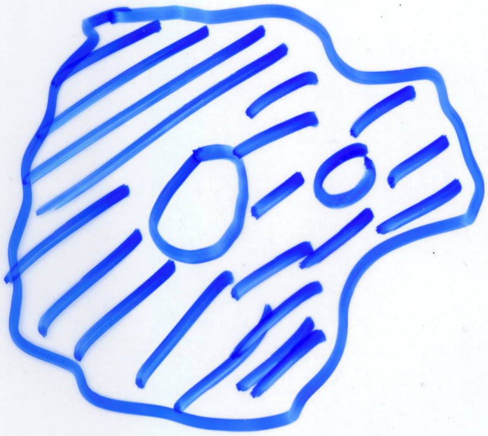
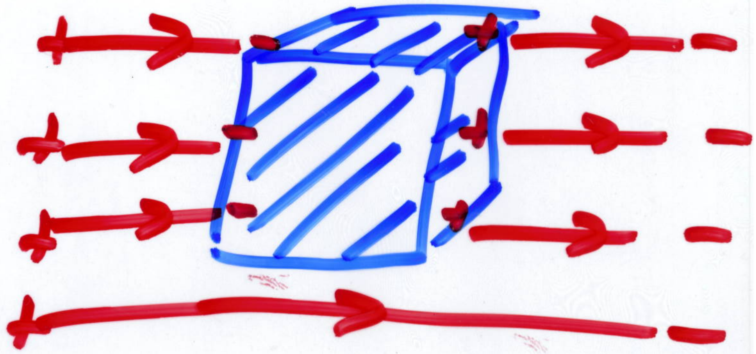
Conductors

a material in which electrons can move freely.

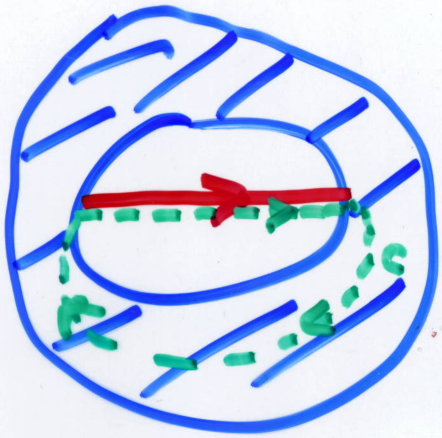
1) In the bulk of a conductor, the electric field is zero.





2) A closed hollow conductor will shield its interior from electric fields due to charges outside the conductor.

2) Proof: Assume there is a field line in the hollow.



$\vec{\nabla}\times\vec{E} = 0$

$\Rightarrow \oint_C \vec{E}\cdot d\vec{l} = 0$

$\Rightarrow \int_A^B \vec{E}\cdot d\vec{l}$ independent of path

$\Rightarrow$ There is a potential

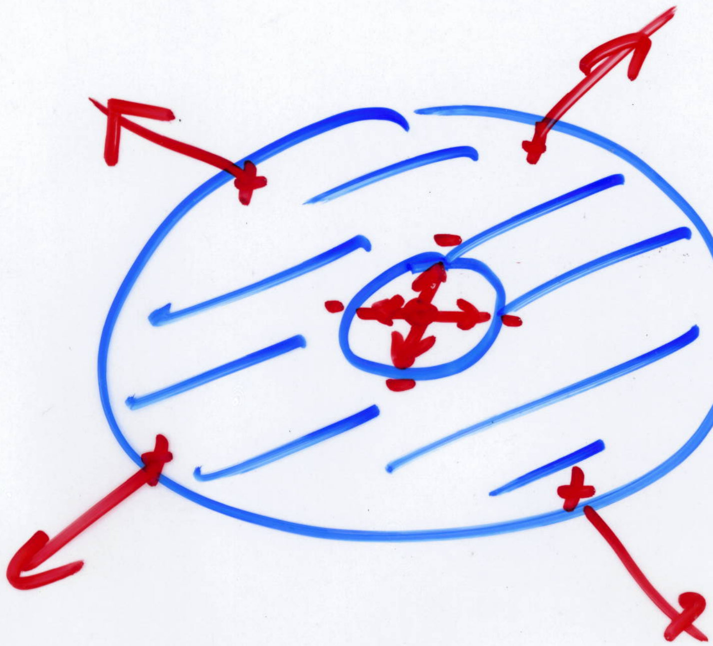
$\vec{E}(\vec{r}) = -\vec{\nabla} V(\vec{r})$

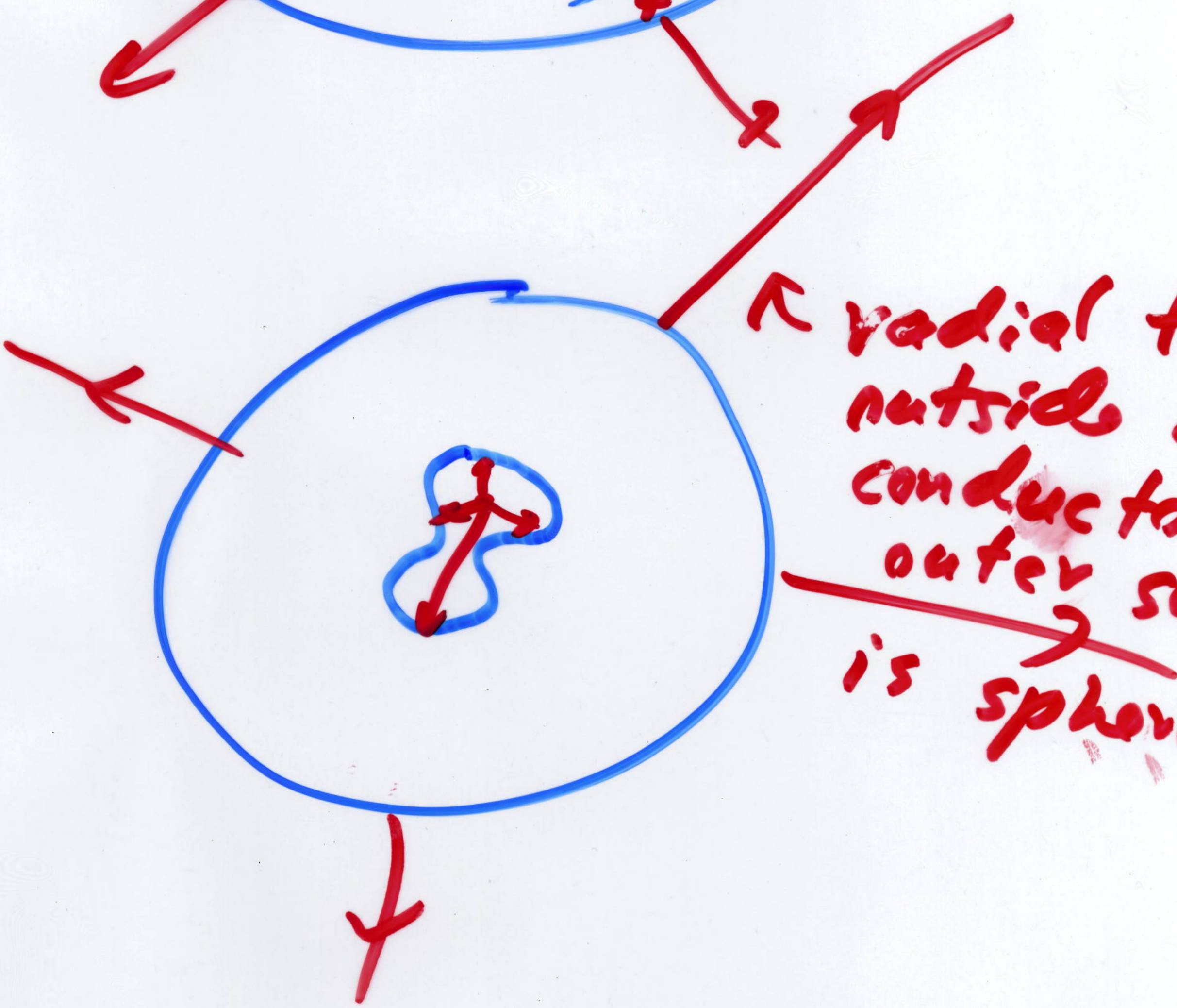
$$\oint_C \vec{E}\cdot d\vec{l} = 0 = \int_{\substack{\text{along}\\\text{field}\\\text{line}}} \vec{E}\cdot d\vec{l} + \int_{\substack{\text{in}\\\text{bulk}\\\text{of conductor}}} \underbrace{\vec{E}}_{0}\cdot d\vec{l}$$

$$0 = \int_{\substack{\text{Along}\\\text{line}}} |\vec{E}||d\vec{l}|$$

So no red field line as drawn.

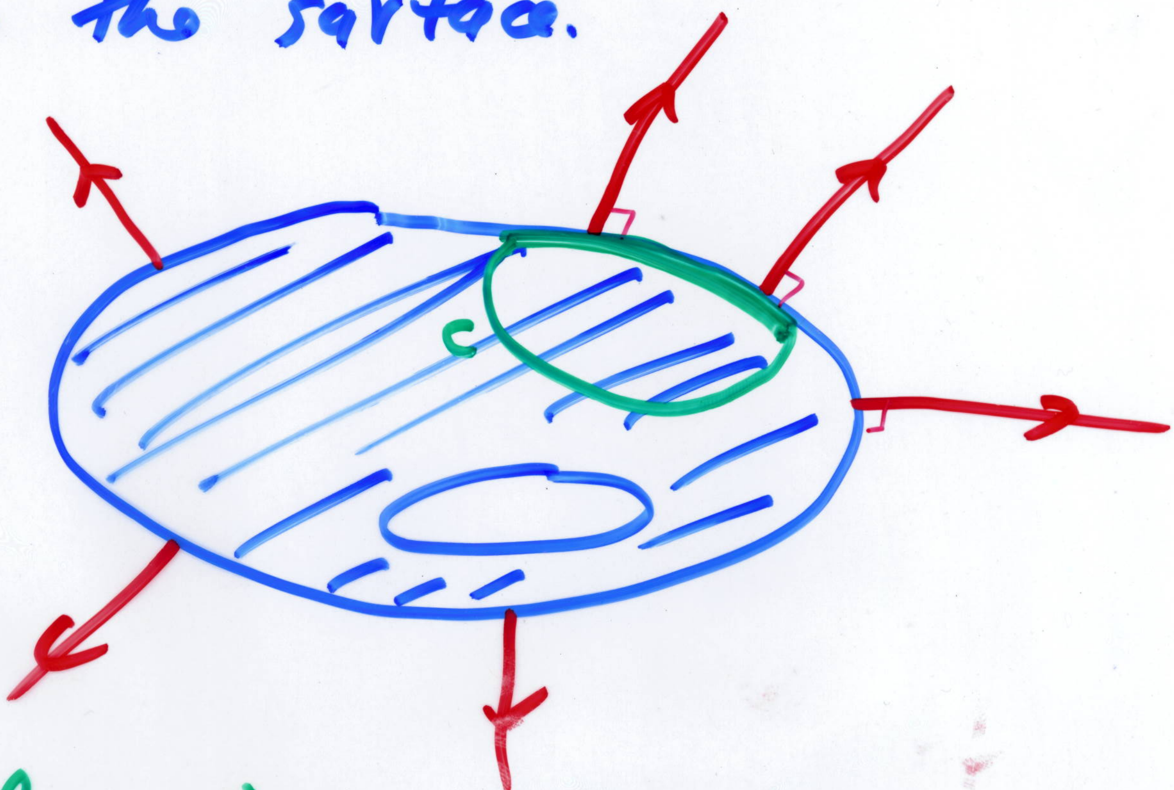
3) A hollow conductor will <u>not</u> shield the outside from charges placed in the cavity.





radial field outside if conductor's outer surface is spherical.

4) The electric field at the surface of a conductor is perpendicular to the surface.



$$\oint_C \vec{E}\cdot d\vec{l} = 0 = \int\limits_{\text{along surface}} \vec{E}\cdot d\vec{l} + \int\limits_{\text{in bulk}} \underbrace{\cancel{\vec{E}}}_{0}\cdot d\vec{l}$$

$\Rightarrow \vec{E}\cdot d\vec{l} = 0$ on surface

$\vec{E}$ is normal to surface

Uniqueness Theorem for $V(\vec{r})$

Divergence (Green)(Gauss) Theorem

$$\iiint_V \vec{\nabla}\cdot\vec{B}(\vec{r})\,dV = \oiint_S \vec{B}(\vec{r})\cdot d\vec{a}$$

Choose $\vec{B}(\vec{r}) = V(\vec{r})\left[\vec{\nabla} V(\vec{r})\right]$

$$\iiint_V \left[V(\vec{r})\,\nabla^2 V(\vec{r}) + \left(\vec{\nabla} V(\vec{r})\right)\left(\vec{\nabla} V(\vec{r})\right)\right] dV$$

$$= \oiint_S V(\vec{r})\left(\vec{\nabla} V(\vec{r})\cdot d\vec{a}\right)$$

Assume two solutions $V_1(\vec{r})$ and $V_2(\vec{r})$

$V(\vec{r})$ on S also known (Boundary conditions)

Poisson's Equation

$$\left.\begin{aligned} \nabla^2 V_1(\vec{r}) &= -\frac{\rho(\vec{r})}{\epsilon_0} \\ \nabla^2 V_2(\vec{r}) &= -\frac{\rho(\vec{r})}{\epsilon_0} \end{aligned}\right\} \text{inside } S$$

$$V_1(\vec{r}) = V_2(\vec{r}) = V(\vec{r}) \quad \text{on } S$$

Superposition $\Rightarrow C(\vec{r}) = V_1(\vec{r}) - V_2(\vec{r})$

$$\nabla^2 C(\vec{r}) = \nabla^2 V_1 - \nabla^2 V_2 = \frac{-\rho}{\epsilon_0} + \frac{\rho}{\epsilon_0} = 0$$

$\nabla^2 C = 0$ Laplace's Eq.

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on Boundary S, $C = V_1 - V_2 = V - V$

$$C = 0$$

$$\iiint_V \left[C \cancel{\nabla^2 C}^{\,0} + (\vec{\nabla} C)\cdot(\vec{\nabla} C) \right] dV = \oint\!\!\oint_S \cancel{C}^{\,0} \left[\vec{\nabla} C \cdot d\vec{a} \right]$$

$$\iiint_V |\vec{\nabla} C|^2 \, dV = 0$$

$$\Longrightarrow \vec{\nabla} C = 0$$

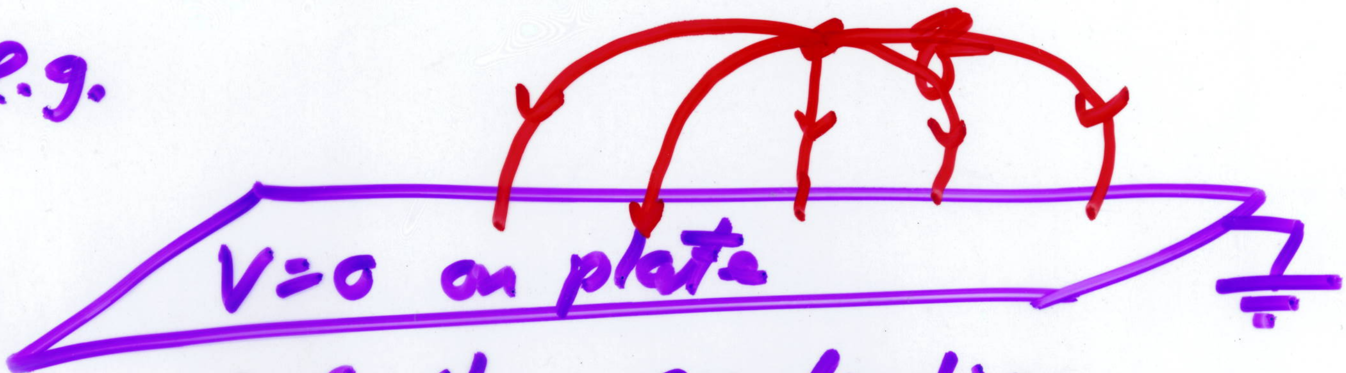
$$\Longrightarrow \vec{\nabla} V_1(\vec{r}) = \vec{\nabla} V_2(\vec{r})$$

$$\Longrightarrow V_1(\vec{r}) = V_2(\vec{r}) + \underline{\text{Const}}.$$

Method of Images

(for finding $V(\vec{r})$)

e.g.



infinite conducting grounded plate