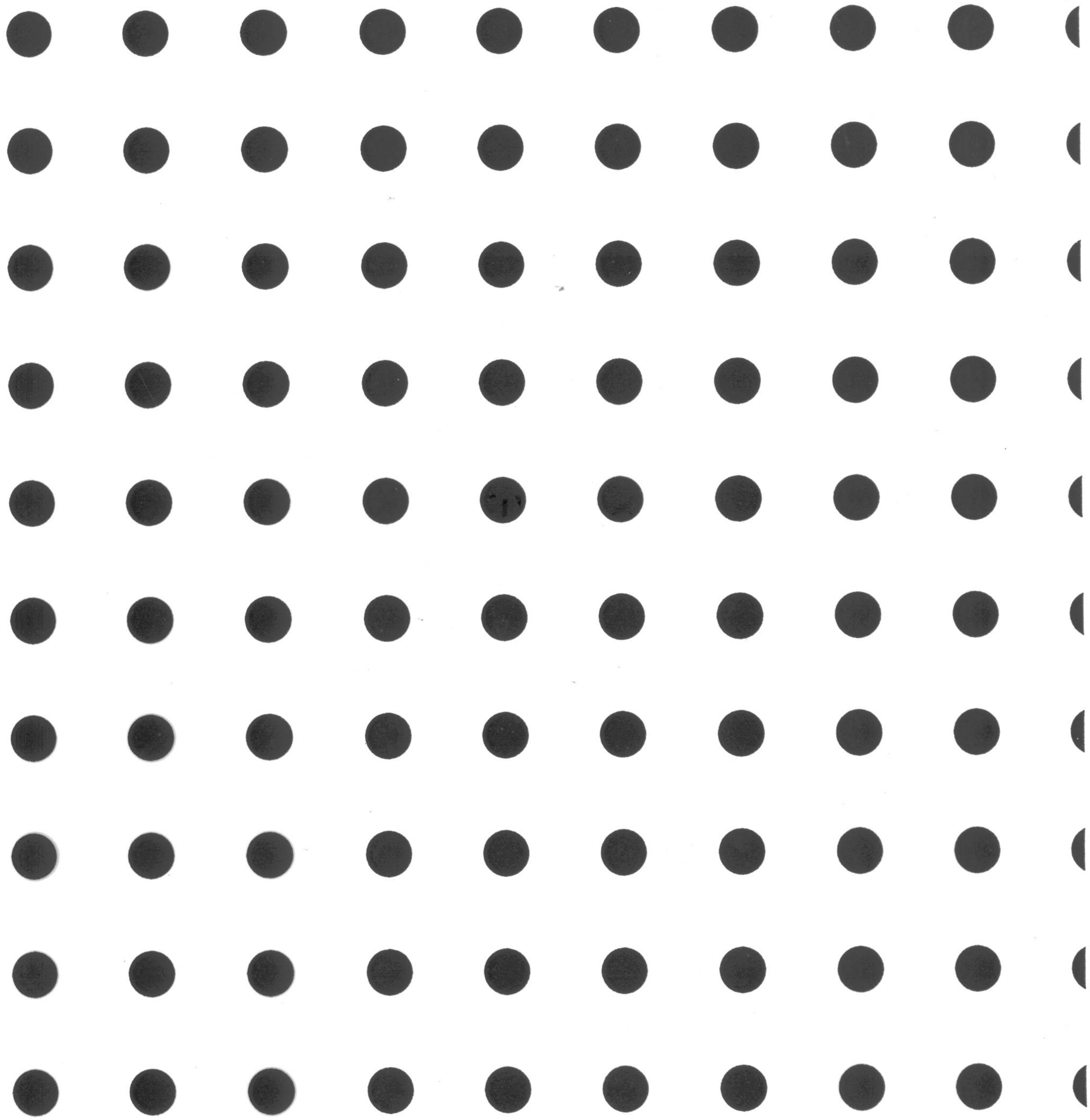

path difference

$2d \sin\theta$

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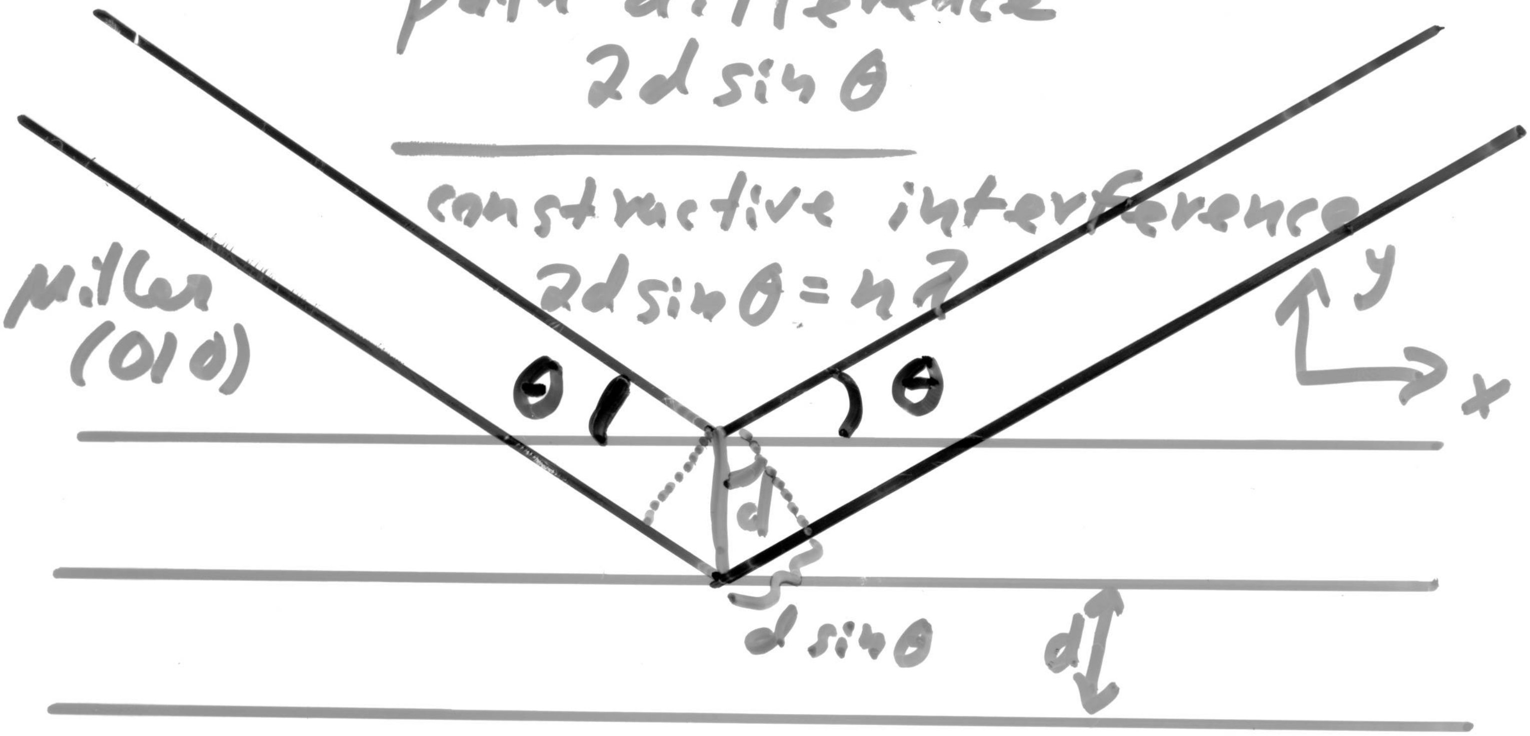
constructive interference

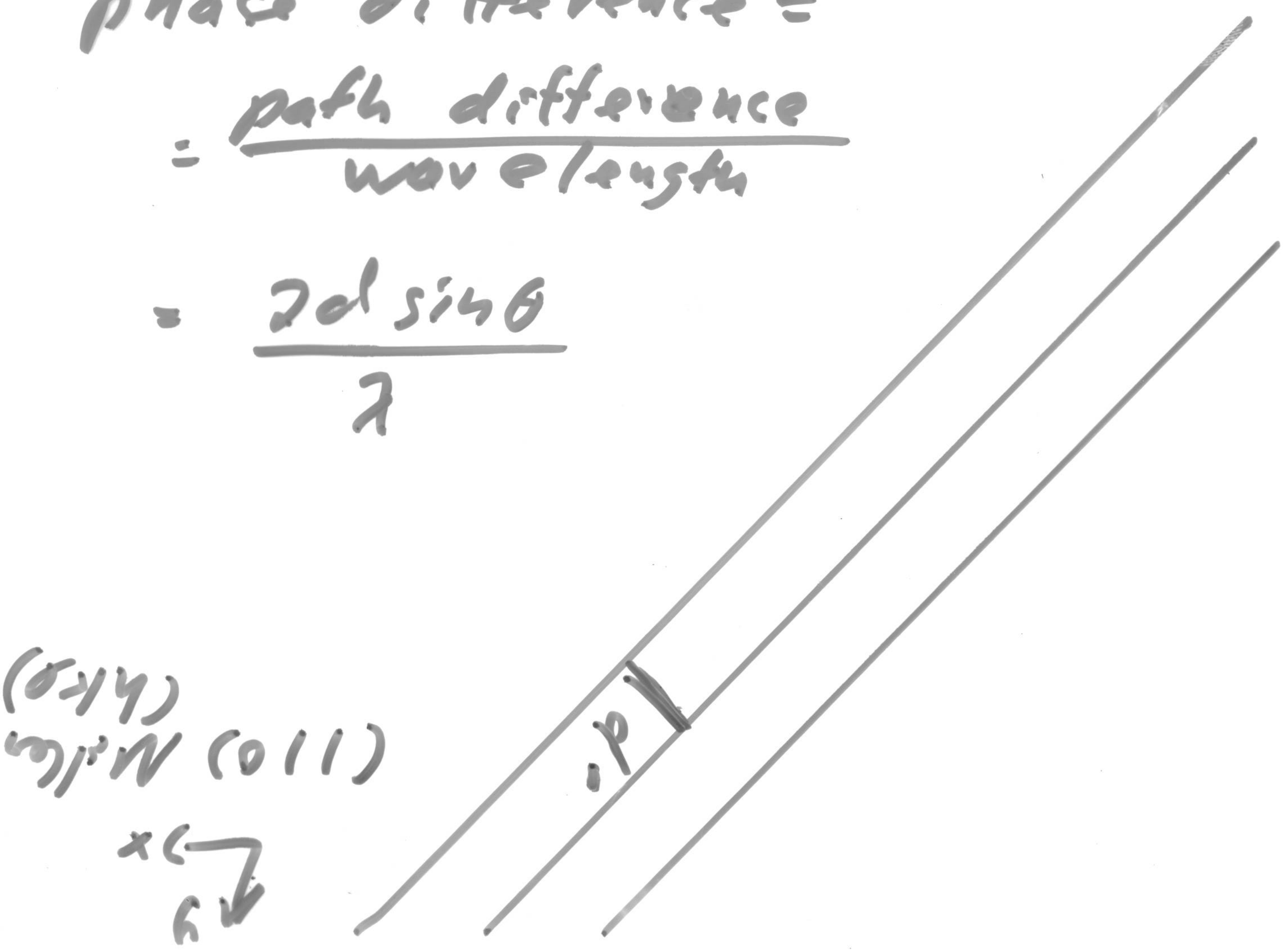
$2d \sin\theta = n\lambda$



phase difference =

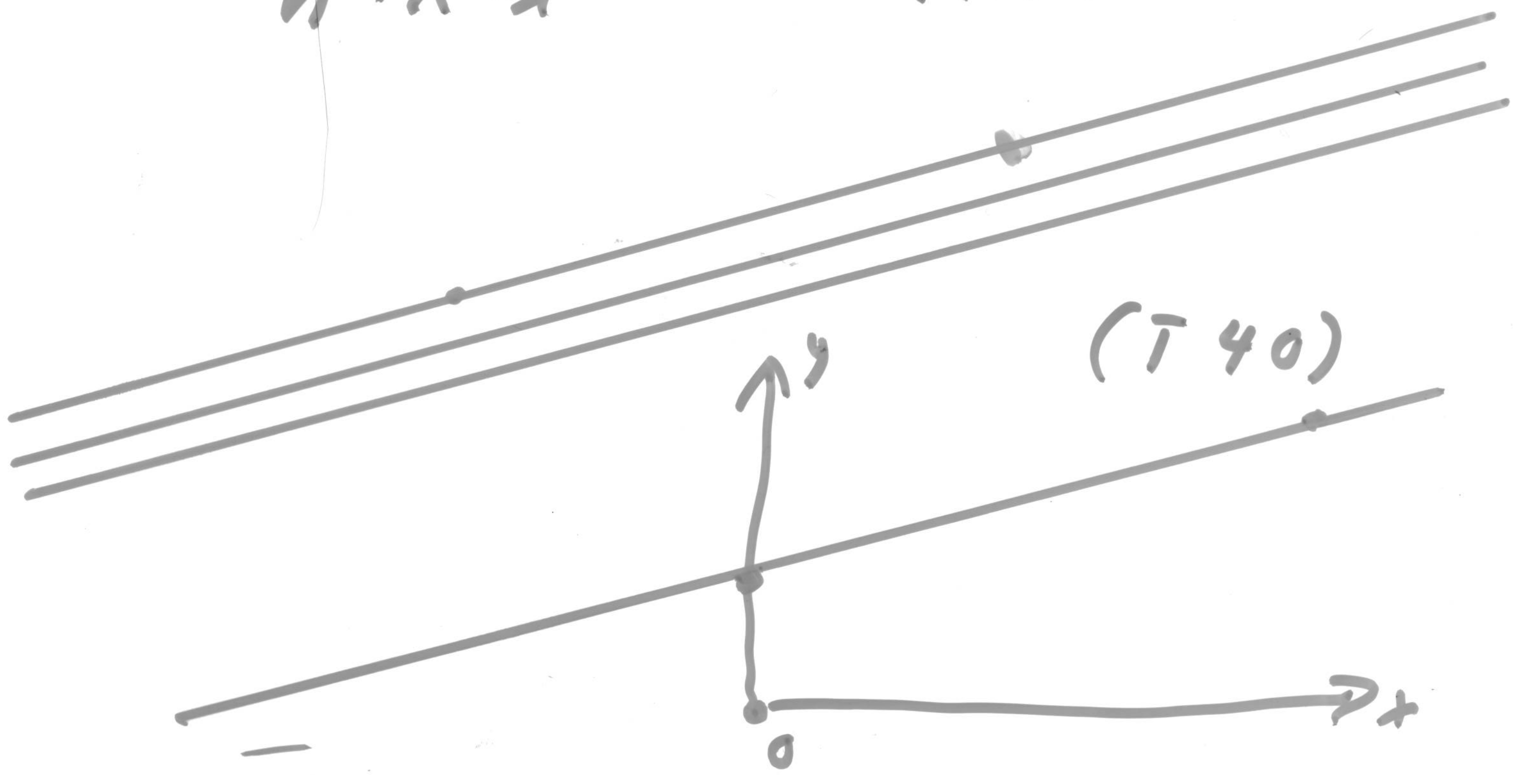
$$= \frac{\text{path difference}}{\text{wavelength}}$$

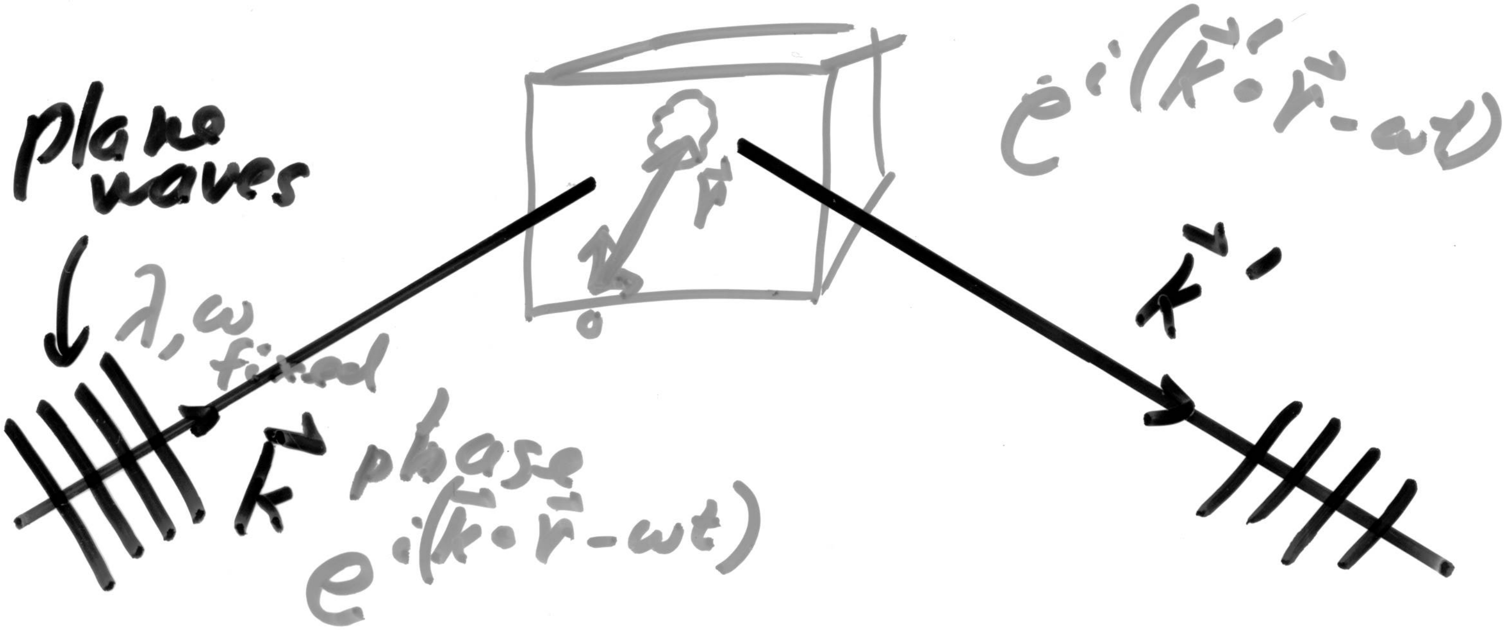
$$= \frac{2d \sin\theta}{\lambda}$$



$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

$(h k l)$ $\boxed{(\bar{1} 4 0)}$

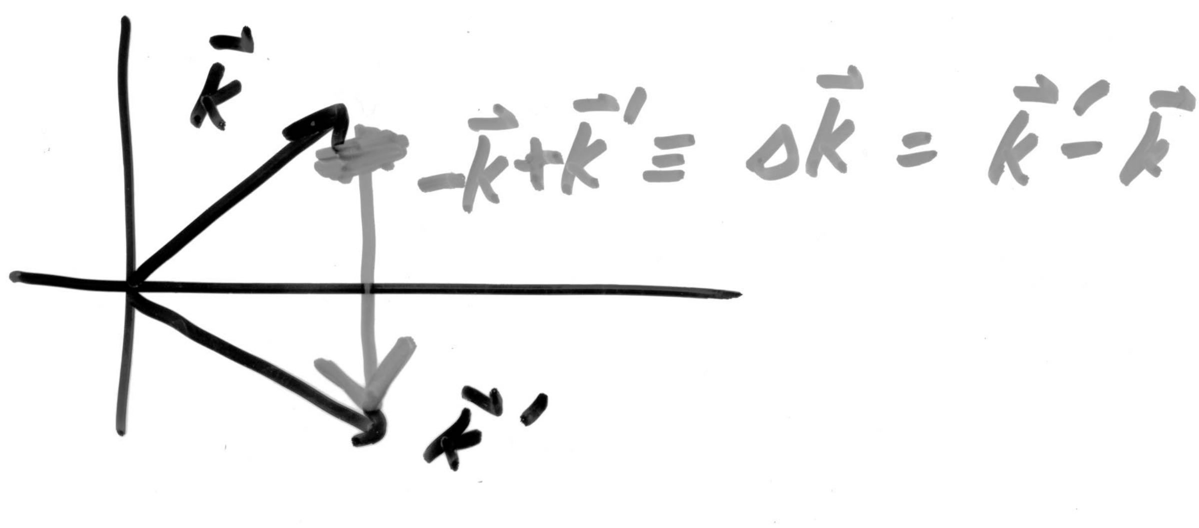


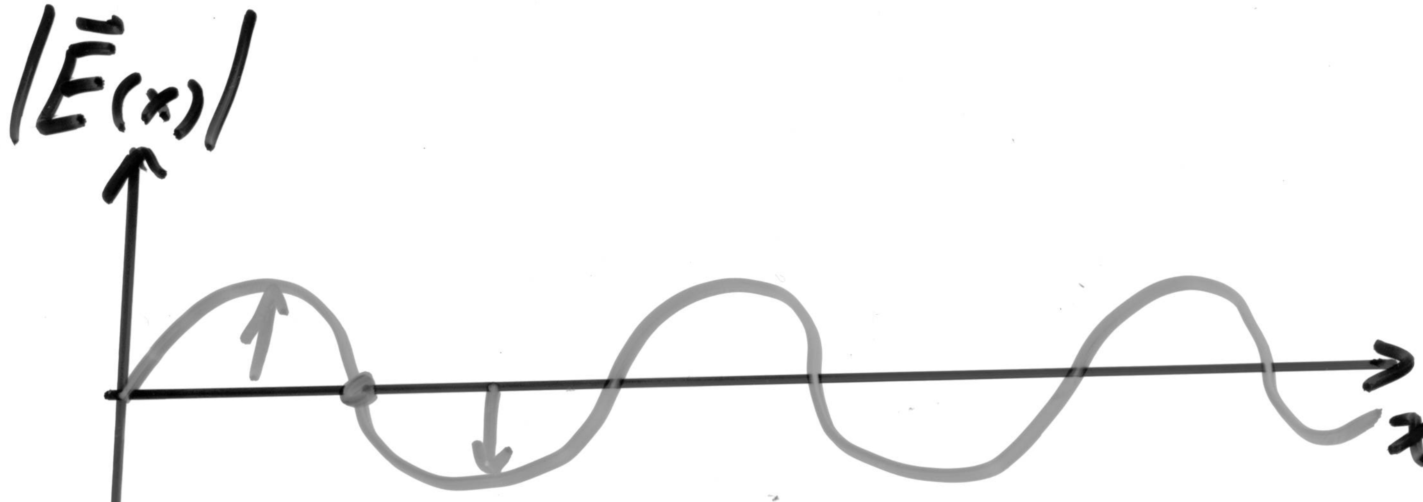


$k = |\vec{k}|$ = wavenumber = $\frac{2\pi}{\lambda}$

Elastic scattering – probe has the same energy coming out that it had going in

$$|\vec{k}'| = |\vec{k}| = k$$





Take real part of complex exponential to get the physical field

The phase difference between the incoming and outgoing waves is

$$e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}$$

Scattering amplitude

$$F = \iiint_{\text{whole crystal}} n(\vec{r})\, e^{i\underbrace{(\vec{k}-\vec{k}')}_{-\Delta\vec{k}}\cdot\vec{r}}\, dV$$

substitute Fourier series from last time

$$n(\vec{r}) = \underbrace{\sum_{p_1}\sum_{p_2}\sum_{p_3}}_{\vec{p}} \tilde{n}_{\vec{p}}\, e^{i\vec{G}\cdot\vec{r}}$$

$$\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \vec{p}$$

$$\vec{G} = \nu_1 \vec{b}_1 + \nu_2 \vec{b}_2 + \nu_3 \vec{b}_3$$

vector in the reciprocal lattice

$$\vec{T} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3$$

translation vector in the direct lattice.

$$F = \iiint \sum_{\vec{p}} \tilde{n}_{\vec{p}}\, e^{i\vec{G}\cdot\vec{r}}\, e^{i(\Delta\vec{k})\cdot\vec{r}}\, dV$$

$$= \iiint \sum \tilde{n}\, e^{i(\vec{G}-\Delta\vec{k})\cdot\vec{r}}\, dV$$

If $(\vec{G}-\Delta\vec{k})\cdot\vec{r} \neq 0$, then the scattered waves will interfere destructively (randomly) and there will be no scattered beam.

If $\vec{G} = \Delta\vec{k}$ then $e^{i(\vec{G}-\Delta\vec{k})\cdot\vec{r}} = e^0 = 1$

$\Rightarrow$ constructive interference.

$\Rightarrow$ scattered radiation when

$\vec{k}' - \vec{k}$ = reciprocal lattice vector

Scattering condition

$$\boxed{\Delta\vec{k} = \vec{G}} = \nu_1\vec{b}_1 + \nu_2\vec{b}_2 + \nu_3\vec{b}_3$$

$\vec{k}' - \vec{k} = \vec{G}$ dot with $\vec{k}$

$$\vec{k}'\cdot\vec{k} - \vec{k}\cdot\vec{k} = \vec{G}\cdot\vec{k}$$

$\vec{k} + \vec{G} = \vec{k}'$ dot with itself

$$(\vec{k}+\vec{G})\cdot(\vec{k}+\vec{G}) = \vec{k}'\cdot\vec{k}'$$

$$\vec{k}\cdot\vec{k} + \vec{G}\cdot\vec{G} + 2\vec{k}\cdot\vec{G} = \vec{k}'\cdot\vec{k}'$$

$$k^2 + G^2 + 2\vec{k}\cdot\vec{G} = k^2 \Rightarrow \boxed{2\vec{k}\cdot\vec{G} + \vec{G}^2 = 0}$$

$$G^2 = 2\vec{k}\cdot\vec{G}$$

Laue Equations

$\Delta\vec{k} = \vec{G}$ dot product with $\begin{cases}\vec{a}_1\\ \vec{a}_2\\ \vec{a}_3\end{cases}$

$$\vec{a}_1\cdot\Delta\vec{k} = \left(v_1\vec{b}_1 + v_2\vec{b}_2 + v_3\vec{b}_3\right)\cdot\vec{a}_1$$

$$\vec{a}_i\cdot\vec{b}_j = 2\pi\,\delta_{ij}$$

$$\left.\begin{aligned}\vec{a}_1\cdot\Delta\vec{k} &= 2\pi v_1\\ \vec{a}_2\cdot\Delta\vec{k} &= 2\pi v_2\\ \vec{a}_3\cdot\Delta\vec{k} &= 2\pi v_3\end{aligned}\right\}$$