At the Brillouin zone boundaries

$k = \pm \frac{\pi}{a}$,

$\omega_{optical} = \sqrt{\frac{2C}{m_2}}$ $\qquad m_1 \geq m_2$

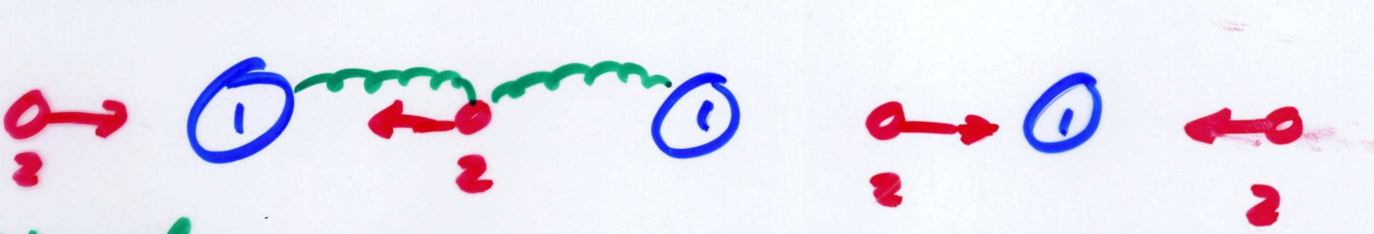
$\omega_{acoustic} = \sqrt{\frac{2C}{m_1}}$

and $\left.\frac{d\omega}{dk}\right|_{k = \pm\frac{\pi}{a}} = 0 = v_{group}$

What does the motion look like?

$\omega_{optical}$ is independent of m_1

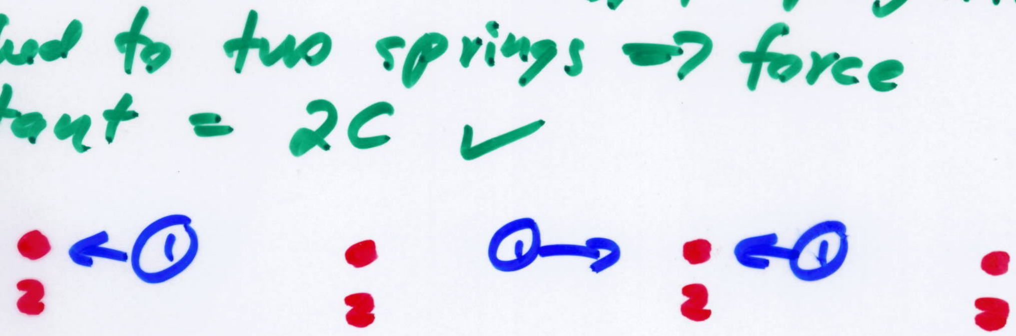
$\Rightarrow$ m_1 frozen



Standing wave, no energy propagation ✓

m_2 attached to two springs $\Rightarrow$ force constant $= 2C$ ✓

$\omega_{acoustic}$

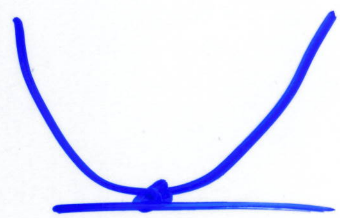


K 3.5 $\qquad$ cgs $k=1$

$$U(R) = N\left(-\frac{kq^2\alpha}{R} + \frac{A}{R^n}\right)$$

$$0 = \frac{\partial U}{\partial R}\bigg|_{R_0} = N\left(\frac{kq^2\alpha}{R^2} - \frac{nA}{R^{n+1}}\right)\bigg|_{R_0} = 0$$

$$\frac{q^2\alpha}{R_0^2} = \frac{nA}{R_0^{n+1}}$$



$$R_0^n = \frac{nA}{q^2\alpha}R_0 \leftarrow$$

$$U(R_0 - R_0\delta) = U(R_0) + \cancel{\frac{dU}{dR}\bigg|_{R_0}}^{\,0}(\delta R_0)$$

$$+\frac{1}{2}\frac{d^2U}{dR^2}\bigg|_{R_0}(\delta R_0)^2$$

K 3.6 Tab 6 7

$k_e = 8.99 \times 10^{9}$ MKS

actual KCl ($z=6$)

$z\lambda = 2.05 \times 10^{-8}$ erg $\leftarrow$

$\rho = 0.326$ Å $= .326 \times 10^{-10}$ m

$\lambda = \frac{z\lambda}{6} = \frac{2.05}{6} \times 10^{-8}$ erg $= 3.417 \times 10^{-9}$ erg

$= 3.417 \times 10^{-16}$ J

1 erg $= 10^{-7}$ J

$\boxed{q = 1.60219 \times 10^{-19} C}$

ZnS $\quad \alpha = 1.6381$

$z = 4$

$$U = N\left(z\lambda e^{-R/\rho} - \frac{\alpha q^2 k}{R}\right)$$

$$\left.\frac{dU}{dR}\right|_{R_0} = 0 \implies R_0^2 e^{-R_0/\rho} = \frac{\rho k \alpha q^2}{z\lambda}$$

Non-dimensionalize

$$\left(\frac{R_0}{\rho}\right)^2 e^{-R_0/\rho} = \frac{k\alpha q^2}{\rho z \lambda}$$

$$x^2 e^{-x} = \frac{k\alpha q^2}{\rho z \lambda} \implies x = 9.2104$$

$R_0 = x\rho = 3.00259$ Å

Binding energy per atom

$$\frac{U}{N} = z\lambda e^{-R_0/\rho} - \frac{k\alpha q^2}{R_0} = -1.12202\times10^{-18}\,\mathrm{J}$$

$$\frac{U}{Nkq^2} = -4.86329\times10^{9}$$

NaCl structure

ρ same, λ same, $z = 6$, $\alpha = 1.747565$, q same

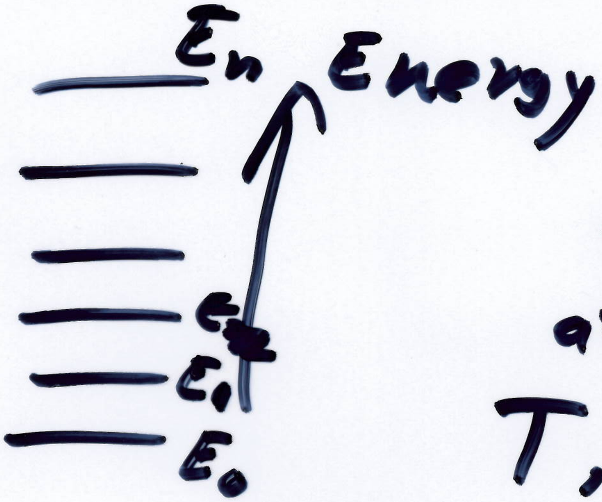
$$x^2 e^{-x} = \frac{k\alpha q^2}{\rho z \lambda} \longrightarrow x = 9.64297$$

$$R_0 = x\rho = 3.14361\ \text{Å}$$

$$\frac{U}{N} = z\lambda e^{-R_0/\rho} - \frac{k\alpha q^2}{R_0} = -1.14955\times10^{-18}\,\mathrm{J}$$

$$\frac{U}{Nkq^2} = -4.98261\times10^{9}$$

Statistical Mechanics



If a system is in thermal equilibrium at absolute temperature T, then the probability that the system has energy E_n is

$$P_n \propto e^{-\frac{E_n}{k_B T}}$$ Boltzmann distribution

$$\sum_{n=0}^{N} P_n = 1$$

$$P_n = \frac{e^{-\frac{E_n}{k_B T}}}{\sum_{p=0}^{N} e^{-\frac{E_p}{k_B T}}}$$

Simple harmonic oscillator

$$E_n = (n + \tfrac{1}{2})\hbar\omega$$

The number of phonons in state n is N_n

$$\frac{N_n}{\sum_{p=0}^{\infty} N_p} = \frac{e^{-\frac{(n+\frac{1}{2})\hbar\omega}{k_B T}}}{\sum_{p=0}^{\infty} e^{-\frac{(p+\frac{1}{2})\hbar\omega}{k_B T}}}$$

Elliptical Orbit Period

$$V(r) = -\frac{k}{r}$$

$$dt = \frac{dr}{\sqrt{\frac{2}{m}\left(E - V(r) - \frac{\ell^2}{2mr^2}\right)}}$$

$$t = \sqrt{\frac{m}{2k}} \int_{r_{min}}^{r} \frac{r'\,dr'}{\sqrt{r' - \frac{r'^2}{2a} - \frac{a(1-e^2)}{2}}}$$

change variable $r' = a(1 - e\cos\psi')$

ψ = eccentric anomaly

$\psi = 0$ at perihelion $= r_{min}$

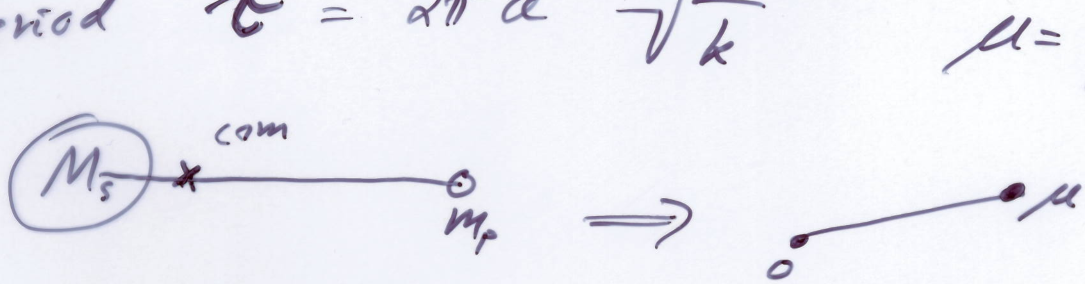
$\psi = \pi$ at aphelion $= r_{max}$

$$t = \sqrt{\frac{ma^3}{k}} \int_0^{\psi} (1 - e\cos\psi')\,d\psi'$$

Integrate ψ from 0 to 2π.

Period $\tau = 2\pi a^{3/2} \sqrt{\frac{m}{k}}$ ← reduced mass μ

$$\mu = \frac{m_s m_p}{m_s + m_p}$$



For planets orbiting the Sun $f = -\frac{G M_s m_p}{r^2}$

$k = G M_s m_p$

s = sun
p = planet