PHYS 4392 - Lecture Notes

March 27, 2026 | Southern Methodist University

> ***Homework Policy:*** *Homework is due by* ***2:00 PM every Wednesday****. Submit via email to scalise@smu.edu.*

1 Recap: Generalized Lorentz Force Law

Consider a particle (sometimes called a *dyon*) with mass *m*, electric charge *q*, and magnetic charge *g*. The equation of motion for such a particle in electromagnetic fields is given by the generalized Lorentz force law:

$$m\, d^2r / dt^2 = q (E + v \times B) + g (B - v \times E / c^2)$$

Here, d^2r/dt^2 denotes the acceleration (Newton dot notation is also used in class). The first term, $q(E + v \times B)$, is the standard Lorentz force on an electric charge. The second term, $g(B - v \times E/c^2)$, is the analogous force on a hypothetical magnetic monopole.

> ***Note:*** *No magnetic monopoles have been observed in nature to date. However, this formalism is useful for understanding electromagnetic duality between electric and magnetic fields. Magnetic monopoles remain an active area of theoretical and experimental research in high-energy physics.*

2 Can Magnetic Forces Do Work?

A central question in electromagnetism is whether the magnetic force can do work on different types of charges and current distributions. The answer depends on the nature of the object carrying the charge.

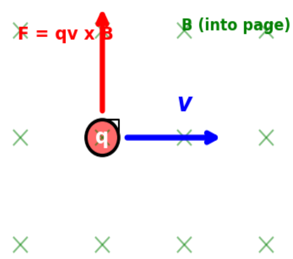
2.1 On Electric Charges (Electric Monopoles, *q*)

The magnetic force on a free electric charge is $F_{mag} = q\, v \times B$. Because the cross product **v x B** is always perpendicular to the velocity **v**, this force can never do work on a free electric charge. We prove this directly:

$$W = \int F \cdot dl = \int q(v \times B) \cdot v\, dt = 0$$

since (v x B) · v = v · (v x B) = 0 (scalar triple product with two identical vectors)

Magnetic Force is Perpendicular to Velocity



F is perpendicular to v, so F . v = 0, therefore W = 0

Key Result: The magnetic force qv x B does no work on free electric charges. It changes the direction of motion but never the speed.

2.2 On Magnetic Charges (Magnetic Monopoles, *g*)

If magnetic monopoles existed, the force on a magnetic charge in a magnetic field would be **F = gB**. Unlike the electric-charge case, this force is *not* necessarily perpendicular to the velocity, so **magnetic forces can do work on magnetic monopoles**.

2.3 On Magnetic Dipoles

Magnetic forces **can do work on magnetic dipoles**. A magnetic dipole in a non-uniform magnetic field experiences a force **F = grad(m · B)**, where **m** is the magnetic dipole moment. This force can have a component along the direction of motion, allowing it to do work.

Example: The Electron

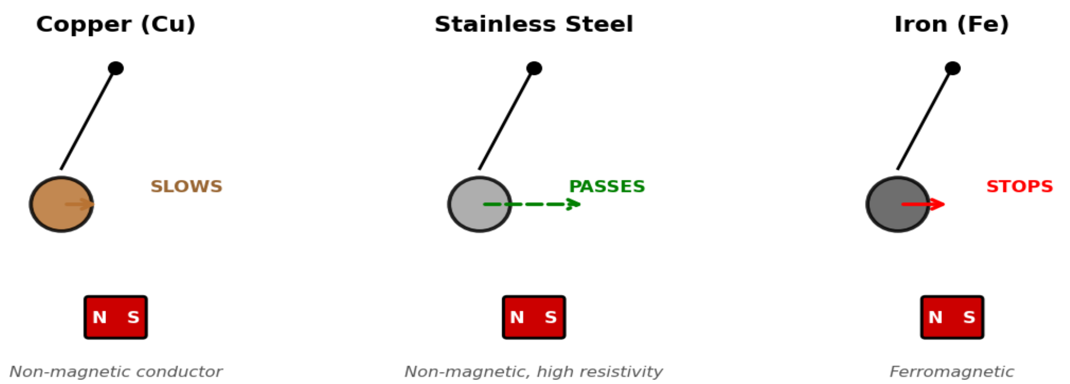
- Electric charge: q = 1.6 x 10^{-19} C
- Magnetic monopole charge: g = 0 (no magnetic charge)
- Rest mass: m = 511 keV/c^2 = 9.109 x 10^{-31} kg
- The electron has a magnetic dipole moment (from intrinsic spin) but no electric dipole moment.

3 In-Class Demonstrations

3.1 Pendulum and Magnet Experiment

A pendulum made of different materials is swung through the pole region of a strong magnet. The behavior depends on the electromagnetic properties of the material:

Pendulum Experiment: Effect of a Strong Magnet on Different Materials



Material	Behavior	Explanation
Copper (Cu)	Slows and stops	Cu is an excellent conductor. A changing magnetic flux through the Cu induces eddy currents (via Faraday's law), which produce a retarding force opposing the motion (Lenz's law).
Stainless Steel (austenitic)	Passes through largely unaffected	Austenitic stainless steel is non-magnetic and has relatively high electrical resistivity, so eddy currents are weak and insufficient to noticeably brake the pendulum.
Iron (Fe)	Stops immediately	Iron is ferromagnetic and strongly attracted to the magnet. The magnetic domains in Fe align with the external field, creating a large net attractive force.

The underlying physics for the copper pendulum is Faraday's law of induction:

curl E = - dB / dt

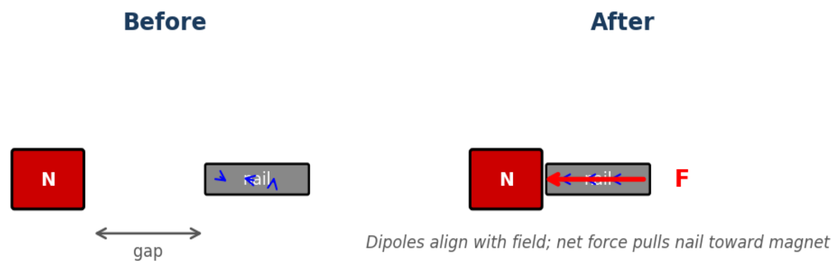
As the conducting pendulum moves through a spatially non-uniform magnetic field, the time-varying flux through the conductor induces an electric field, which drives eddy currents. These currents create their own magnetic field that opposes the change in flux (Lenz's law), resulting in a braking force on the pendulum. Crucially, it is the *induced electric field* that does the work on the charges in the conductor, not the magnetic field directly.

3.2 Nail and Magnet (Magnetic Dipole Interaction)

A nail placed near a magnet is attracted and moves toward it. The individual atomic magnetic dipoles in the nail (which are initially randomly oriented) align with the external field. Once aligned, the gradient of the magnetic field exerts a net force on the dipole:

$$F = \text{grad}(\, m \cdot B\,)$$

Magnetic Force on a Magnetic Dipole (Nail)



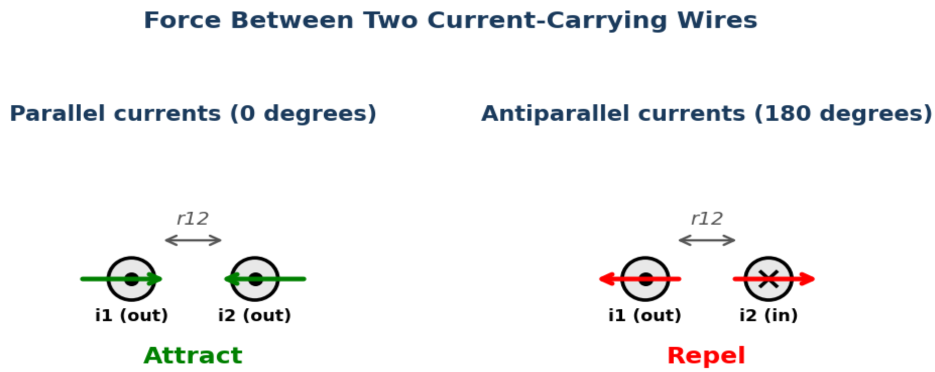
This is another example of a magnetic force doing work on a magnetic dipole in a non-uniform field. The nail accelerates toward the magnet, gaining kinetic energy.

4 Force Between Current-Carrying Wires

In the class experiment, we examined the force that one current-carrying wire exerts on another. For two long, parallel wires of length *l* carrying currents i_1 and i_2, separated by a distance r_{12}, the force per unit length on wire 2 due to wire 1 is:

$$F_{1\ on\ 2} = (\mu_0 / 4\pi) \cdot 2\, l\, i_1\, i_2 \cos(\theta) / r_{12} \quad \text{(in the direction of r-hat}_{21}\text{)}$$

where θ is the angle between the two current directions, and **r-hat$_{21}$** is the unit vector pointing from wire 2 toward wire 1.



Configuration	θ	cos θ	Effect
Parallel currents	0 degrees	+1	Attractive force
Perpendicular currents	90 degrees	0	No force (in this approximation)
Antiparallel currents	180 degrees	-1	Repulsive force

4.1 Analogy to Coulomb's Law

The force between two current-carrying wires is the magnetic analog of Coulomb's law for electric charges:

Coulomb's Law (Electric)	Ampere's Force Law (Magnetic)
$F = (1/4\pi\varepsilon_0)\, q_1 q_2 / r_{12}^2$ (in the direction of r-hat$_{12}$)	$F = (\mu_0/4\pi)\, 2l\, i_1 i_2 \cos\theta / r_{12}$ (in the direction of r-hat$_{21}$)

Like charges repel; unlike charges attract.	Parallel currents attract; antiparallel currents repel.

4.2 Derivation from the Lorentz Force

The force between two wires can be derived from the Lorentz force **F = qv x B**, applied to a current-carrying wire (**F = i l x B**):

Step 1: The magnetic field produced by a long straight wire carrying current i_1 at a perpendicular distance r is:

$$\mathbf{B_1 = (\mu_0 / 4\pi) \cdot 2\, i_1 / r}$$

Step 2: The force on a length l of wire 2 (carrying current i_2) in the field of wire 1 is:

$$\mathbf{F = i_2\, l \cdot B_1 = (\mu_0 / 4\pi) \cdot 2\, l\, i_1\, i_2 / r_{12}}$$

This recovers exactly the force law stated above for the case of parallel wires ($\theta = 0$). The $\cos(\theta)$ factor arises when the currents are not parallel.

5 Key Constants

Quantity	Symbol	Value
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T · m / A
Permittivity of free space	ε_0	8.854×10^{-12} F / m
Speed of light	c	2.998×10^{8} m/s
Coulomb constant	k_e	8.988×10^{9} N · m^2 / C^2
Elementary charge	e	1.602×10^{-19} C
Electron mass	m_e	511 keV/c^2 = 9.109×10^{-31} kg

6 Summary of Key Takeaways

1. The generalized Lorentz force for a dyon (electric charge q, magnetic charge g) is: **$F = q(E + v \times B) + g(B - v \times E/c^2)$**.

2. Magnetic forces do **no work** on free electric charges, since **$F = qv \times B$** is always perpendicular to **v**.

3. Magnetic forces **can** do work on magnetic monopoles (**$F = gB$**) and on magnetic dipoles (**$F = \mathrm{grad}(m \cdot B)$**).

4. Eddy current braking (as in the Cu pendulum) is ultimately driven by the *induced electric field* from Faraday's law, not by the magnetic field directly.

5. The force between current-carrying wires, **$F = (\mu_0/4\pi)(2l\, i_1 i_2 \cos\theta / r_{12})$**, is the magnetic analog of Coulomb's law.

6. Parallel currents attract; antiparallel currents repel.