Magnetic Dipoles, Polarization, and the Auxiliary Field *H*

PHYS 4392 — Electromagnetism | Lecture of April 17, 2026

1. Three Constructions of a Dipole

There are three idealized constructions that produce a point dipole as a limit of a finite charge or current configuration. The electric dipole is built from two opposite charges; the Gilbert magnetic dipole is its hypothetical analogue using magnetic monopoles; and the Ampèrian dipole is built from a current loop, which is the only one that corresponds to physical reality.

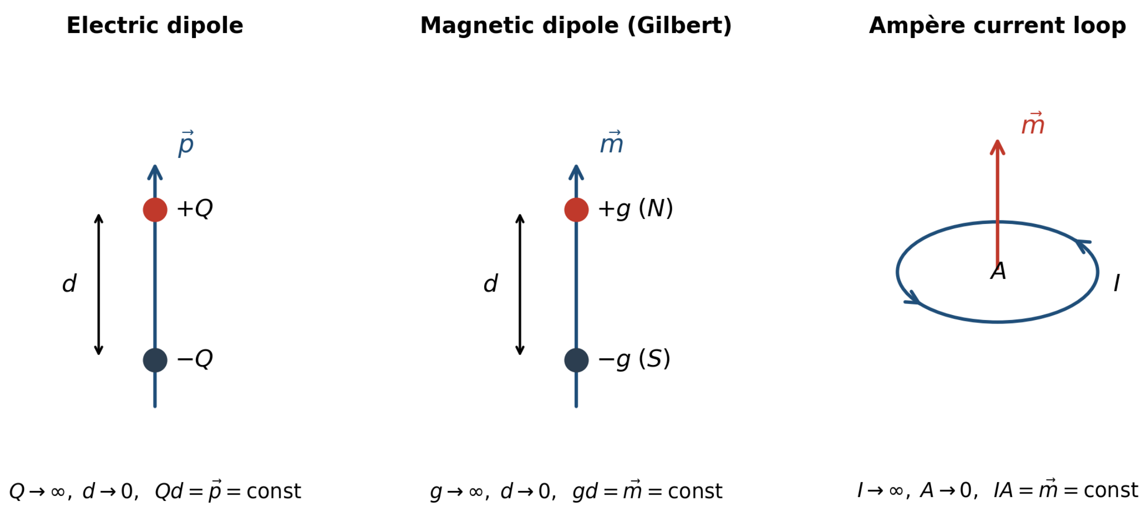

Figure 1. The three idealized dipole limits. In each case, the source strength is taken to infinity while the size goes to zero, holding the dipole moment fixed.

Vector potential of a magnetic dipole

For an Ampèrian point dipole at the origin with magnetic moment m, the vector potential is

$$\vec{A}_{\text{dipole}}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

Magnetic field of a dipole

Taking the curl of A gives the dipole field. **This identity should be committed to memory for the exam.**

$$\vec{B}_{\text{dipole}}(\vec{r}) = \nabla\times\vec{A}_{\text{dipole}} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

If one demands that the volume-averaged field over a small sphere centered on the dipole equal the field at the center, a delta-function term must be added. The complete expression is

$$\vec{B}_{\text{dipole}}(\vec{r}) \;=\; \frac{\mu_0}{4\pi}\,\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r}-\vec{m}\right] \;+\; \frac{2}{3}\,\mu_0\,\vec{m}\,\delta^3(\vec{r})$$

2. Sources of the Electric Fields

Inside matter, the total charge density splits into a **free** piece (mobile carriers we control) and a **bound** piece (the response of the medium):

$$\rho_{\text{total}} \;=\; \rho_{\text{free}} \;+\; \rho_{\text{bound}}$$

Sources of E, D, and P

The three electric field-like quantities have distinct sources:

$$\nabla\cdot\vec{E} \;=\; \frac{\rho_{\text{total}}}{\varepsilon_0}\,, \qquad \nabla\cdot\vec{D} \;=\; \rho_{\text{free}}\,, \qquad \nabla\cdot\vec{P} \;=\; -\,\rho_{\text{bound}}$$

The displacement field D sees only free charge; the polarization field P — the electric dipole moment per unit volume — sources (with a minus sign) the bound charge.

$$\left[\vec{E}\right] = \frac{V}{m} = \frac{N}{C}, \qquad \left[\vec{D}\right] = \frac{C}{m^2}$$

Polarization picture

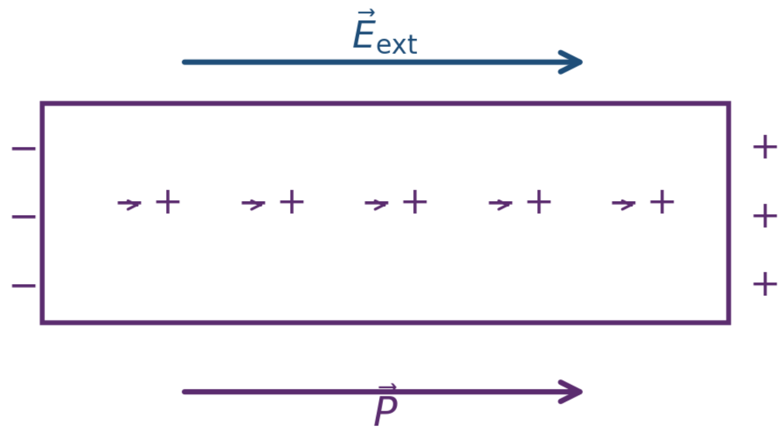

Figure 2. A linear dielectric polarized by an external field. Bound surface charges appear at the ends of the slab, and the polarization P is the volume density of induced electric dipoles.

Combining the divergence equations gives the constitutive relation

$$\varepsilon_0(\nabla\cdot\vec{E}) = \rho_{\text{free}} + \rho_{\text{bound}} \;=\; \nabla\cdot\vec{D} - \nabla\cdot\vec{P} \quad\Longrightarrow\quad \vec{D} \;=\; \varepsilon_0\vec{E} \;+\; \vec{P}$$

Linear response and electric susceptibility

Assuming a linear dielectric, P is proportional to the external field, and we write

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}$$

where χ_e is the electric susceptibility. For ordinary dielectrics $\chi_e > 0$, so the internal field is reduced ($E_{int} < E_{ext}$); these are sometimes called *dia-electrics*. There is no known "para-electric" analogue of paramagnetism (no material with $E_{int} > E_{ext}$). For a metal such as copper, $\chi_e \to \infty$ and the internal field is completely screened to zero.

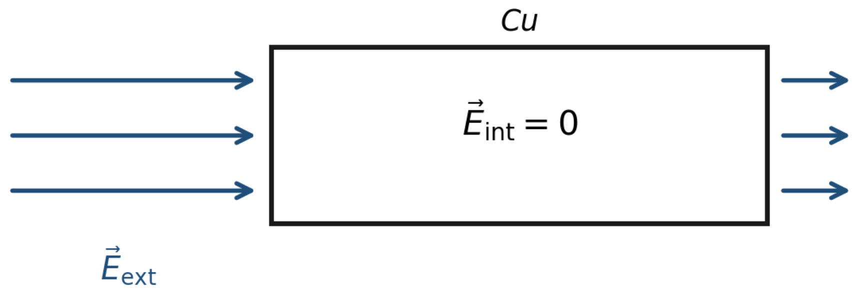

Figure 3. Electrostatic screening in a conductor. Mobile charges redistribute until the interior field vanishes.

Some materials carry a permanent built-in polarization (no external field required). Examples include barium titanate ($BaTiO_3$) and quartz (SiO_2); these are ferroelectrics.

3. Sources of the Magnetic Fields

The magnetic case parallels the electric case, with currents replacing charges. The total current density splits into free and bound parts,

$$\vec{J}_{\text{total}} = \vec{J}_{\text{free}} + \vec{J}_{\text{bound}}$$

The three magnetic field-like quantities have distinct sources:

$$\nabla \times \vec{B} = \mu_0 \vec{J}_{\text{total}}, \qquad \nabla \times \vec{H} = \vec{J}_{\text{free}}, \qquad \nabla \times \vec{M} = \vec{J}_{\text{bound}}$$

The auxiliary field H sees only the free current; the magnetization M — the magnetic dipole moment per unit volume — sources the bound current. Because no magnetic monopoles exist, there is no genuine "monopole density" on the right of any of these — only currents.

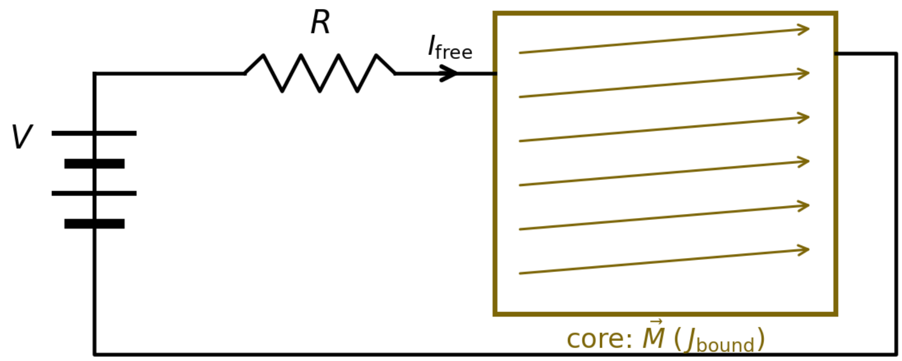

Figure 4. A current-driven solenoid with a magnetizable core. The free current I_{free} flows in the windings; the core responds with a bound current that contributes to B through M.

Constitutive relation between B, H, and M

Eliminating the bound current from Ampère's law gives the magnetic constitutive relation, in direct analogy with $D = \varepsilon_0 E + P$:

$$\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}, \qquad \text{equivalently} \qquad \vec{B} = \mu_0(\vec{H} + \vec{M})$$

Three classes of magnetic response

Materials are classified by how the internal field compares with the applied one. Diamagnets weaken the field slightly; paramagnets enhance it slightly; ferromagnets enhance it dramatically. Diamagnetism is universal — every material has a diamagnetic contribution — but it is overwhelmed whenever a stronger paramagnetic or ferromagnetic response is present:

Diamagnetic	$B_{int} < B_{ext}$	universal; small effect
Paramagnetic	$B_{int} > B_{ext}$	unpaired electrons; small effect
Ferromagnetic	$B_{int} >> B_{ext}$	ordered domains; large effect

Hierarchy rule of thumb: every material is at least diamagnetic. If the paramagnetic response dominates the diamagnetic, we call it a paramagnet; if the ferromagnetic response dominates the paramagnetic, we call it a ferromagnet. Paramagnetism and ferromagnetism are the same kind of effect — aligned moments — differing only in *strength.*

4. Which Fields Are Measurable?

Of the six fields we have discussed, only some are directly measurable. The auxiliary fields *D* and *M* are bookkeeping devices — useful, but neither is a fundamental field one can read off a probe without some additional information about the medium.

Measurable	Not directly measurable
E — from a test charge ***B*** — from a test current loop ***H*** — if the free current is known *(in vacuum, $B = \mu_0 H$)*	***D*** — bookkeeping for free charge ***M*** — not a field but a density of microscopic moments: $M = m$/Volume

Summary: E and B are the physical fields. H is convenient in problems where the free current is the natural input (e.g. a solenoid driven by an external source), and D is convenient when the free charge is the natural input. M and P are densities of microscopic moments, not fields in the same sense.