In[68]:=
```
n[e_] = 1 / (Exp[e / kt] - 1)
```

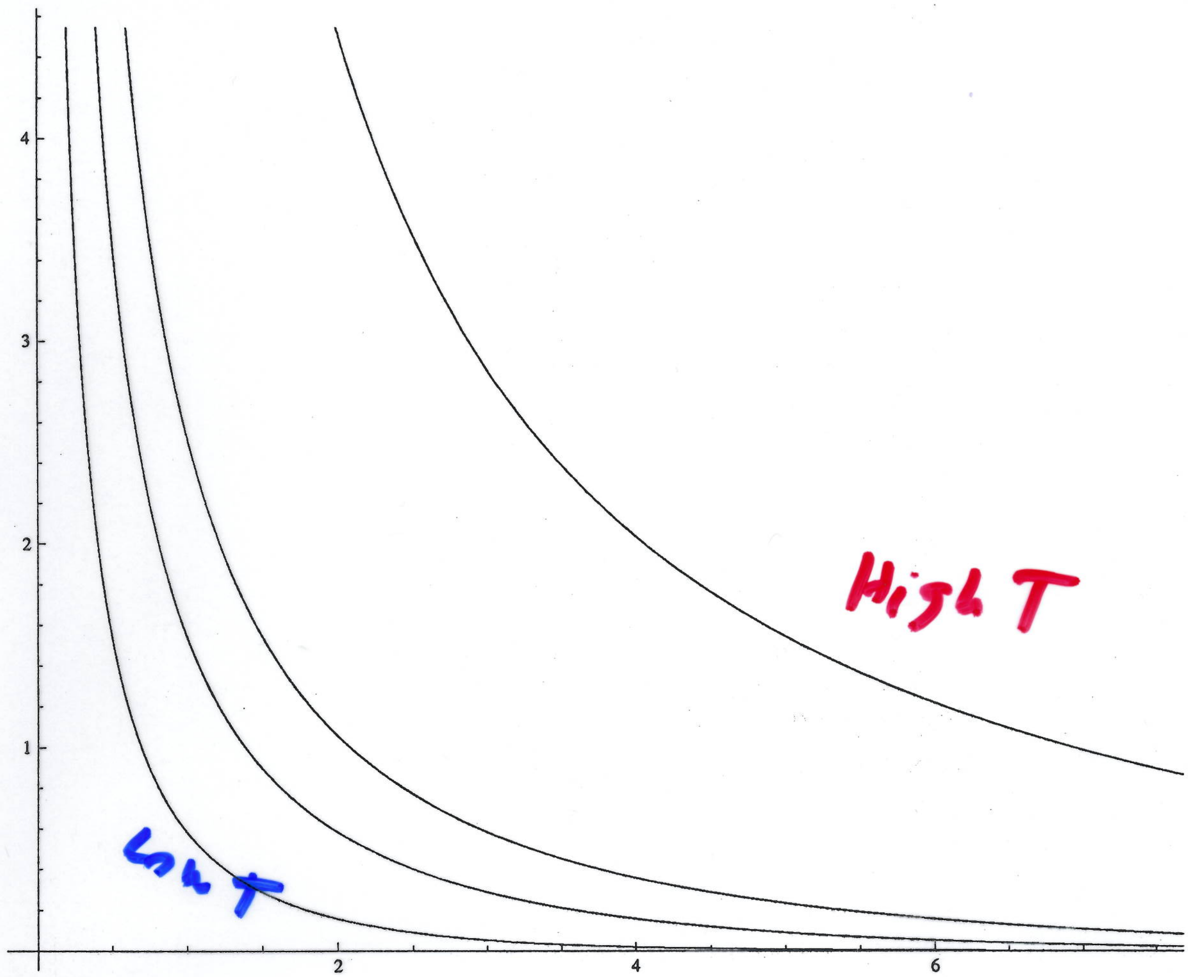
Out[68]=

$$\frac{1}{-1 + e^{e/kt}}$$

In[119]:=
```
kt = {1, 2, 3, 10};
```

In[120]:=
```
Plot[n[e], {e, 0, 10}]
```

Out[120]=



In[156]:=
```
f[e_] =
    1 / (Exp[(e - u) / kbt] + 1);
```

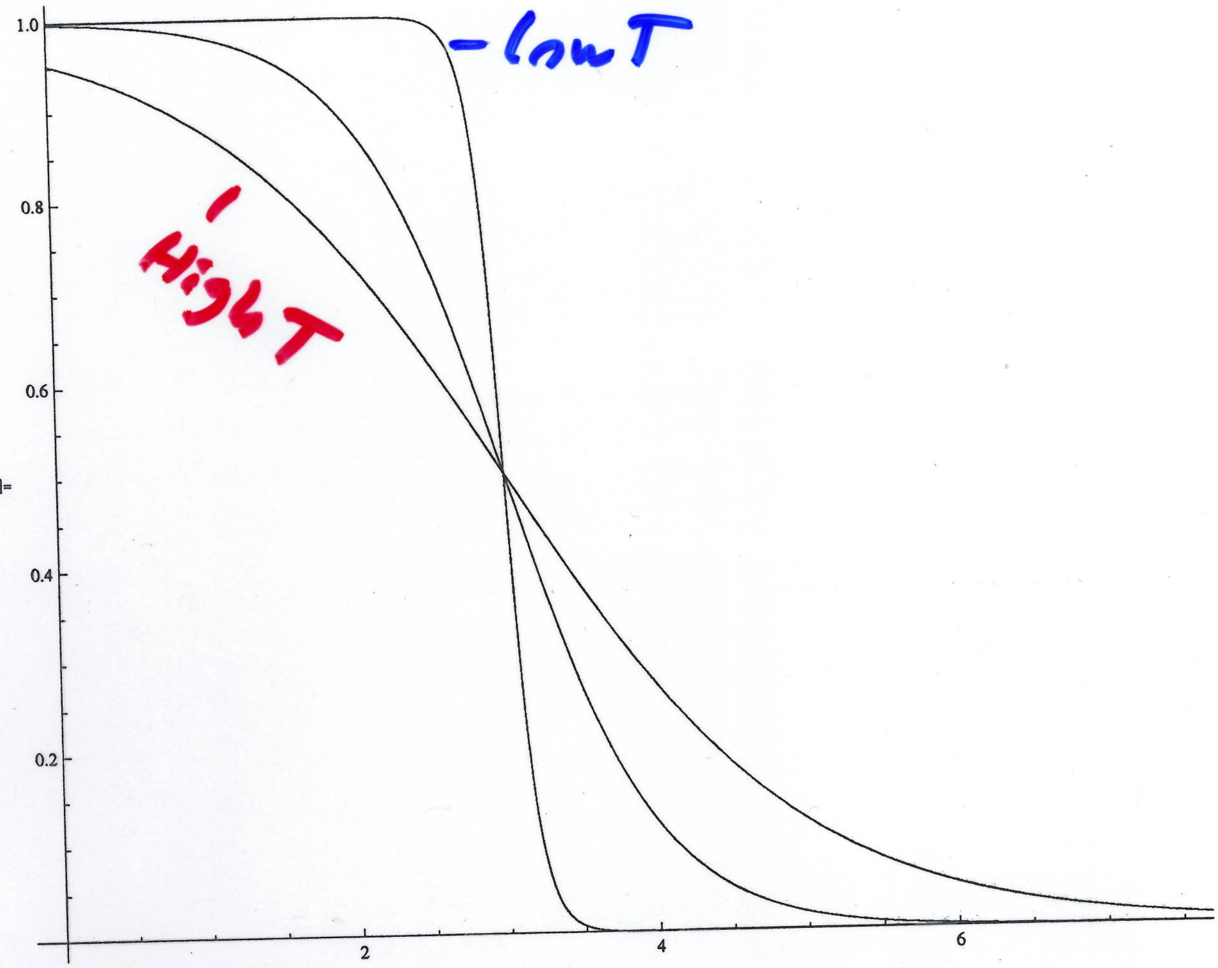
In[155]:=
```
u = {3, 3, 3}; kbt = {.1, .5, 1};
```

↑ μ = chemical potential

$\mu = 3$ fixed

In[157]:= `Plot[f[e], {e, 0, 10}]`

Out[157]=



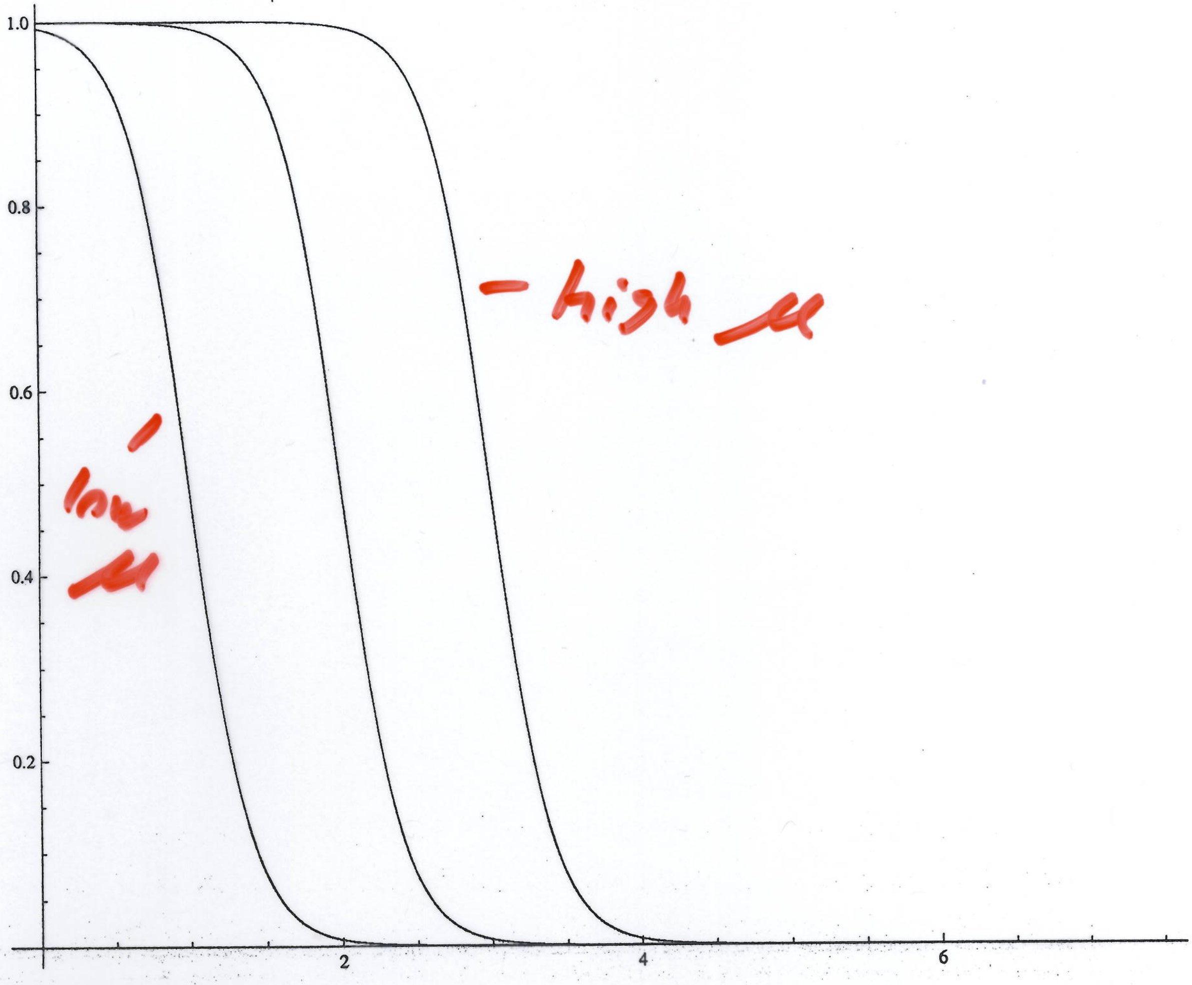
In[171]:= `f[e_] = 1 / (Exp[(e - u) / kbt] + 1);`

In[170]:= `u = {1, 2, 3}; kbt = {.2, .2, .2};`

T = fixed

In[172]:= `Plot[f[e], {e, 0, 10}]`

Out[172]=



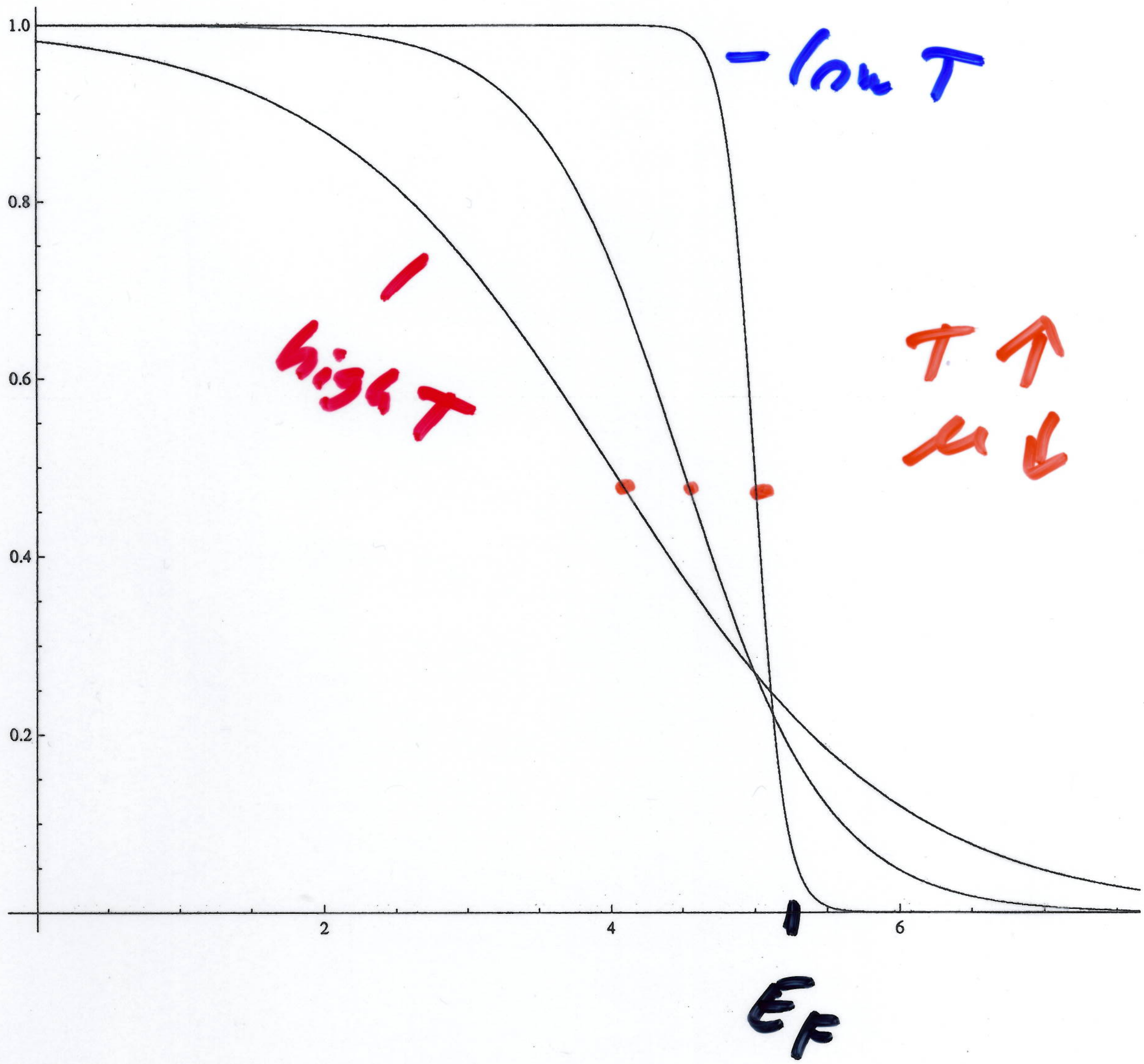
In[193]:= `f[e_] = 1 / (Exp[(e - u) / kbt] + 1);`

In[192]:= `u = {5, 4.5, 4}; kbt = {.1, .5, 1};`

N = fixed

In[194]:= `Plot[f[e], {e, 0, 10}]`

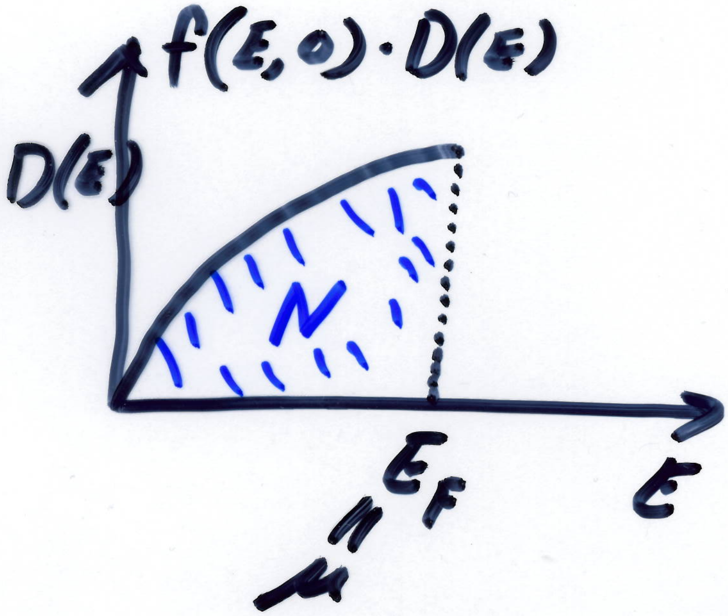
Out[194]=

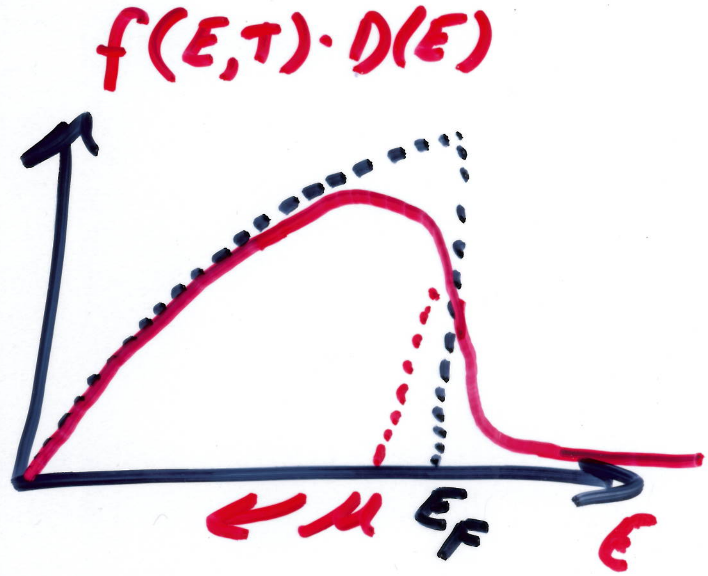


Heat Capacity of electron gas

At $T = 0$ $\qquad\qquad$ $T \neq 0$

$$f(E,T)\Big|_{T=0} = \begin{cases} +1, & E < E_F \\ 0, & E > E_F \end{cases}$$





$$\mu = E_F\left[1 - \frac{1}{3}\left(\frac{\pi k_B T}{2E}\right)^2 + \dots\right]$$

$$C_{el} = \frac{d_\Delta U_{el}}{dT}$$

not zero even if $T=0$

$$\Delta U_{el} = U_{el}(T) - U_{el}(0)$$

$$= \int_{E=0}^{\infty} E\, f(E,T)\cdot D(E)\, dE - \int_{E=0}^{\infty} E\, f(E,0)\cdot D(E)\, dE$$

$$\Delta U_{el} = \int_{E=0}^{\infty} E\, f(E,T)\, D(E)\, dE - \int_{E=0}^{E_F} E\, D(E)\, dE$$

$$= \int_{E=0}^{E_F} E\, f(E,T)\, D(E)\, dE + \int_{E=E_F}^{\infty} E\, f(E,T)\, D(E)\, dE - \int_{E=0}^{E_F} E\, D(E)\, dE$$

$$= \int_{E=E_F}^{\infty} E\, f(E,T)\, D(E)\, dE - \int_{E=0}^{E_F} E\left[1 - f(E,T)\right] D(E)\, dE$$

$$N = \int_{E=0}^{\infty} f(E,T)\, D(E)\, dE = \int_{E=0}^{E_F} D(E)\, dE$$

multiply both sides by E_F

$$\int_{E=0}^{\infty} E_F\, f(E,T)\, D(E)\, dE = \int_{E=0}^{E_F} E_F\, D(E)\, dE$$

$$\int_{E=0}^{E_F} E_F\, f(E,T)\, D(E)\, dE + \int_{E=E_F}^{\infty} E_F\, f(E,T)\, D(E)\, dE = \int_{E=0}^{E_F} E_F\, D(E)\, dE$$

$$\int_{E=E_F}^{\infty} E_F\, D(E)\, f(E,T)\, dE = \int_{E=0}^{E_F} E_F\left[1 - f(E,T)\right] D(E)\, dE$$

$$\Delta U_{el} = \underbrace{\int_{E=E_F}^{\infty} (E-E_F) D(E) f(E,T)\, dE}_{} + \underbrace{\int_{E=0}^{E_F} (E_F - E)[1 - f(E,T)] D(E)\, dE}_{}$$

energy to move electrons from E_F to $E > E_F$

this is positive

energy to move electrons from $E < E_F$ to E_F

this is positive

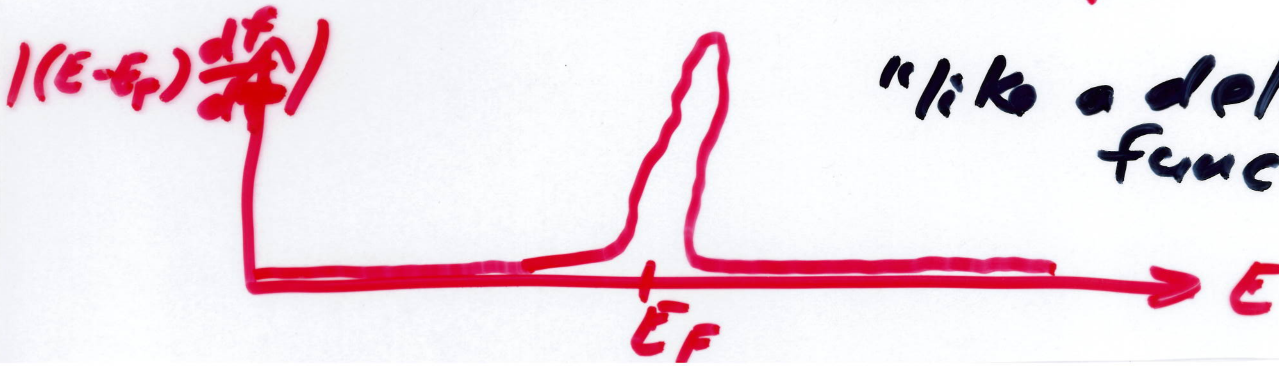
$$C_{el} = \frac{d\Delta U_{el}}{dT}$$, only T dependence is in $f(E,T)$

$$\Delta U_{el} = \int_{E=0}^{\infty} (E-E_F) f(E,T) D(E)\, dE + \int_{E=0}^{E_F} (E_F - E) D(E)\, dE$$

$$C_{el} = \frac{d\Delta U_{el}}{dT} = \int_{E=0}^{\infty} \underline{(E-E}_F) D(E) \underline{\left[\frac{df(E,T)}{dT}\right]} dE$$

for $k_B T \ll E_F$ in metals,

$\left|(E-E_F)\frac{df}{dT}\right|$ is large only near E_F



"like a delta function"

approximate $D(E) \approx D(E_F)$

$$C_{el} \approx D(E_F) \int_{E=0}^{\infty} (E - E_F) \frac{df}{dT} dE$$

under the same assumption (low T)

$\mu \approx E_F$

$$f(E,T) = \frac{1}{e^{\frac{E-\mu}{k_B T}} + 1} \approx \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

$$\frac{df}{dT} = \frac{e^{\frac{E-E_F}{k_B T}}}{\left[e^{\frac{E-E_F}{k_B T}} + 1\right]^2} \frac{E-E_F}{(k_B T)^2} k_B$$

Define $x = \frac{E - E_F}{k_B T}$ dimensionless

$dx = \frac{dE}{k_B T}$ when $E = 0$
$x = -\frac{E_F}{k_B T}$

when $E = \infty$, $x = \infty$

$$C_{el} \approx D(E_F)\, k_B \int_{E=0}^{\infty} x^2 \frac{e^x}{[e^x+1]^2}\, dE$$

$$= k_B^2 T\, D(E_F) \int_{x=\frac{-E_F}{k_B T}}^{\infty} x^2 \frac{e^x}{[e^x+1]^2}\, dx$$

Replace the lower limit by $x = -\infty$

e^x is already ~ 0 at $x = \frac{-E_F}{k_B T}$

$$\int_{x=-\infty}^{+\infty} x^2 \frac{e^x}{[e^x+1]^2}\, dx = \frac{\pi^2}{3}$$

$$\int_{x=-\infty}^{-\frac{E_F}{kT}} x^2 \frac{e^x}{[e^x+1]^2}\, dx \approx 0$$

as long as $k_B T \ll E_F$

$$C_{el} \approx \frac{\pi^2}{3} k_B^2 D(E_F)\, T \propto T$$

Remember $C_{v\,\text{phonons}} \propto T^3$

$\Big\}$ low T

$$D(E) = \frac{dN}{dE} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

$$N = \frac{V}{3\pi^2}\left(\frac{2mE}{\hbar^2}\right)^{3/2}$$

\# states with energy $< E$

$$D(E) = \frac{3N}{2E} \rightarrow D(E_F) = \frac{3N}{2E_F} = \frac{3N}{2k_B T_F}$$

$$C_{el} = \frac{\pi^2}{2} k_B \cdot N \frac{T}{T_F}$$

$m_{th} > m_{free}$

$$C_{TOTAL} = C_{el} + C_{v\,\text{phonon}} = \gamma T + A T^3$$