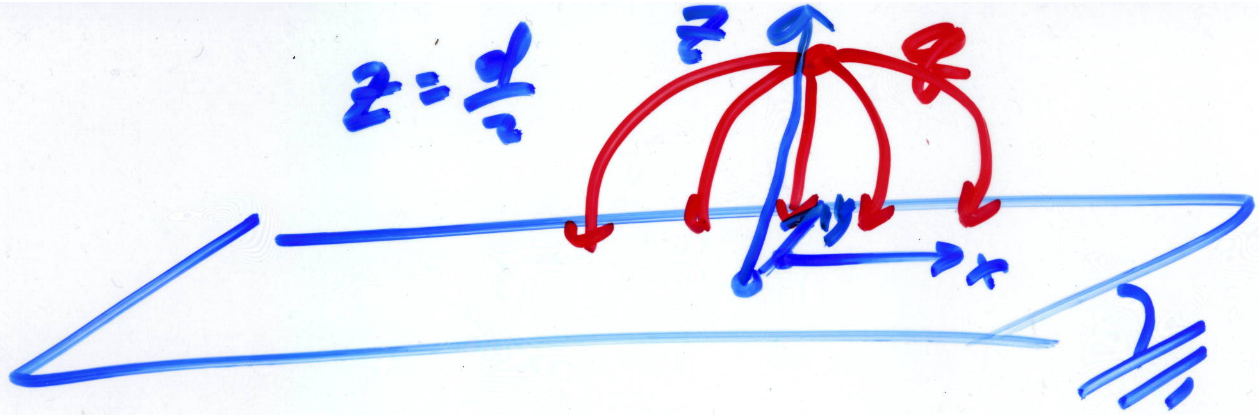


Solve Poisson's Equation in the upper half space ($z > 0$)

$$\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\epsilon_0}$$

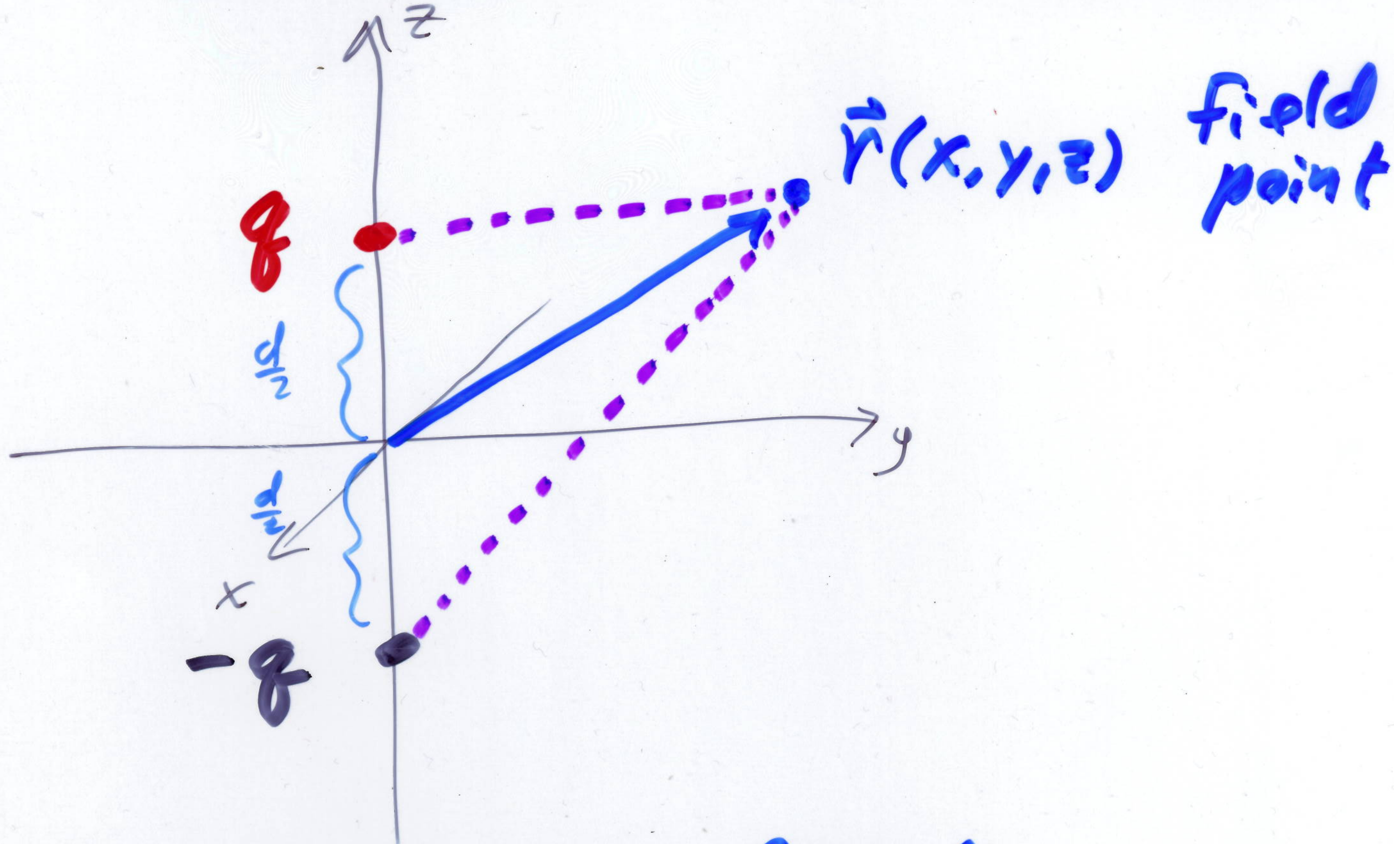
$$\rho(\vec{r}) = q\,\delta(x)\,\delta(y)\,\delta(z - \tfrac{d}{2})$$

$$= q\,\delta(x-0)\,\delta(y-0)\,\delta(z - \tfrac{d}{2})$$

Dimensions

$$\frac{Q}{L^3} = Q \cdot \frac{1}{L} \cdot \frac{1}{L} \cdot \frac{1}{L}$$

$$\int_{x=-\infty}^{+\infty} \delta(x-a)\,dx = 1$$



$$\rho(\vec{r}\,') = q\,\delta(x')\,\delta(y')\,\delta\!\left(z'-\frac{d}{2}\right) - q\,\delta(x')\,\delta(y')\,\delta\!\left(z'+\frac{d}{2}\right)$$

$$V(\vec{r}) = \underset{\substack{\text{All}\\\text{space}}}{\iiint} \frac{k\,\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,dV'$$

$$V(x,y,z) = \frac{kq}{\sqrt{x^2+y^2+\left(z-\frac{d}{2}\right)^2}} - \frac{kq}{\sqrt{x^2+y^2+\left(z+\frac{d}{2}\right)^2}}$$

satisfies Poission $\nabla^2 V = \frac{-\rho}{\epsilon_0}$, BCs

$\partial^2/\partial x^2 + \frac{\partial^2}{\partial y^2} + \partial^2/\partial z^2\, V(x,y,z) = \rho/\epsilon_0$

Poisson's Eq. $\nabla^2 V(\vec{r}) = \dfrac{-\rho(\vec{r})}{\epsilon_0}$

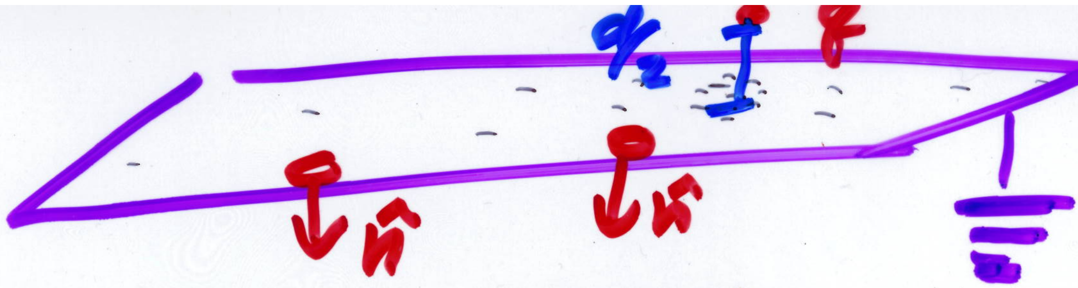
2nd order, linear, <u>non</u>-homogeneous partial differential equation.

Need to solve Laplace's Eq.

$$\nabla^2 V(\vec{r}) = 0$$

with boundary conditions

$$V(x, y, 0) = 0 \; ; \; V(x, y, \infty) = 0$$



$z=0$

surface charge density $\sigma(x,y)$

$$V(x,y,z) = \frac{kq}{\sqrt{x^2+y^2+(z-\frac{d}{2})^2}} - \frac{kq}{\sqrt{x^2+y^2+(z+\frac{d}{2})^2}}$$

Electric field

$$\vec{E}(x,y,z) = -\vec{\nabla}V(x,y,z)$$

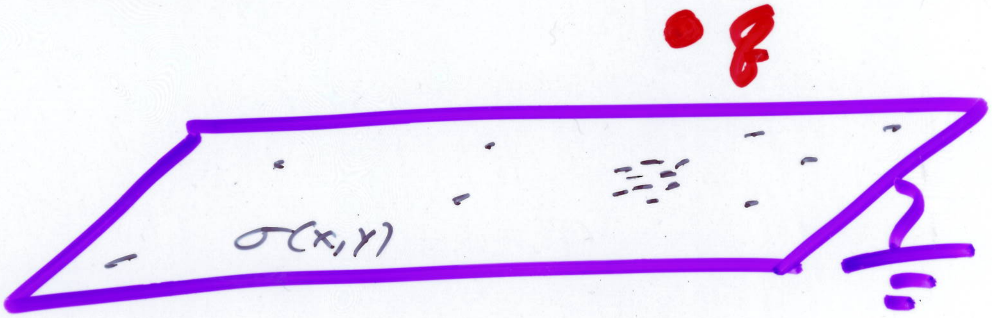
$$\hat{n}\cdot\vec{E}\Big|_{z=0} = \frac{\sigma}{\epsilon_0}$$

$$\hat{n} = -\hat{k} = -\hat{e}_z = -\hat{e}_3$$

$$\sigma(x,y) = \frac{-qd}{2\pi\left(\sqrt{x^2+y^2+d^2}\right)^3}$$

$$\int_{x=-\infty}^{+\infty}\int_{y=-\infty}^{+\infty}\sigma(x,y)\,dx\,dy = -q \checkmark$$

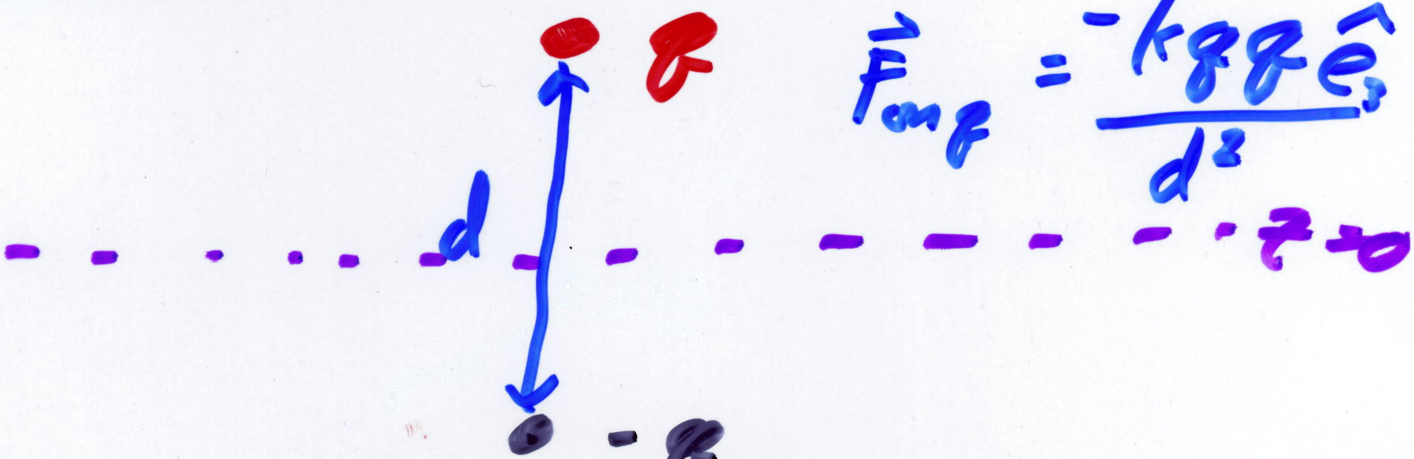
Find the force on real charge q



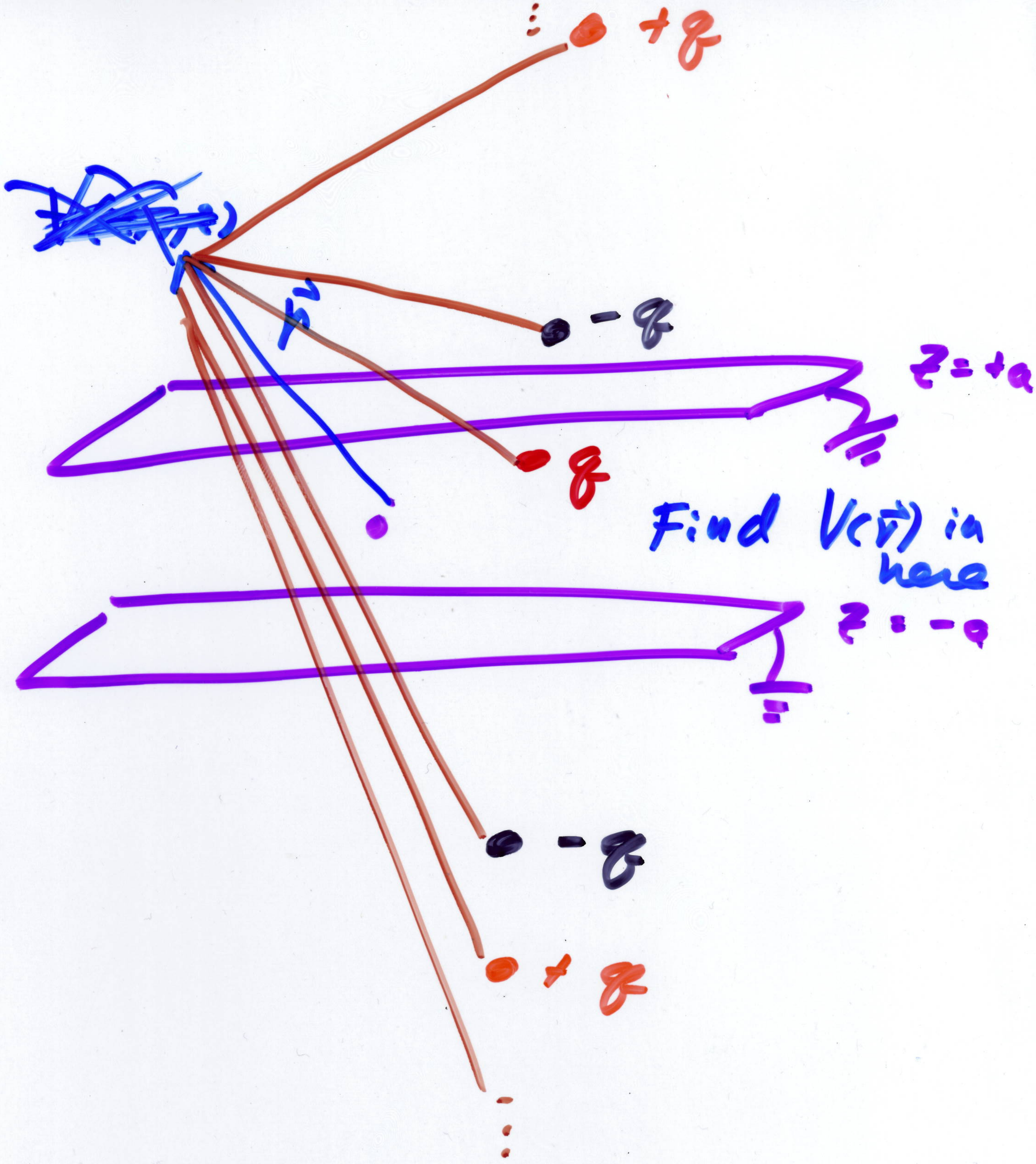
Hard way

$$\vec{F}_{on\,q} = \sum_{i=1}^{N} \frac{k q q_i (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

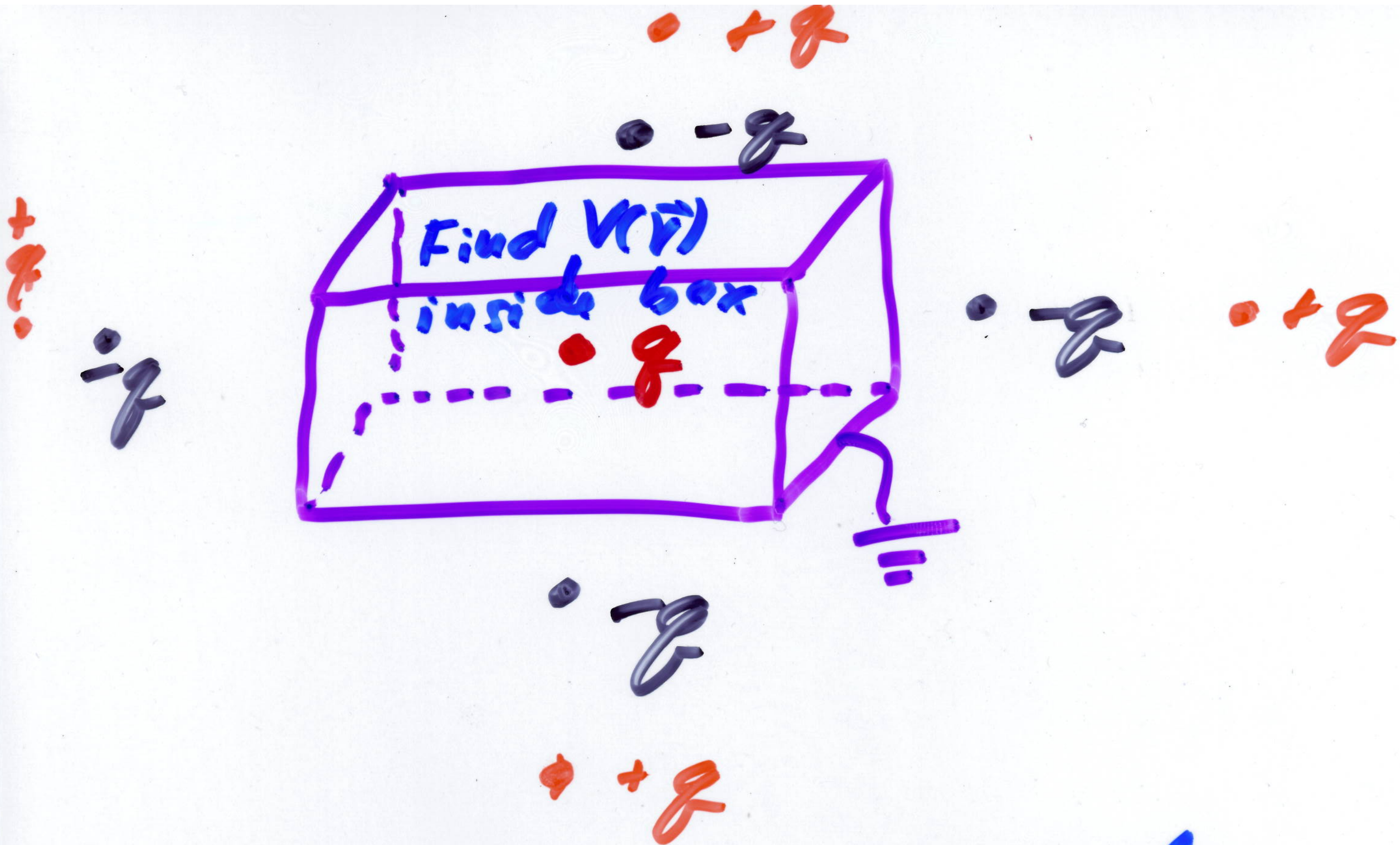
$$\rightarrow \iint_{\substack{z=0 \\ \text{plane}}} \frac{k q \, \sigma(x',y') \, dx' dy' (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$



$$\vec{F}_{on\,q} = \frac{-k q q \, \hat{e}_z}{d^2}$$



+q
−q
$\vec{r}$
z = +a
q
Find $V(\vec{r})$ in here
z = −a
−q
+q



$$\nabla^2 V(\vec{r}) = \frac{-\rho}{\epsilon_0}$$

$\leftarrow$ real, in box