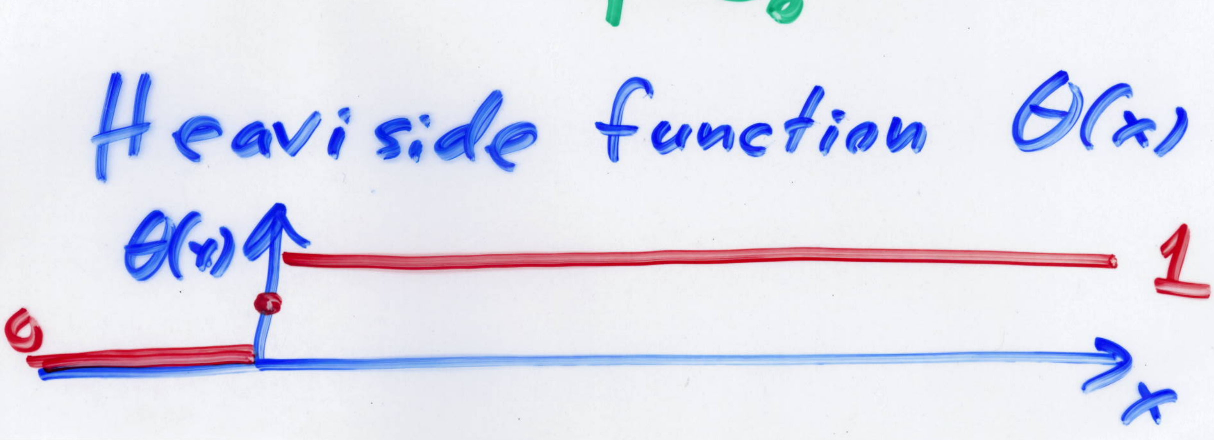
$$\oiint_S \vec{E}(\vec{r}) \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$4\pi E(r)\, r^2 = \frac{0}{\epsilon_0}$$

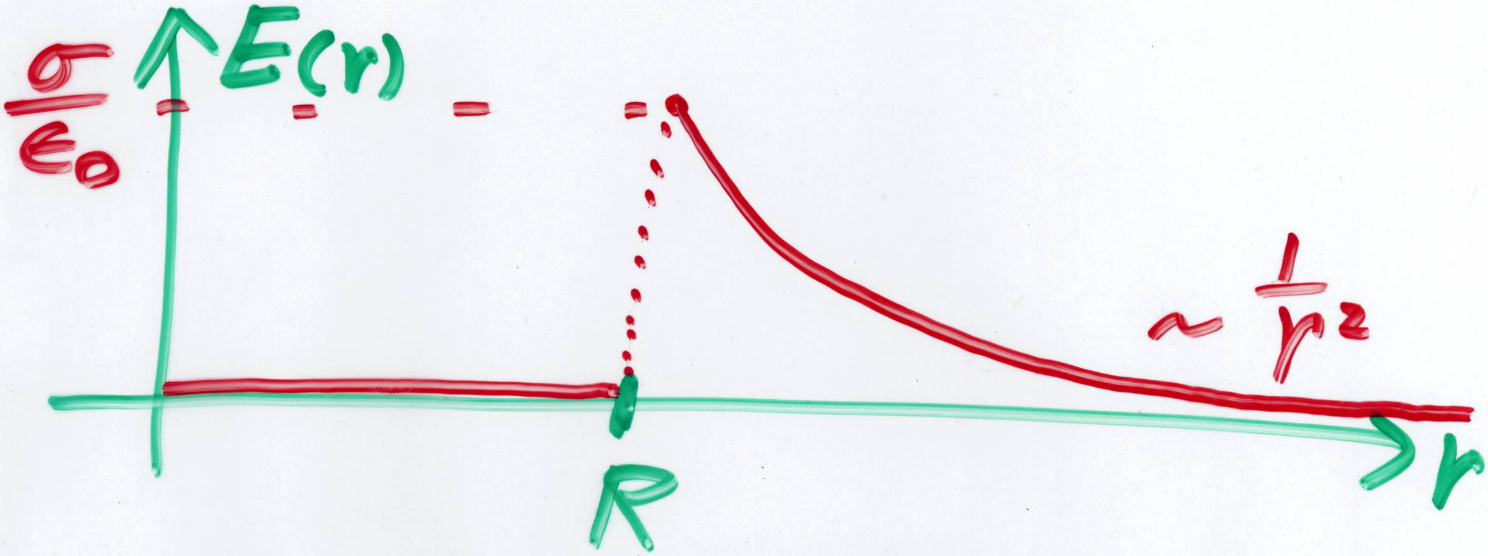
$$\Rightarrow \boxed{E(r) = 0 \qquad r < R}$$

$$\vec{E}(\vec{r}) = \begin{cases} 0 & , \; r < R \\ \dfrac{R^2 \sigma}{r^2 \epsilon_0} \hat{e}_r & , \; r > R \end{cases}$$

Heaviside function $\Theta(x)$



$$\vec{E}(\vec{r}) = \frac{R^2\sigma}{r^2\epsilon_0}\hat{e}_r \; \Theta(r-R)$$



Electrostatic Potential = Voltage

$$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$$

$\vec{\nabla} \times \vec{E}(\vec{r}) = 0$
integral is path-independent

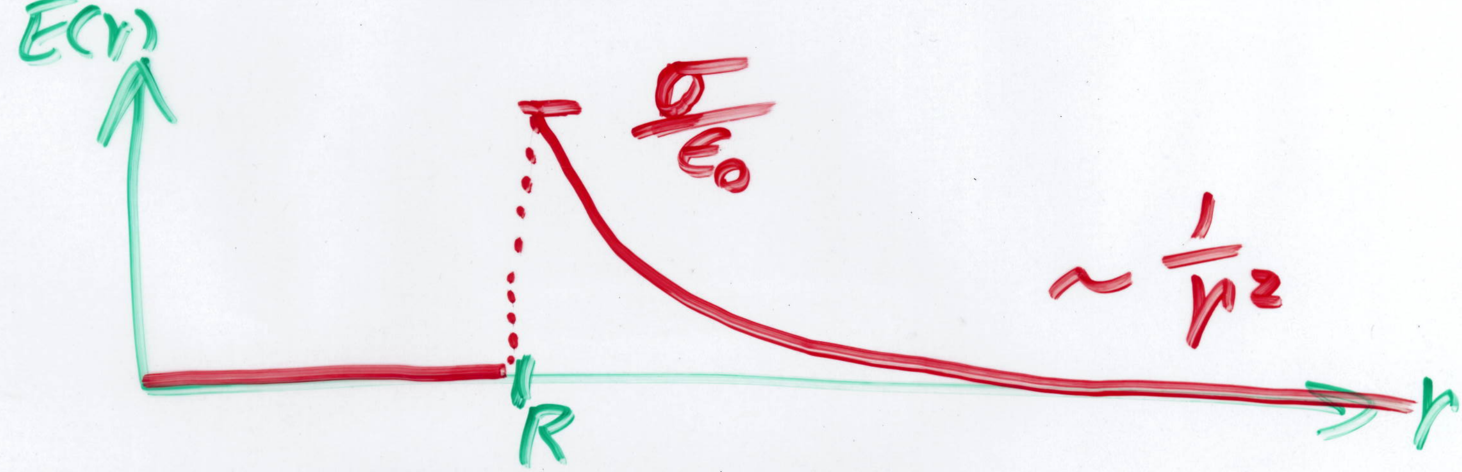
$$V_2 - V_1 = -\int_{1,C}^{2} \vec{E}(\vec{r}) \cdot \vec{ds}$$

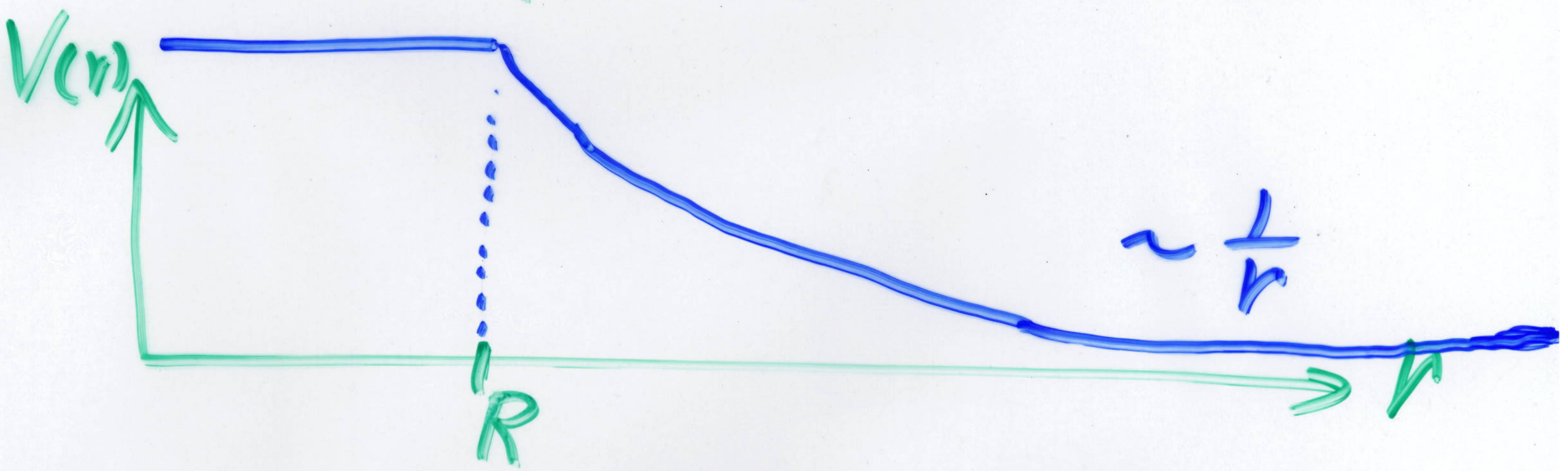
Convention 2 to be $r \to \infty$

$$V(\infty) = 0$$

$$V(r) = V(\infty) + \int_{r'=r}^{\infty} \vec{E}(\vec{r}') \cdot dr'\hat{e}_r$$

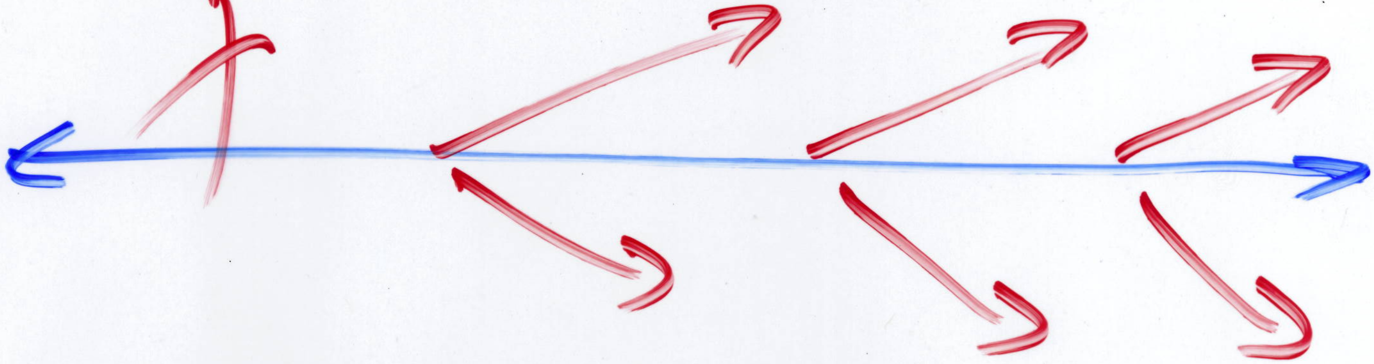
$$V(r) = 0 + \int_{r'=r}^{\infty} \frac{R^2\sigma}{r'^2\epsilon_0}\,\Theta(r'-R)\,dr'$$

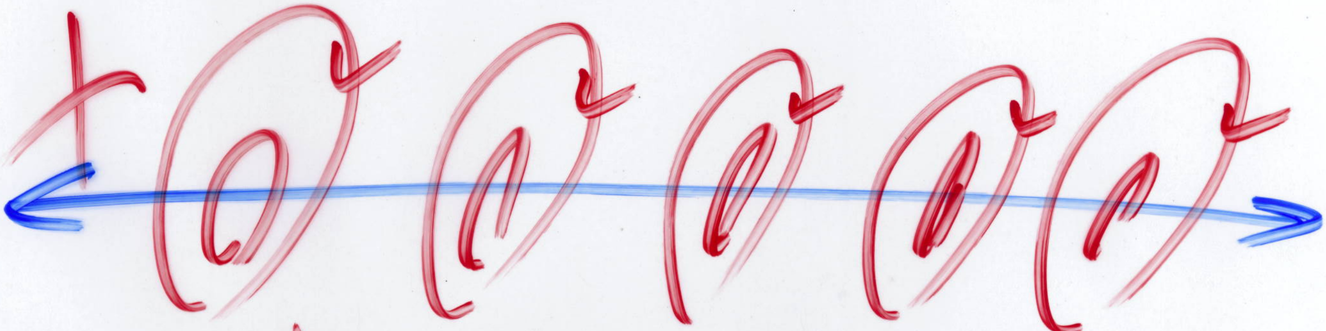




$$V(r) = \begin{cases} \dfrac{R^2\sigma}{r\epsilon_0}, & r \geq R \\[2ex] \dfrac{R\sigma}{\epsilon_0}, & r \leq R \end{cases}$$

What does $\vec{E}$ look like

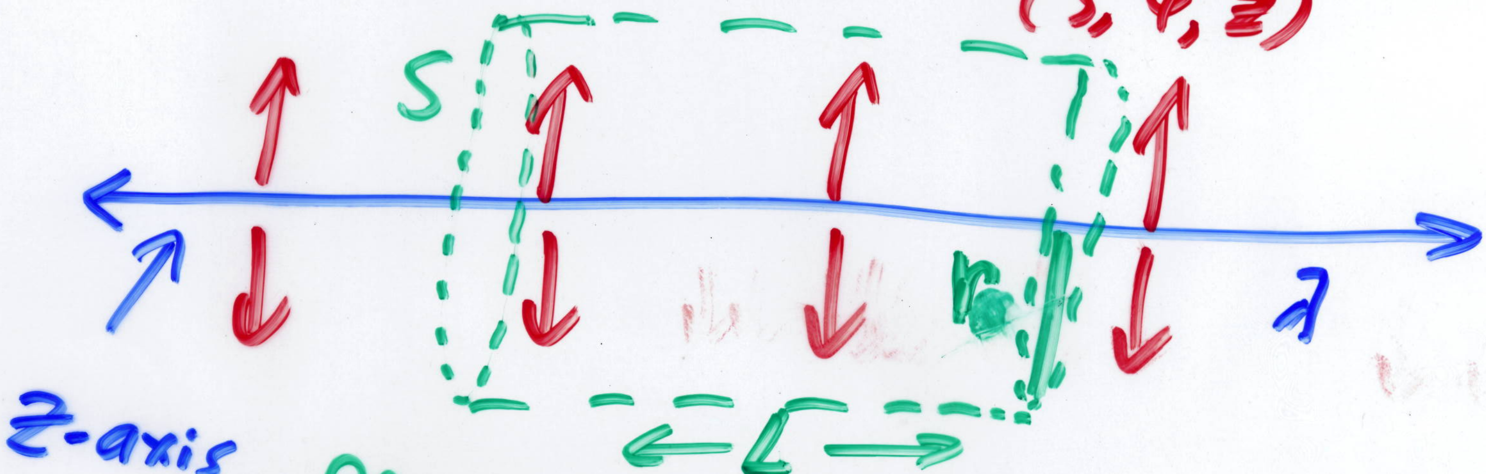




$\vec{\nabla} \times \vec{E} = 0$

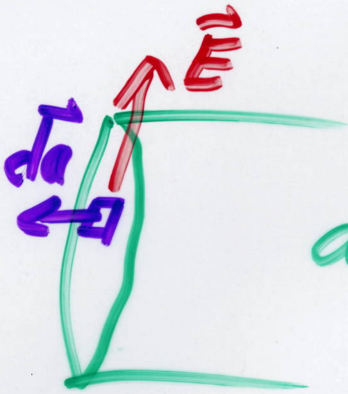
$\vec{E}$lectric field for a line of charge
constant linear charge density λ.
Use Gauss' Law.

cylindrical polar coordinates
(s, φ, z)



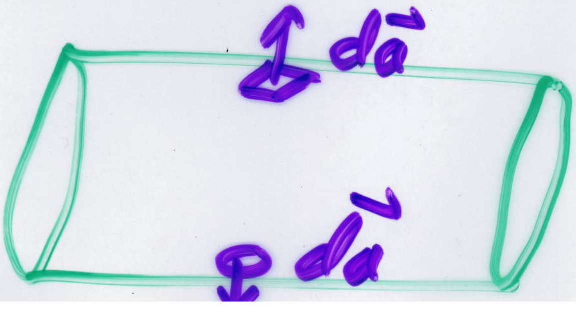
$$\oint_S \vec{E}(\vec{r}) \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

Left endcap



and right endcap

Lateral surface area



$$E(r) \underbrace{\iint da}_{\text{lateral area}} = E(r)\, 2\pi r L = \frac{Q_{enc}}{\epsilon_0}$$

$$= \frac{\lambda L}{\epsilon_0}$$

$$\vec{E}(r) = \frac{\lambda}{2\pi \epsilon_0 r} \hat{e}_s$$

$V(r) = ?$ with convention $V(\infty) = 0$

$$V(r) = \frac{-\lambda}{2\pi\epsilon_0} \ln(r) \leftarrow$$

$$V(b) - V(a) = \frac{-\lambda}{2\pi\epsilon_0}\left[\ln(b) - \ln(a)\right]$$

$$\Delta V = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$