Electromagnetic Theory

PHYS 4392 — After Spring Break Review

Spring 2026 Study Guide

1. Electrostatics Review

This section covers the foundational relationships between electrostatic potentials, electric fields, and the partial differential equations that govern them. We work exclusively in
MKS (SI) units: meter, kilogram, second.

Key Definitions

Symbol / Term	Description
$V(r)$	Electrostatic scalar potential — a scalar field that is a function of position r
$E(r)$	Electric field vector — obtained from the negative gradient of the potential
$\rho(r)$	Charge density distribution as a function of position
ε_0	Permittivity of free space (8.854×10^{-12} F/m)

Fundamental Relations

Electric field from potential:

$$E(r) = -\nabla V(r)$$

The electric field points in the direction of steepest decrease of the potential.

Poisson's Equation (non-homogeneous):

$$\nabla^2 V(r) = -\rho(r) / \varepsilon_0$$

This relates the potential to the source charge distribution. When charges are present, the Laplacian of V is non-zero.

Laplace's Equation (homogeneous, charge-free regions):

$$\nabla^2 V(r) = 0$$

This is a 2nd-order, linear, homogeneous partial differential equation. The general solution involves two arbitrary constants determined by boundary conditions.

Physical Analogy: Soap Film on a Wire Frame

Consider a soap film stretched between inner and outer wire frames (viewed from above).

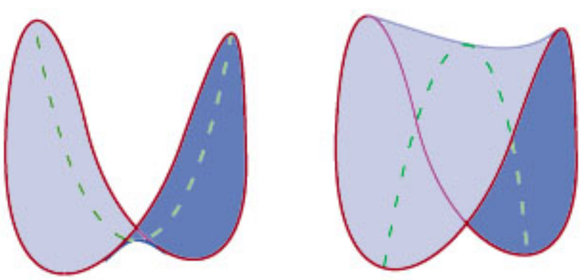
The surface the soap film forms minimizes its potential energy — it follows the path of least potential.

This is directly analogous to the geometry of the electrostatic potential V(x,y) satisfying the Laplace equation.

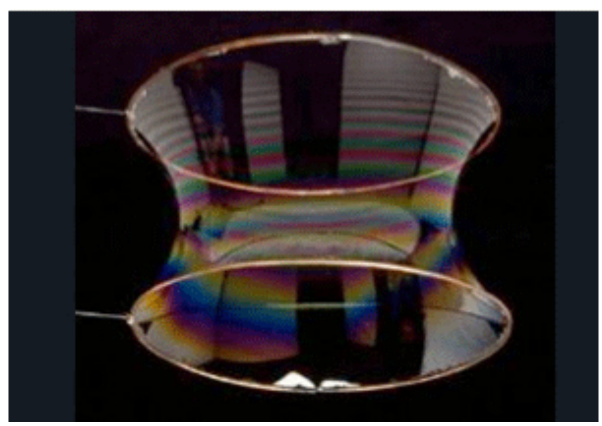
Note: This analogy holds approximately when the boundary conditions are not too extreme ("wild").

Unit Systems

Symbol / Term	Description
MKS (SI)	meter, kilogram, second — used throughout this course
CGS	centimeter, gram, second — common in older texts and some subfields



A curve and two possible saddle-shaped film surfaces



2. Connections to Earlier Coursework

The equations in electrodynamics build on concepts from introductory physics. Here is how the key ideas connect.

From Mechanics (PHYS 1303)

Newton's Second Law:

$$\Sigma F = ma = m\, d^2r(t)/dt^2$$

This is a 2nd-order linear ODE — the same mathematical structure appears throughout electromagnetic theory, where the field equations are 2nd-order PDEs.

From Electricity & Magnetism (PHYS 1304)

Lorentz Force Law (force on an electric charge q):

$$F_q = qE + q(v \times B)$$

Here q is a point electric charge (the electric monopole). The first term is the electric force; the second is the magnetic force on a moving charge.

3. Time-Dependent Fields & Magnetic Monopoles

Time-Dependent Fields

In full electrodynamics, both the electric and magnetic fields depend on position and time:

$$E(r, t) \quad \text{and} \quad B(r, t)$$

These fields are always coupled — they depend on the same variables and evolve together. This coupling is central to electromagnetic wave propagation and radiation.

Antennas and Radiation

When charges accelerate (e.g., oscillating in an antenna), they radiate electromagnetic waves. The antenna structure (HF, UHF, etc.) determines what frequencies are emitted and received. Key idea: the radiation fields E and B are always perpendicular to each other and to the direction of propagation.

Magnetic Monopoles (Hypothetical)

By analogy with the electric monopole (point charge q), we can introduce a hypothetical magnetic monopole with magnetic charge g. The force on such a charge would be:

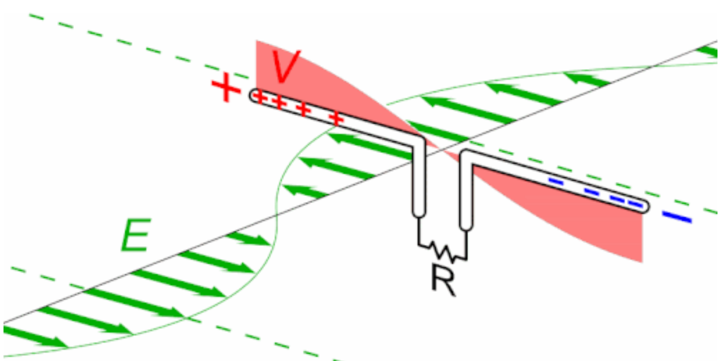
$$F_g = gB - g(v \times E)/c^2$$

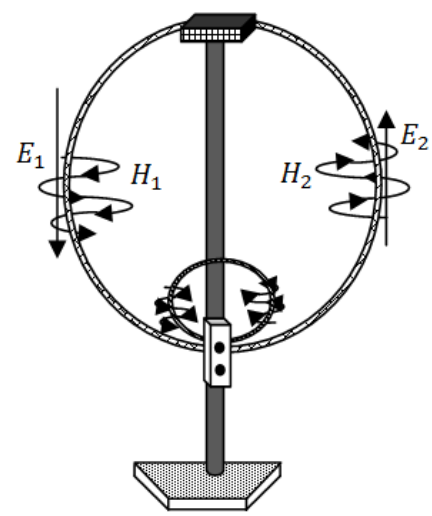
This is the magnetic dual of the Lorentz force law. Note the sign difference and the factor of $1/c^2$.

Units of Magnetic Charge

[q] = coulombs (C)
[g] = ampere·meters (A·m)
Note: The unit of g is NOT webers (Wb = T·m²).

While magnetic monopoles have not been experimentally observed, Dirac showed that their existence would explain the quantization of electric charge. The field lines of a magnetic monopole radiate outward from a “north pole” just as electric field lines radiate from a positive charge.





4. Maxwell's Equations (Differential Form)

Maxwell's equations unify all of classical electrodynamics into four coupled PDEs. In MKS (SI) units and differential form:

Equation Name	Differential Form	Physical Meaning
Gauss's Law (E)	$\nabla \cdot E(r,t) = \rho(r) / \varepsilon_0$	Electric charges produce electric field divergence
Gauss's Law (B)	$\nabla \cdot B(r,t) = 0$	No magnetic monopoles (in standard EM)
Faraday's Law	$\nabla \times E(r,t) = -\partial B(r,t)/\partial t$	Time-varying B fields induce E fields
Ampère–Maxwell Law	$\nabla \times B = \mu_0 J(r) + \mu_0\varepsilon_0 \partial E(r,t)/\partial t$	Currents and displacement current produce B fields

The Displacement Current

The last term in the Ampère–Maxwell law deserves special attention:

$$\mu_0 \varepsilon_0 \partial E(r,t)/\partial t \quad \leftarrow \text{displacement current density}$$

Maxwell's key insight was adding this displacement current term. Without it, Ampère's law is inconsistent with charge conservation. With it, the equations predict electromagnetic waves propagating at speed $c = 1/\sqrt{(\mu_0\varepsilon_0)}$.

Key Takeaways

1. Gauss's Law (E): Charges are sources of E-field divergence.
2. Gauss's Law (B): There are no magnetic charges ($\nabla \cdot B = 0$ always).
3. Faraday's Law: Changing B fields produce curling E fields.
4. Ampère–Maxwell: Currents AND changing E fields produce curling B fields.
5. The displacement current completes the symmetry and enables wave solutions.

Quick Reference: Equation Summary

Electrostatics

$$E = -\nabla V$$

$$\nabla^2 V = -\rho/\varepsilon_0 \quad \text{(Poisson)}$$

$$\nabla^2 V = 0 \quad \text{(Laplace)}$$

Mechanics & Forces

$$\Sigma F = m\, d^2r/dt^2 \quad \text{(Newton)}$$

$$F = qE + q(v \times B) \quad \text{(Lorentz)}$$

$$F = gB - g(v \times E)/c^2 \quad \text{(Magnetic monopole, hypothetical)}$$

Maxwell's Equations

$$\nabla \cdot E = \rho/\varepsilon_0$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\partial B/\partial t$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0\, \partial E/\partial t$$