Boundary Condition for Linear Dielectrics.

$$\vec{\nabla}\times\vec{E} = 0$$

$$0 = \iiint_V \vec{\nabla}\times\vec{E}\, dV = \oiint_S \hat{n}\times\vec{E}\, da$$ Stokes' Theorem

$$= \oiint_S \left[\hat{n}_1\times\vec{E}_1 + \hat{n}_2\times\vec{E}_2\right] da = 0$$

$$\Rightarrow \hat{k}\times(\vec{E}_1 - \vec{E}_2)\, 0 \Longrightarrow \boxed{\begin{matrix} E_{1y} = E_{2y} \\ E_{1x} = E_{2x} \end{matrix}}$$

$$\not\Rightarrow E_{1z} = E_{2z}$$

The tangential components of the electric field are continuous across the boundary.

$$\vec{\nabla}\cdot\vec{D} = \rho_{free} = 0$$ if no free charge in problem

$$0 = \iiint_V \vec{\nabla}\cdot\vec{D}\,dV = \oiint_S \hat{n}\cdot\vec{D}\,da$$

Gauss, Green, Divergence Theorem

$$= \oiint(\hat{n}_1\cdot\vec{D}_1 + n_2\cdot\vec{D}_2)\,da = 0$$

$$\hat{k}\cdot(\vec{D}_1 - \vec{D}_2) \Longrightarrow \boxed{D_{1z} = D_{2z}}$$

The normal components of the displacement are continuous across the boundary.

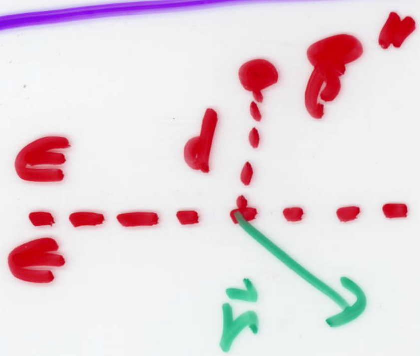
Find $V(\vec{r})$ in the upper half space.

$\vec{r}_1{}' = (0,0,d)$ $\quad$ $\vec{r}_2{}' = (0,0,-d)$

$$V_1(\vec{r}) = \frac{kq}{|\vec{r}-\vec{r}_1{}'|} + \frac{kq'}{|\vec{r}-\vec{r}_2{}'|}$$

$$V_1(\vec{r}) = \frac{kq}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{kq'}{\sqrt{x^2+y^2+(z+d)^2}}$$

for $z \geq 0$



$$V_2(\vec{r}) = \frac{kq''}{\frac{\epsilon}{\epsilon_0}\sqrt{x^2+y^2+(z-d)^2}}$$

$$V_2(\vec{r}) = \frac{1}{4\pi\epsilon}\,\frac{q''}{\sqrt{x^2+y^2+(z-d)^2}}$$

not ϵ_0

for $z \leq 0$

Apply the boundary condition

$$E_{1x} = E_{2x} \Longrightarrow V_1 = V_2$$

On the interface $z=0$.

$$V_1\Big|_{z=0} = \frac{kq}{\sqrt{x^2+y^2+d^2}} + \frac{kq'}{\sqrt{x^2+y^2+d^2}}$$

$$V_2\Big|_{z=0} = \frac{kq''}{\frac{\epsilon}{\epsilon_0}\sqrt{x^2+y^2+d^2}}$$

$$\boxed{q + q' = \frac{q''}{\epsilon/\epsilon_0}}$$

Now we need the displacement.

In region 1 ($z \geq 0$) $\vec{D} = \epsilon_0 \vec{E} = -\vec{\nabla} V_1 \epsilon_0$

In region 2 ($z \leq 0$) $\vec{D} = \epsilon \vec{E} = \epsilon(-\vec{\nabla} V_2)$

$$D_{1z} = D_{2z}$$

$$\epsilon_0 \frac{\partial}{\partial z} V_1 \bigg|_{z=0} = -\epsilon \frac{\partial}{\partial z} V_2 \bigg|_{z=0}$$

$$\frac{kq(-d)}{(x^2+y^2+d^2)^{3/2}} + \frac{kq'(d)}{(x^2+y^2+d^2)^{3/2}} = \frac{kq''(-d)}{(x^2+y^2+d^2)^{3/2}}$$

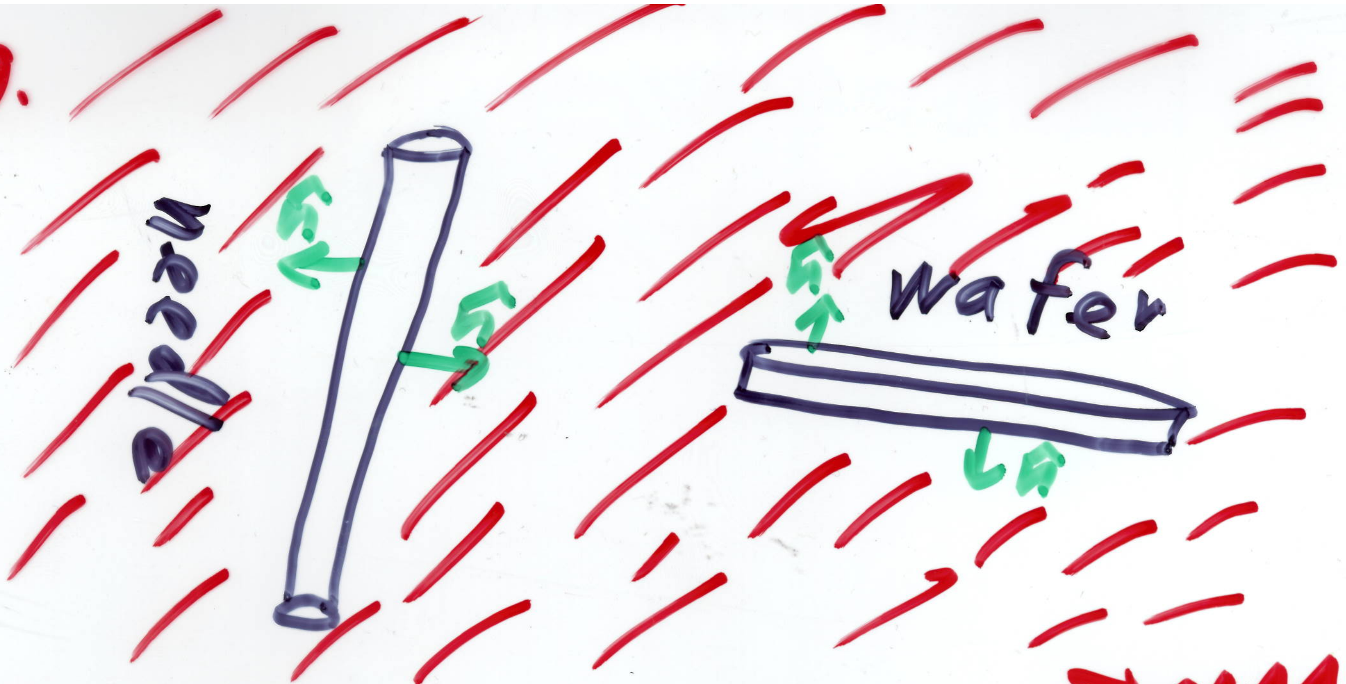
$$\boxed{q - q' = q''}$$

$q' = -\frac{\epsilon - \epsilon_0}{\epsilon + \epsilon_0} q$ | $q'' = \frac{2\epsilon}{\epsilon + \epsilon_0} q$

metalic limit

$\epsilon \to \infty$ $q' \to -q$ ✓

E.g.



Uniform electric field $\vec{E}_0 \uparrow\uparrow\uparrow\uparrow\uparrow$

Uniform polarization field $\vec{P}_0 \uparrow\uparrow\uparrow\uparrow$

Uniform displacement field $\vec{D}_0 = \epsilon_0 \vec{E}_0 + \vec{P}_0$

$\uparrow\uparrow\uparrow\uparrow\uparrow$

Find $\vec{E}_{in}$, $\vec{P}_{in}$, and $\vec{D}_{in}$.

needle	wafer
$\vec{E}_{in} = \vec{E}_0$	$\vec{D}_{in} = \vec{D}_0$
$\vec{P}_{in} = 0$	$\vec{P}_{in} = 0$
$\vec{D}_{in} = \epsilon_0 \vec{E}_{in} + \cancelto{0}{\vec{P}_{in}}$	$\vec{E}_{in} = \dfrac{\vec{D}_{in} - \cancelto{0}{\vec{P}_{in}}}{\epsilon_0}$
$\vec{D}_{in} = \underline{\epsilon_0 \vec{E}_0} = \underline{\vec{D}_0 - \vec{P}_0}$	$\vec{E}_{in} = \underline{\dfrac{\vec{D}_0}{\epsilon_0}} = \underline{\vec{E}_0 + \dfrac{\vec{P}_0}{\epsilon_0}}$