1. An infinitely long cylindrical shell of radius a carries current I along the axis of the cylinder ($\hat{z}$) . In what follows, you may need to use generalized functions (distributions) and the curl and divergence in cylindrical coordinates:

$$\vec{\nabla} \times \vec{f}(s, \phi, z) = \left(\frac{1}{s} \frac{\partial f_z}{\partial \phi} - \frac{\partial f_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial f_s}{\partial z} - \frac{\partial f_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial (s f_\phi)}{\partial s} - \frac{\partial f_s}{\partial \phi} \right) \hat{z}$$

$$\vec{\nabla} \cdot \vec{f}(s, \phi, z) = \frac{1}{s} \frac{\partial (s f_s)}{\partial s} + \frac{1}{s} \frac{\partial f_\phi}{\partial \phi} + \frac{\partial f_z}{\partial z}$$

   (a) What is the current density vector field $\vec{J}(\vec{r})$ everywhere?

   (b) What is the magnetic vector field $\vec{B}(\vec{r})$ everywhere?

   (c) What is the vector potential field $\vec{A}(\vec{r})$ in Coulomb gauge everywhere? Make sure you check $s < a$, $s > a$, and $s = a$.