Electric Potential for a point dipole at the origin $\vec{r} = \vec{r} - 0$

$$V(\vec{r}) = \frac{k\,\vec{p}\cdot\vec{r}}{r^3} = \frac{k\,p\cos\theta}{r^2}$$

Point dipole at source point $\vec{r}\,'$

$$V(\vec{r}) = \frac{k\,\vec{p}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} = \frac{k\,\vec{p}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}$$

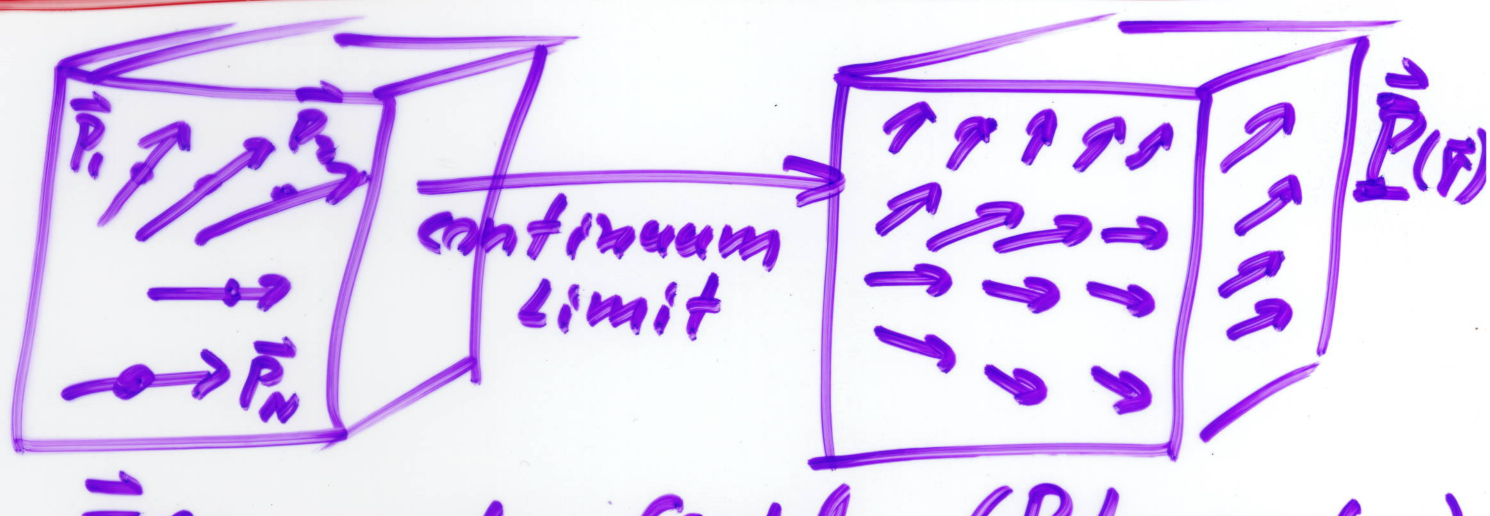
Griffiths: $\vec{\mathscr{r}} \equiv \vec{r} - \vec{r}\,'$

$$\hat{\mathscr{r}} \equiv \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|}$$

Many point dipoles $\vec{p}_i$ at source points $\vec{r}_i\,'$, $i = 1 \ldots N$

$$V(\vec{r}) = \sum_{i=1}^{N} \frac{k\,\vec{p}_i\cdot(\vec{r}-\vec{r}_i\,')}{|\vec{r}-\vec{r}_i\,'|^3}$$

$\rho(\vec{r}')$ scalar field
monopole density
= charge per unit volume



$\vec{P}(\vec{r})$ vector field (Polarization)
dipole density
= dipole moment per unit volume.

Potential due to Polarization field

$$V(\vec{r}) = \iiint \frac{k\,\vec{P}(\vec{r}\,')\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, d\tau'$$

volume element $dx'dy'dz'$

$$\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3} = -\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}\,'|} = +\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}\,'|}$$

with respect to field coordinate $\hat{e}_x\frac{\partial}{\partial x} + \hat{e}_y\frac{\partial}{\partial y} + \ldots$

w.r.t. source coordinates $\hat{e}_x\frac{\partial}{\partial x'} + \hat{e}_y\frac{\partial}{\partial y'} + \ldots$

$$V(\vec{r}) = \iiint k\,\vec{P}(\vec{r}\,')\cdot\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}\,'|}\, d\tau'$$

$$= \iiint k\,\vec{\nabla}'\cdot\left(\frac{\vec{P}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\right)d\tau' - \iiint k\,\frac{\vec{\nabla}'\cdot\vec{P}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\,d\tau'$$

Use Green's Thm

$$V(\vec{r}) = \oint\!\!\int \frac{k\,\hat{n}'\cdot\vec{P}(\vec{r}')}{|\vec{r}-\vec{r}'|}\,da'$$

$$+ \iiint \frac{k\left(-\vec{\nabla}'\cdot\vec{P}(\vec{r}')\right)}{|\vec{r}-\vec{r}'|}\,d\tau'$$

This looks like

$$V(\vec{r}) = \iint \frac{k\,\sigma(\vec{r}')}{|\vec{r}-\vec{r}'|}\,da'$$

$$+ \iiint \frac{k\,\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}\,d\tau'$$

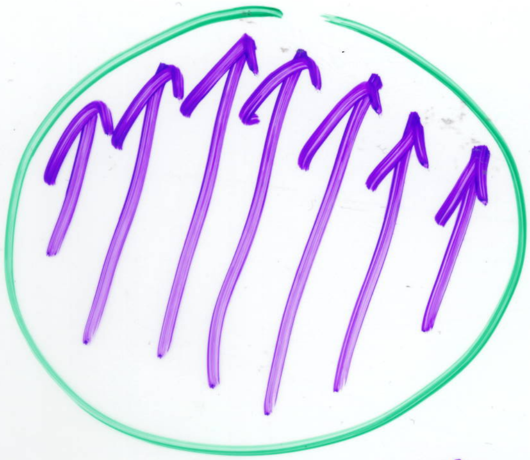
Bound Surface charge density is

$$\sigma_b(\vec{r}') = \hat{n}'\cdot\vec{P}(\vec{r}')$$

Bound volume charge density is

$$\rho_b(\vec{r}') = -\vec{\nabla}'\cdot\vec{P}(\vec{r}')$$

Problem: Sphere of ~~constant~~ uniform Polarization Find $V(\vec{r})$ everywhere



$\vec{P}(\vec{r}')$

$$\rho_b(\vec{r}') = -\vec{\nabla}' \cdot \vec{P}(\vec{r}') = 0$$

$$\sigma_b(\vec{r}') = \hat{r}' \cdot \vec{P}(\vec{r}') = P\cos\theta$$

similar to problem already solved.

inside $V(r,\theta) = E_0 r\cos\theta \rightarrow \frac{P}{3\epsilon_0} r\cos\theta$

$$= \frac{P}{3\epsilon_0} z$$

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outside

$$V(r,\theta) = E_0 \frac{R^3}{r^2}\cos\theta \rightarrow \frac{P}{3\epsilon_0}\frac{R^3}{r^2}\cos\theta$$