Physics 4392 Professor Scalise
Homework Assignment #14

1. A particle of charge q_1 at instantaneous position $\vec{r}_1$ is moving in an inertial frame with velocity $\vec{v}_1$. It gives rise to a magnetic field (in cgs units) at $\vec{r}$

$$\vec{B}_1(\vec{r}) = q_1 \frac{\vec{v}_1}{c} \times \frac{\vec{r} - \vec{r}_1}{|\vec{r} - \vec{r}_1|^3}$$

A second particle of charge q_2 at position $\vec{r}_2$ moves with velocity $\vec{v}_2$ and experiences a force due to the first particle

$$\vec{F}_{12} = q_2 \frac{\vec{v}_2}{c} \times \vec{B}_1(\vec{r}_2)$$

   (a) Show that $\vec{F}_{12} \neq -\vec{F}_{21}$ in general.

   (b) Under what conditions does the weak form of Newton's third law hold? (The weak form means that the force vectors are equal in magnitude and opposite in direction. The strong form means that in addition the force vectors are collinear.)

2. A circular cylindrical solenoid of radius a and infinite length has n turns of wire per length which carries current I.

   (a) Find the current density $\vec{J}(\vec{r})$. (You will need delta functions.)

   (b) Find the vector potential $\vec{A}(\vec{r})$ everywhere. (Make sure it is continuous at $r = a$, otherwise the divergence will be infinite there.)

   (c) Is the Coulomb gauge condition satisfied for your choice of $\vec{A}(\vec{r})$?

   (d) Find the magnetic field $\vec{B}(\vec{r})$ everywhere.

   (e) Find the self-inductance per unit volume.