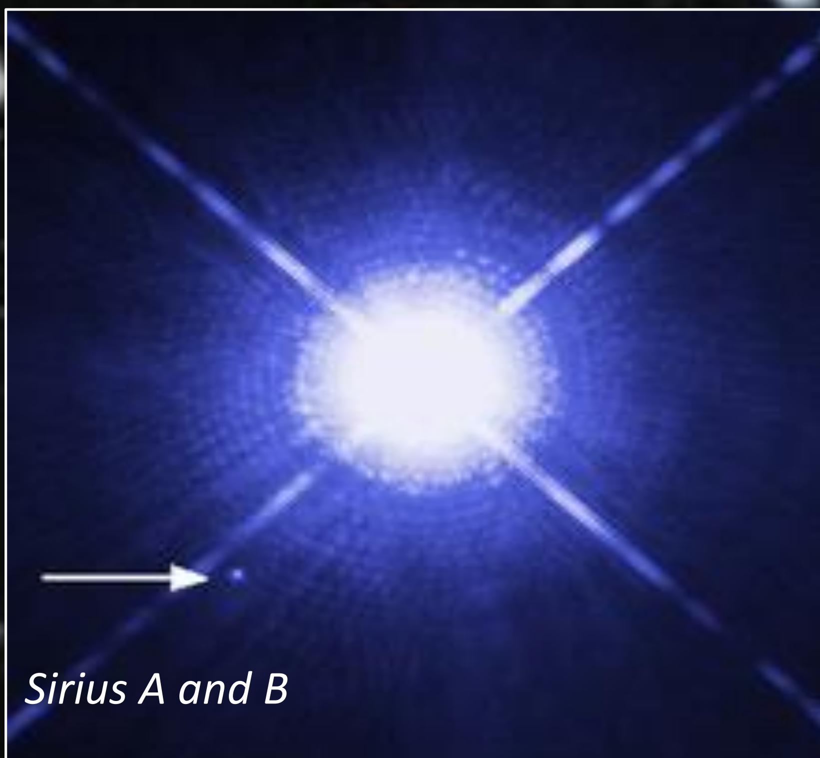
White Dwarfs and Electron Degeneracy



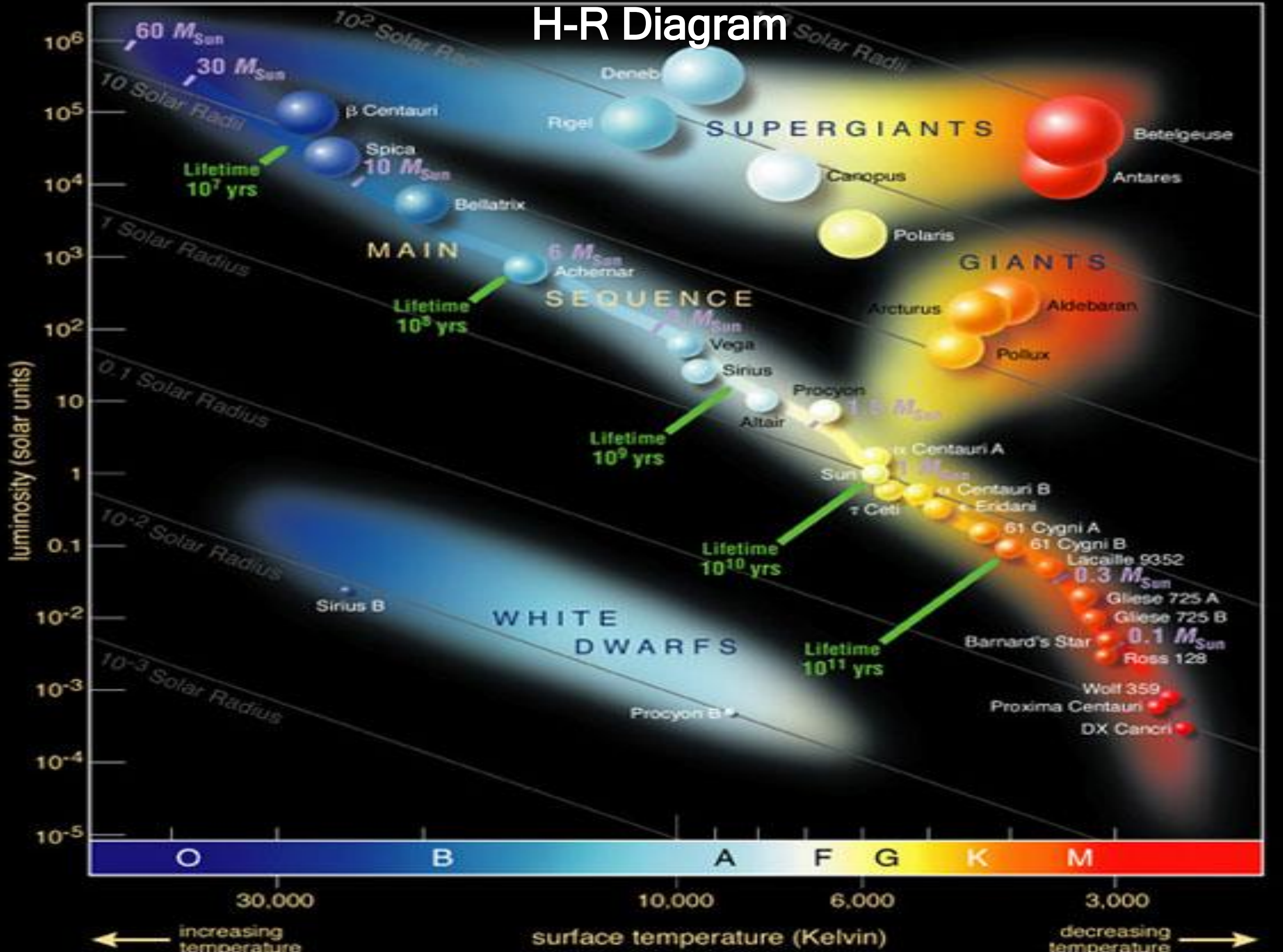
Sirius A and B

Farley V. Ferrante
Southern Methodist University

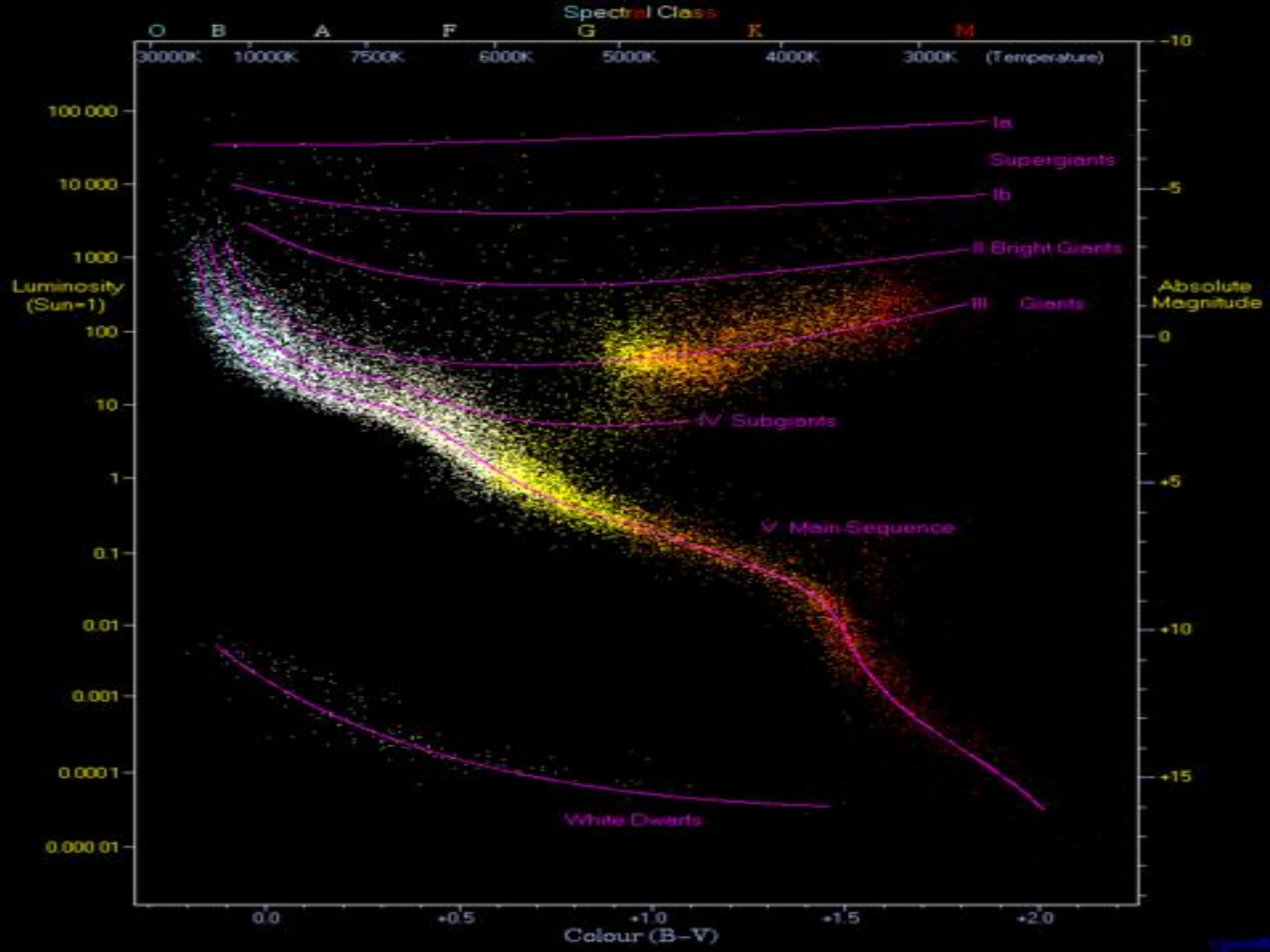
Outline

- Stellar astrophysics
- White dwarfs
  - Dwarf novae
  - Classical novae
  - Supernovae
- Neutron stars

H-R Diagram



60 M Sun
30 M Sun
10 M Sun
6 M Sun
0.3 M Sun
0.1 M Sun
10 Solar Radii
1 Solar Radius
0.1 Solar Radius
10-2 Solar Radius
10-3 Solar Radius
SUPERGIANTS
GIANTS
MAIN SEQUENCE
WHITE DWARFS
Lifetime 10^7 yrs
Lifetime 10^8 yrs
Lifetime 10^9 yrs
Lifetime 10^10 yrs
Lifetime 10^11 yrs
Deneb
Rigel
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Altair
Procyon
Canopus
Polaris
Betelgeuse
Antares
Arcturus
Aldebaran
Pollux
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B
luminosity (solar units)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^-2
10^-3
10^-4
10^-5
O
B
A
F
G
K
M
30,000
10,000
6,000
3,000
increasing temperature
surface temperature (Kelvin)
decreasing temperature


Spectral Class
O
B
A
F
G
K
M
30000K
10000K
7500K
6000K
5000K
4000K
3000K
(Temperature)
Luminosity (Sun=1)
100 000
10 000
1000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
-10
-5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B−V)

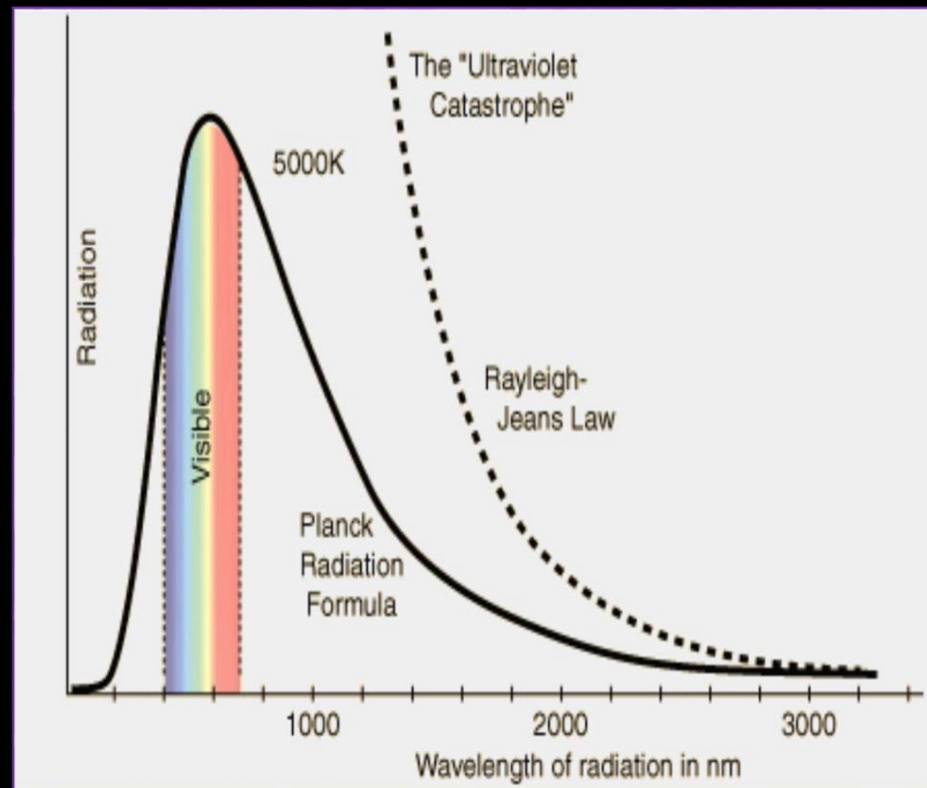
Stellar Astrophysics

- Stefan-Boltzmann Law:

$$F_{bol} = \sigma T^4; \sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67x10^{-5} ergs^{-1} cm^{-2} K^{-4}$$

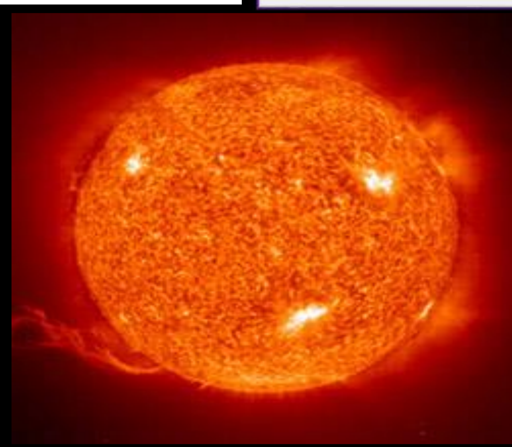
  - Effective temperature of a star: temperature of a black body with the same luminosity per surface area
  - Stars can be treated as black body radiators to a good approximation
  - Effective surface temperature can be obtained from the B-V color index with the Ballesteros equation:

$$T = 4600\left(\frac{1}{0.92(B-V)+1.70} + \frac{1}{0.92(B-V)+0.62}\right)$$



- Luminosity:

$$L = 4\pi r_*^2 \sigma T_E^4$$



Life History of Stars

Mass	Core Details	Comments
> $0.08M_{sun}$	Low mass ball of gas, not hot enough for hydrogen fusion	Stars in this mass range are not stars, but brown dwarfs of spectral type L and T.
$0.08M_{sun} < M < 0.5M_{sun}$	Fusion of H -> ^{4}He. Star is never hot enough to fuse ^{4}He to ^{12}C or ^{16}O.	Stars in this mass range are M on the main sequence. End up white dwarfs made of helium.
$0.5M_{sun} < M < 5M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O. Center is not hot enough to fuse ^{12}C and ^{16}O.	Stars in this mass range are A, F, G and K on the main sequence. End up white dwarfs made of ^{12}C and ^{16}O.
$5M_{sun} < M < 7M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O -> ^{20}Ne and ^{24}Mg.	Stars in this mass range are B on main sequence. End up as white dwarfs made of ^{20}Ne and ^{24}Mg.
$M > 7M_{sun}$	Fusion of H -> ^{4}He -> ^{12}C and ^{16}O -> ^{20}Ne and ^{24}Mg -> heavier elements .	Stars in this mass range are O on the main sequence. End up as neutron stars or black holes.

White dwarf

- Core of solar mass star
- Degenerate gas of oxygen and carbon
- No energy produced from fusion or gravitational contraction

Hot white dwarf NGC 2440. The white dwarf is surrounded by a "cocoons" of the gas ejected in the collapse toward the white dwarf stage of stellar evolution.

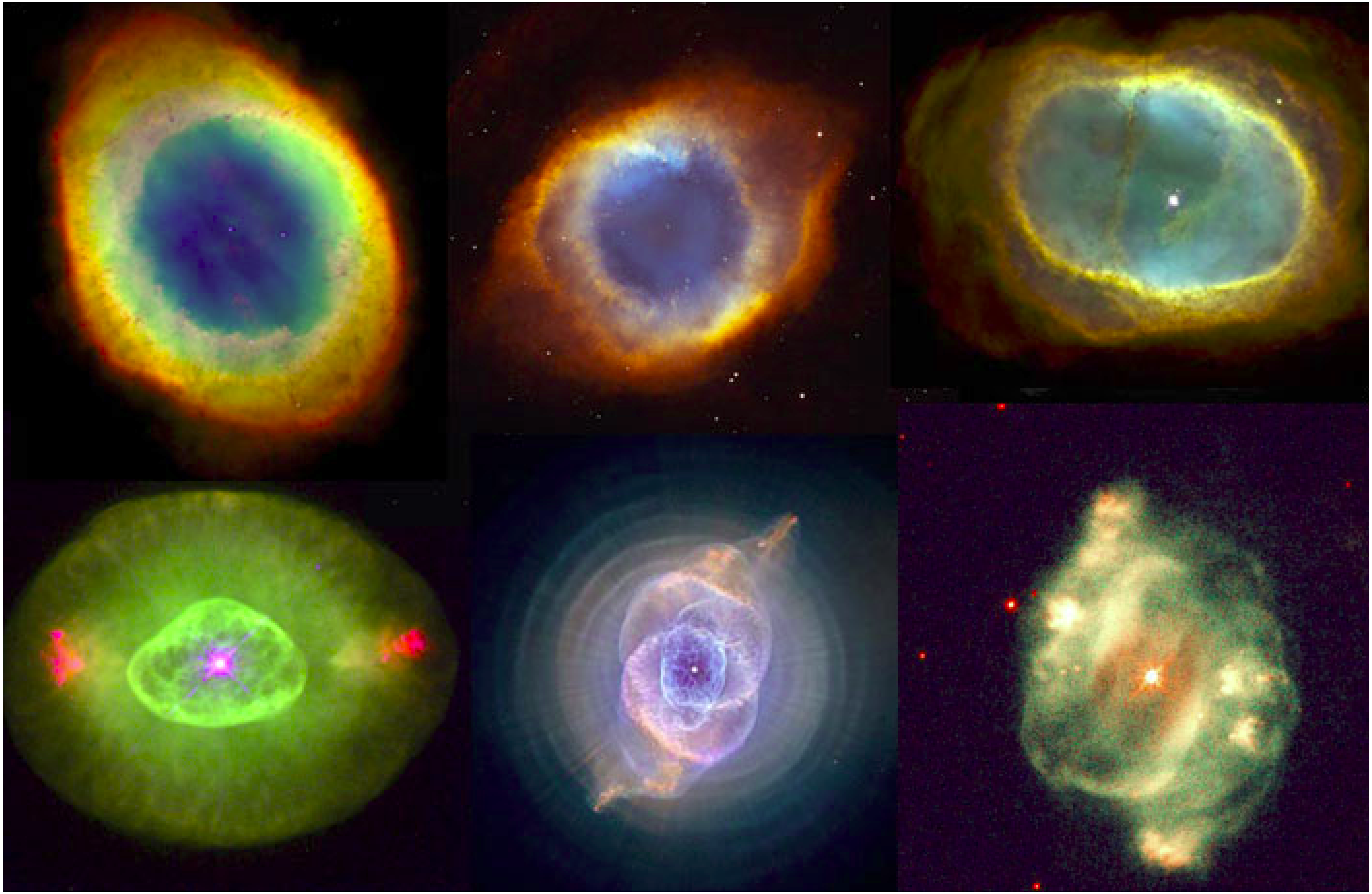
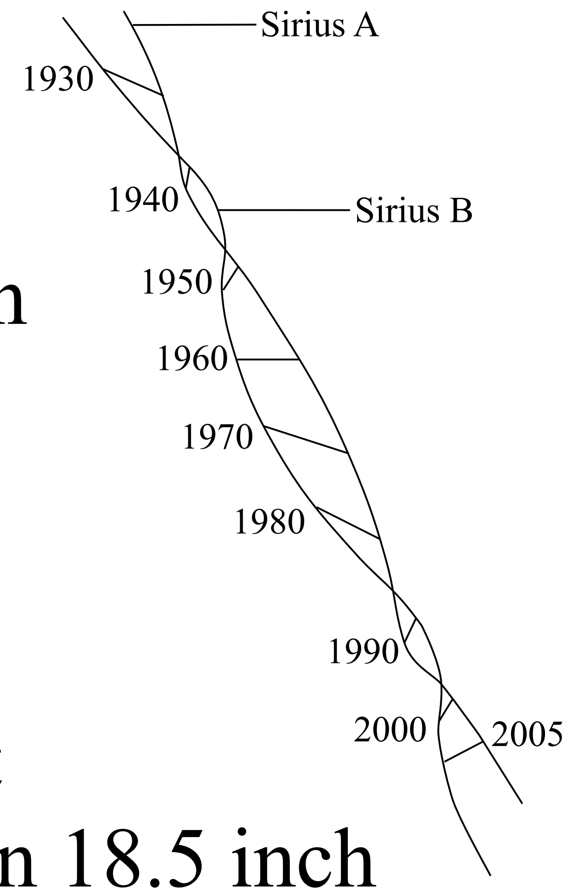
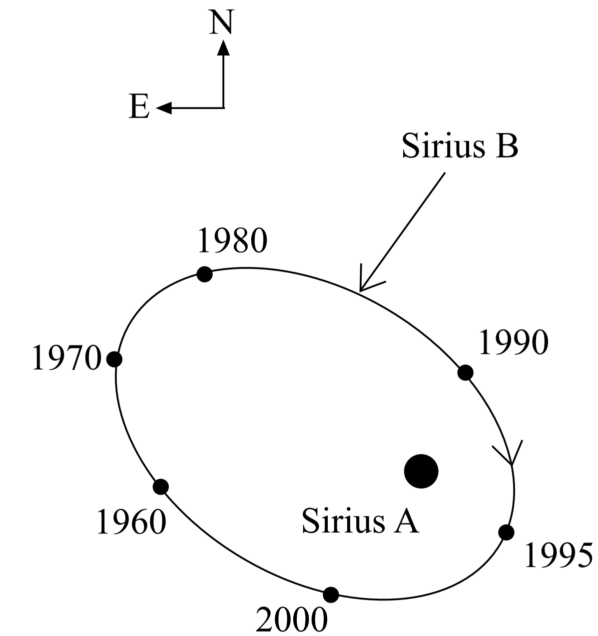


Figure 4.2 Several examples of planetary nebulae, newly formed white dwarfs that irradiate the shells of gas that were previously shed in the final stages of stellar evolution. The shells have diameters of $\approx 0.2 - 1$ pc. Photo credits: M. Meixner, T.A. Rector, B. Balick et al., H. Bond, R. Ciardullo, NASA, NOAO, ESA, and the Hubble Heritage Team

White Dwarfs





Sirius B is a white dwarf companion to Sirius A.

In 1844 German astronomer Friedrich Bessel deduced the existence of a companion star from changes in the proper motion of Sirius.

In 1862, astronomer Alvan Clark first observed the faint companion using an 18.5 inch refractor telescope at the Dearborn Observatory.

In 1915 Walter Adams observed the spectrum of the star, determining it was a faint whitish star. This lead astronomers to conclude it was a white dwarf.

Matter at Quantum Densities

As stars evolve, their cores contract and the core density increases. At some point the distance between the atoms is smaller than their de Broglie wavelengths and classical assumptions can no longer be used.

Recall: **de Broglie Wavelength**

$$\lambda = \frac{h}{p} = \frac{h}{(2mE)^{1/2}} \approx \frac{h}{(3mkT)^{1/2}}$$

Since,

$$p = mv$$

$$p = \sqrt{2mE}$$

$$E_K = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$E \sim \frac{3kT}{2}$$

mean energy of a particle

Question: Which will reach the quantum domain first, electrons or protons?

Although both electrons and protons share the same energy, electrons have smaller mass and longer wavelengths. The electron density will reach the quantum domain first.

When the inter particle spacing is of order 1/2 a de Broglie wavelength, quantum effects will become important.

$$\rho_q \approx \frac{m_p}{(\lambda/2)^3} = \frac{8m_p(3m_e kT)^{3/2}}{h^3}$$

Calculate the quantum density at the center of the sun ($T = 15 \times 10^6$ K).

$$\rho_q \approx \frac{8 \times 1.7 \times 10^{-24}\ \mathrm{g}\ (3 \times 9 \times 10^{-28}\ \mathrm{g} \times 1.4 \times 10^{-16}\ \mathrm{erg\ K^{-1}} \times 15 \times 10^6 \mathrm{K})^{3/2}}{(6.6 \times 10^{-27}\ \mathrm{erg\ s})^3}$$

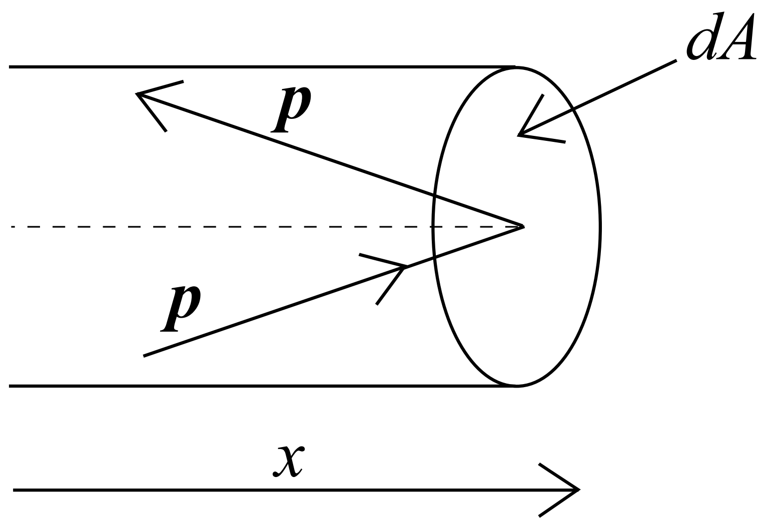
$$p_q = 640\ g\ cm^{-3}$$

The core density of the sun is 150 g cm^{-3}.
Much below the quantum regime.

Pressure Exerted by Ideal Gas

Consider ideal gas particles hitting the sides of a container.

Recall, that particles with momentum p_x impart $2p_x$ to the surface with each reflection.



The force per unit area imparted is then

$$\frac{dF_x}{dA} = \frac{2p_x}{dAdt} = \frac{2p_x v_x}{dAdx} = \frac{2p_x v_x}{dV}$$

where we used:

$$v_x = \frac{dx}{dt}$$

To get the pressure, we sum forces due to particles of all momenta.

$$P = \int_0^\infty dN(p) \frac{p_x v_x}{dV} dp$$

Note: half the particles are not moving towards walls.

Simplify:

$$\boxed{P = \int_0^\infty dN(p) \frac{p_x v_x}{dV} dp}$$

$$p_x v_x = m v_x^2 = \frac{1}{3} m v^2 = \frac{1}{3} p v$$

If we assume the velocities are isotropic: $v_x^2 = v_y^2 = v_z^2$

Substitute:

$$P = \frac{1}{3} \int_0^\infty n(p) p v dp$$

where we used: $dN/dV \equiv n$

For a non-relativistic degenerate gas:

$$\boxed{n_e(p)dp = \begin{cases} 8\pi p^2 \frac{dp}{h^3} & \text{if } |\mathbf{p}| \leq p_f \\ 0 & \text{if } |\mathbf{p}| > p_f \end{cases}}$$

$$\boxed{v = p/m_e}$$

$$P_e = \frac{1}{3} \int_0^{p_f} \frac{8\pi}{h^3} \frac{p^4}{m_e} dp = \frac{8\pi}{3h^3 m_e} \frac{p_f^5}{5}$$

$$= \left(\frac{3}{8\pi}\right)^{2/3} \frac{h^2}{5m_e} n_e^{5/3}$$

$$\boxed{n_e = \frac{8\pi}{3h^3} p_f^3}$$

Finally, noting $n_e = Zn_+ = Z\rho/Am_p$

$$\boxed{P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{\mathcal{Z}}{A}\right)^{5/3} \rho^{5/3}}$$

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{\mathcal{Z}}{A}\right)^{5/3} \rho^{5/3}$$

Comments:

The electron pressure does not depend on temperature.

For a typical white dwarf, $\rho \sim 10^6$ g cm^{-3} and T ~ 10^7 K.
Their Z/A ~ 0.5.

$$P_e \sim \frac{(6.6 \times 10^{-27}\ \mathrm{erg\ s})^2}{20 \times 9 \times 10^{-28}\ \mathrm{g}\ (1.7 \times 10^{-24}\ \mathrm{g})^{5/3}} 0.5^{5/3} (10^6\ \mathrm{g\ cm}^{-3})^{5/3} \ = 3 \times 10^{22}\ \mathrm{dyne\ cm}^{-2}$$

Compare to the thermal pressure of nuclei at this temperature.

$$P = n\ \ kT = 2 \times 10^{20}\ dyne\ cm^{-2}$$

Thus, degenerate electron pressure completely dominates the pressure in these stars.

Properties of White Dwarfs

Mass-Radius Relationship:

Recall the EOS for a degenerate non-relativistic electron gas:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20 m_e m_p^{5/3}} \left(\frac{Z}{A}\right)^{5/3} \rho^{5/3}.$$

The scaling relation for this equation is

$$P \sim b\rho^{5/3} \sim b\frac{M^{5/3}}{r^5}$$

where b is a constant

Recall our scaling relations from the equations of stellar structure:

$$P \sim \frac{GM\rho}{r} \sim \frac{GM^2}{r^4}$$

Equating these pressures yields:

$$r \sim \frac{b}{G} M^{-1/3}$$

What?

Notice: The radius decreases with increasing mass!

Re-derive scaling relations between mass and radius with an index $(4 + \varepsilon)/3$.

$$P \sim b\rho^{5/3} \quad \longrightarrow \quad P \sim \rho^{(4+\epsilon)/3} = \frac{M^{(4+\epsilon)/3}}{r^{(4+\epsilon)}}$$

Equating with pressure from our stellar equations.

$$\boxed{P \sim \frac{GM\rho}{r} \sim \frac{M^2}{r^4}}$$

$$\frac{M^{4/3}M^{\epsilon/3}}{r^4 r^{\epsilon}} = \frac{M^{(4+\epsilon)/3}}{r^{4+\epsilon}} \sim \frac{M^2}{r^4}$$

$$r^{\epsilon} \sim M^{(\epsilon-2)/3}$$

$$r \sim M^{(\epsilon-2)/3\epsilon}$$

When $\epsilon \rightarrow 0$

$$r \rightarrow M^{-\infty} = 0$$

$$r \sim \frac{b}{G} M^{-1/3}$$

A white dwarf with Z/A = 0.5 and M = 1M_{sun} has a radius of ~ 4000 km.

Fully working out the equations of stellar structure gives an equation for radius of

$$\boxed{r_{\mathrm{wd}} = 2.3 \times 10^9 \mathrm{cm} \left(\frac{\mathcal{Z}}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3}}$$

mass increases → radius decreases → density increases → electrons pushed to larger momenta

Mass/radius relation for degenerate star

- Stellar mass = *M;* radius = *R*
- Gravitational potential energy: $$Egr = -\frac{3GM^2}{5R}$$
- Heisenberg uncertainty: $$\Delta x \Delta p \geq \hbar$$
- Electron density: $$n = \frac{3N}{4\pi R^3} \approx \frac{M}{m_p R^3}$$

$$\Delta x \approx n^{-1/3} \qquad \Delta p \approx \frac{\hbar}{\Delta x} \approx \hbar n^{1/3}$$

- Kinetic energy: $$\varepsilon = \frac{p^2}{2m_e} \qquad K = N\varepsilon = \frac{M}{m_p}\varepsilon \approx \frac{\hbar^2 M^{5/3}}{m_e m_p^{5/3} R^2}$$

Mass/radius relation for degenerate star

- Total energy:

$$E = K + U \approx \frac{\text{h}^2 M^{5/3}}{m_e m_p^{5/3} R^2} - \frac{GM^2}{R}$$

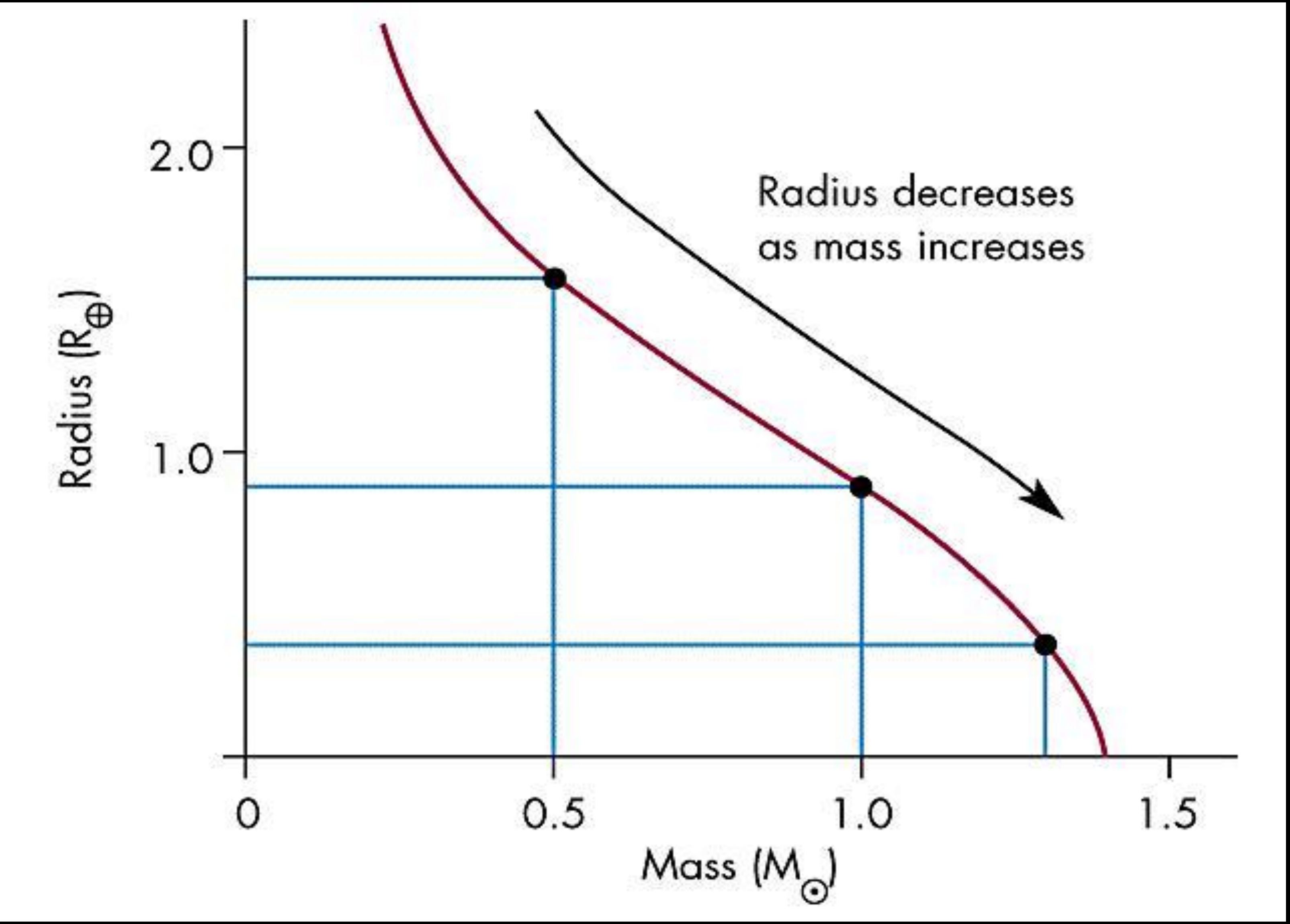
- Find *R* by minimizing *E:*

$$\frac{dE}{dR} \approx -\frac{\text{h}^2 M^{5/3}}{m_e m_p^{5/3} R^3} + \frac{GM^2}{R^2} = 0$$

- Radius decreases as mass increases:

$$R \approx \frac{\text{h}^2 M^{-1/3}}{G m_e m_p^{5/3}}$$

Mass vs radius relation



$$r \rightarrow M^{-\infty} = 0$$

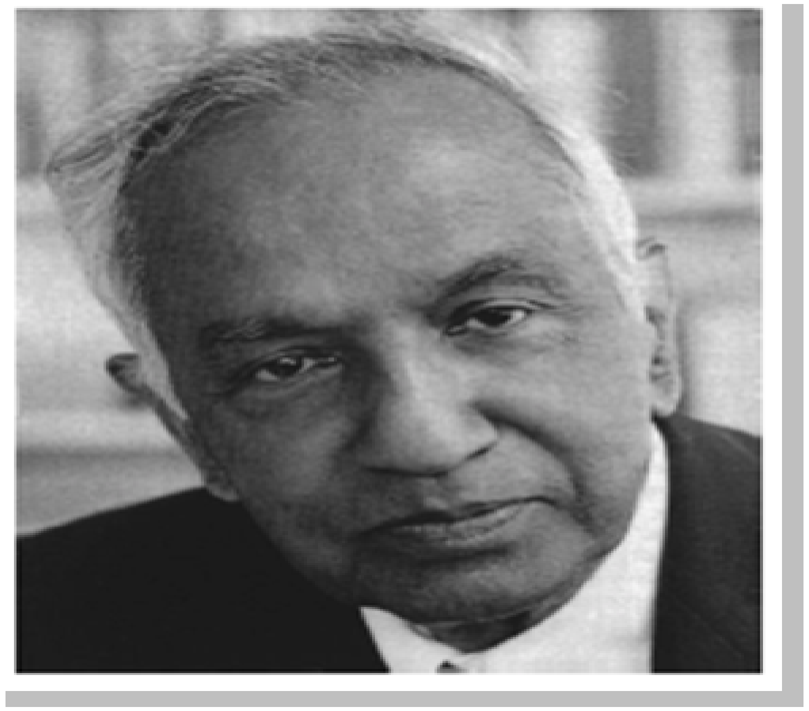
<u>What does it mean?</u>

At masses so high that electrons become ultra-relativistic, the electron pressure is unable to support the star against gravity.

If the density is high enough, degeneracy pressure due to protons and neutrons begins to operate. Stops collapse and produces a neutron star.

<u>Chandrasekhar Mass:</u>

The maximum stellar mass that can be supported by electron degeneracy pressure.



Subrahmanyan Chandrasekhar (1910–1995)

Estimate Chandrasekhar Mass

Start with virial theorem

$$\bar{P}V = -\frac{1}{3}E_{\mathrm{gr}}$$

Substitute the ultra-relativistic electron degeneracy pressure and self gravity

$$\boxed{P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{\mathcal{Z}}{A}\right)^{4/3} \rho^{4/3}}$$

$$\left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{\mathcal{Z}}{A}\right)^{4/3} \rho^{4/3} V \sim \frac{1}{3}\frac{GM^2}{r}$$

$$\boxed{\rho \sim \frac{M}{V}}$$

$$\boxed{V = \frac{4\pi}{3}r^3}$$

Simplify:

$$M \sim 0.11 \left(\frac{\mathcal{Z}}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p$$

Full Solution using Equations of Stellar Structure:

$$\boxed{M_{\mathrm{ch}} = 0.21 \left(\frac{\mathcal{Z}}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{3/2} m_p}$$

Accurately calculated value is 1.4 M_{sun}.

As electron velocities increase, the rates at which momentum transfers approaches c. So, we need to modify the EOS for degenerate electron gas.

$$P_e = \left(\frac{3}{8\pi}\right)^{1/3} \frac{hc}{4m_p^{4/3}} \left(\frac{\mathcal{Z}}{A}\right)^{4/3} \rho^{4/3}$$

← **EOS for an ultra-relativistic degenerate spin-1/2 fermion gas**

Compare to non-relativistic case:

$$P_e = \left(\frac{3}{\pi}\right)^{2/3} \frac{h^2}{20m_e m_p^{5/3}} \left(\frac{\mathcal{Z}}{A}\right)^{5/3} \rho^{5/3}$$

← **EOS for a degenerate non-relativistic electron gas**

Notes: The power index changes.

The electron mass disappears.

For ultra-relativistic particles, the rest mass is negligible.

As we go from small to large white dwarf masses, we transition gradually from non-relativistic to ultra-relativistic.

Notes:

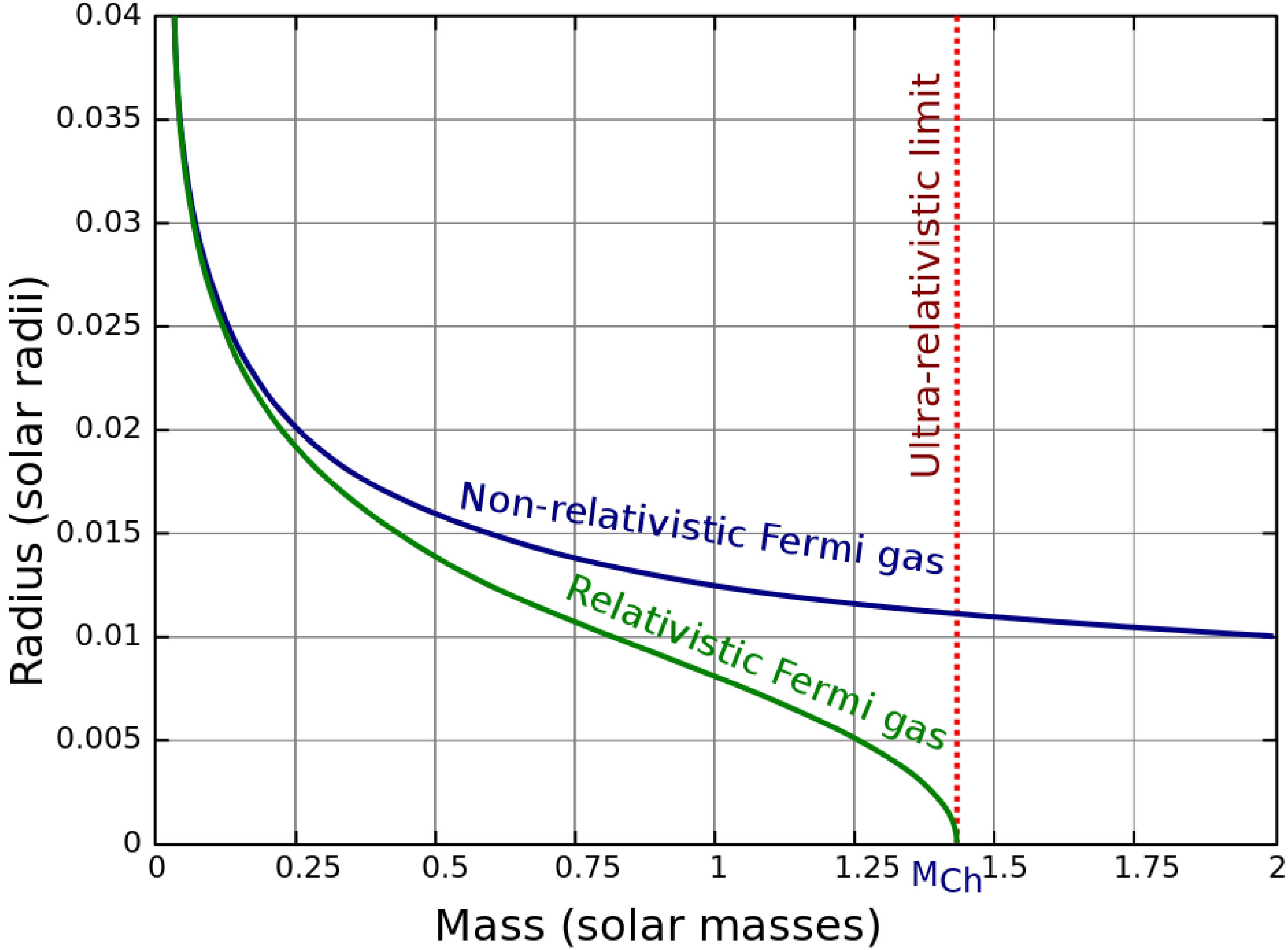
The accurately calculated Chandrashaker mass is 1.4 $M_{sun.}$

No white dwarfs with masses greater than M_{ch} have ever been found.

The lower bound of isolated white dwarfs found is 0.25 $M_{sun.}$
Why is there a lower bound?

The universe is too young! Stars that have mass < 0.8 M_{sun} could produce smaller white dwarfs. However, even if they were formed in the early universe, they have not yet gone though their main sequence lifetime.

Mass vs radius relation



White Dwarf Cooling

The temperature inside a white dwarf is approximately constant with radius. Let's estimate the temperature.

The white dwarf contacts until the degeneracy pressure stops the contraction of the thermal core. Just before equilibrium:

$$E_{\rm th} \sim \frac{1}{2}\frac{GM^2}{r} = \frac{3}{2}NkT$$

What are WD composed of?

Ans: Helium!

$N_{nuclei} = M/4m_H$ and $N_e = M/2m_H$.

$$E_{\rm th} = \frac{3}{2}\frac{M}{m_p}(\frac{1}{2}+\frac{1}{4})kT = \frac{9}{8}\frac{M}{m_p}kT$$

Thus,

$$\frac{1}{2}\frac{GM^2}{r} \sim \frac{9}{8}\frac{M}{m_p}kT \longrightarrow kT \sim \frac{4}{9}\frac{GMm_p}{r}$$

$$kT \sim \frac{4}{9}\frac{GMm_p}{r} \qquad r_{\rm wd} \sim \frac{h^2}{20m_e m_p^{5/3} G}\left(\frac{\mathcal{Z}}{A}\right)^{5/3} M^{-1/3}$$

Put it all together and we have:

$$\boxed{kT \sim \frac{80G^2 m_e m_p^{8/3}}{9h^2}\left(\frac{\mathcal{Z}}{A}\right)^{-5/3} M^{4/3}}$$

For a 0.5 M_{sun} white dwarf this give T ~ 8 x 10^8 K.

The WD is then endpoint in stellar evolution. No nuclear reactions occur. Hence, it cools over time by radiating it's energy. The radiated luminosity is given by

$$L = 4\pi r_{\rm wd}^2 \sigma T_E^4$$

We will assume that white dwarf is at a constant temperature and estimate the cooling time.

$$L = 4\pi r_{\mathrm{wd}}^2 \sigma T_E^4 \qquad E_{th} = \frac{3}{8}\frac{M}{m_p}kT$$

Putting it together:

We only take nuclei, not electrons.

$$4\pi r_{\mathrm{wd}}^2 \sigma T^4 \sim \frac{dE_{\mathrm{th}}}{dt} = \frac{3Mk}{8m_p}\frac{dT}{dt}$$

$$dt = \frac{3Mk}{32\pi\sigma m_p r^2} T^{-4} dT$$

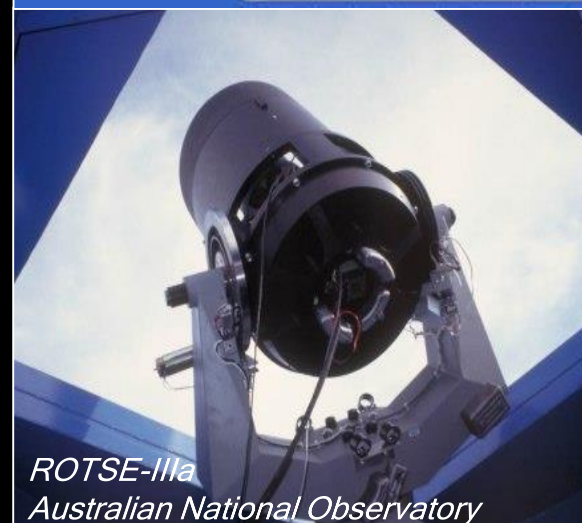
Integral is left to the student. Put in M = $0.5M_{\mathrm{sun}}$ and r_{wd} = 4000 km.

$$\tau_{\mathrm{cool}} \sim \frac{3Mk}{8m_p 4\pi r_{\mathrm{wd}}^2 \sigma 3T^3} = 3\times 10^9 \mathrm{yr}\left(\frac{T}{10^3\mathrm{K}}\right)^{-3}$$

It would take our WD several Gyr to cool to 10^3 K. In reality, the insulating non-degenerate surface layers would result in an even slower cooling rate. Detailed models take this and other effects into account. For carbon/oxyen WD, cooling over 10^{10} yrs only brings temperatures down to 3000 - 4000 K. This explains the high temperatures (and blue/white colors).

ROTSE

- Robotic Optical Transient Search Experiment
- Original purpose: Observe GRB optical counterpart (“afterglow”)
- Observation & detection of optical transients (seconds to days)
- Robotic operating system
  - Automated interacting Linux daemons
  - Sensitivity to short time-scale variation
  - Efficient analysis of large data stream
  - Recognition of rare signals
- Current research:
  - GRB response
  - SNe search (RSVP)
  - Variable star search
  - Other transients: AGN, CV (dwarf novae), flare stars, novae, variable stars, X-ray binaries



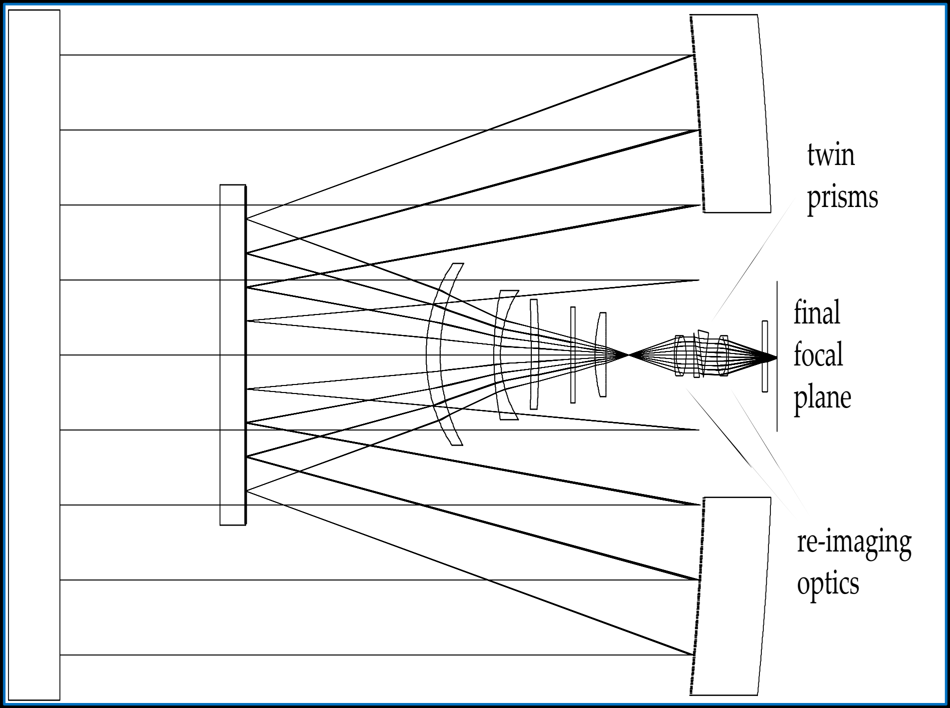
ROTSE-IIIa
Australian National Observatory

ROTSE-I

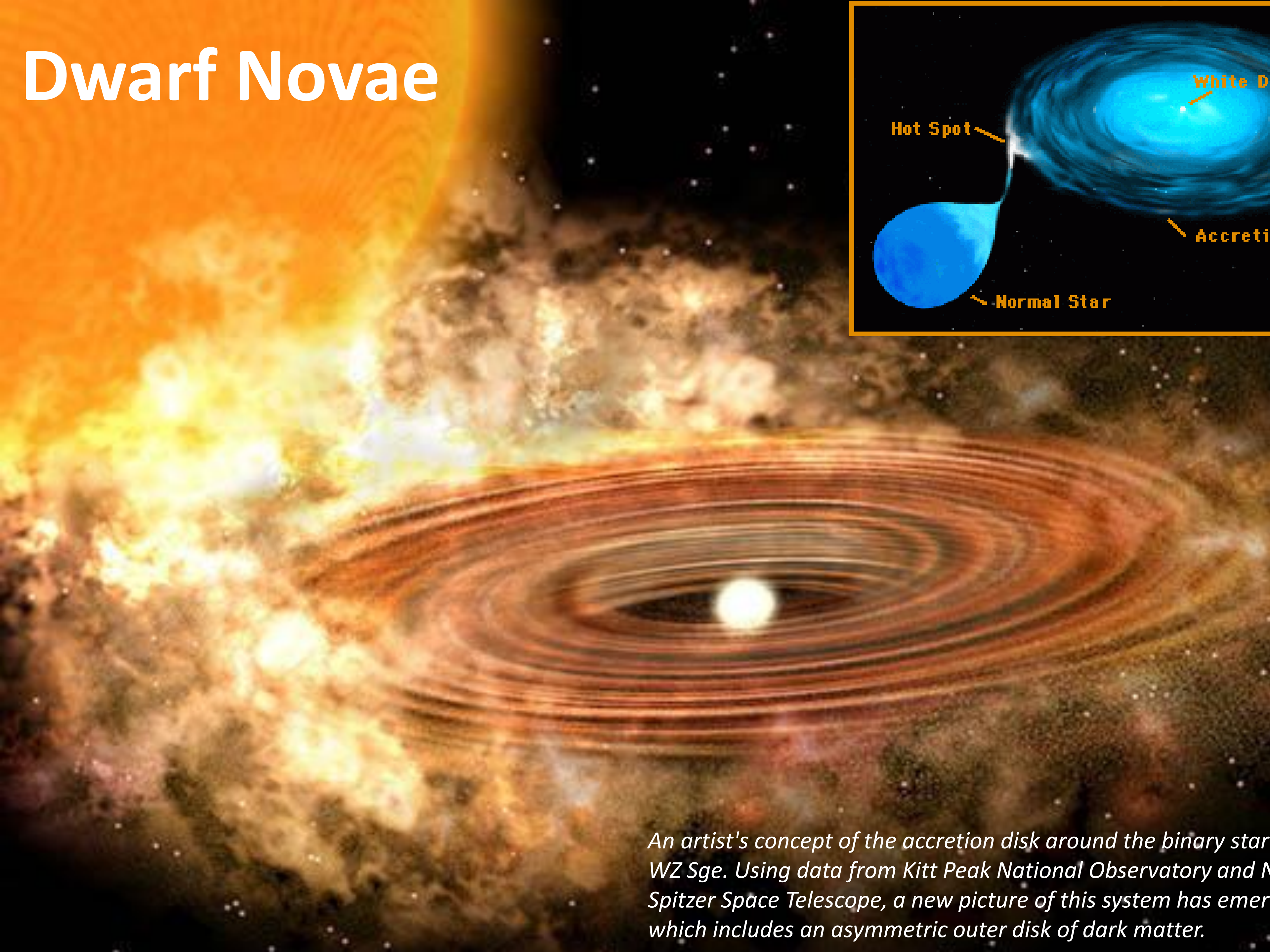
- 1st successful robotic telescope
- 1997-2000; Los Alamos, NM
- Co-mounted, 4-fold telephoto array (Cannon 200 mm lenses)
- CCD
  - 2k x 2k Thomson
  - “Thick”
  - Front illuminated
  - Red sensitive
  - R-band equivalent
  - Operated “clear” (unfiltered)
- Optics
  - Aperture (cm): 11.1
  - f-ratio: 1.8
  - FOV: 16°×16°
- Sensitivity (magnitude): 14-15
  - Best: 15.7
- Slew time (90°): 2.8 s
- 990123: Observed 1st GRB afterglow in progress
  - Landmark event
  - Proof of concept

ROTSE-III

- 2003 – present
- 4 Cassegrain telescopes
- CCD
  - “Thin”
  - Back illuminated
  - Blue-sensitive
  - High QE (UBVRI bands)
  - Default photometry calibrated to R-band
- Optics
  - Aperture (cm): 45
  - f-ratio: 1.9
  - FOV: 1.85°×1.85°
- Sensitivity (magnitude): 19-20
- Slew time: < 10 s



Dwarf Novae

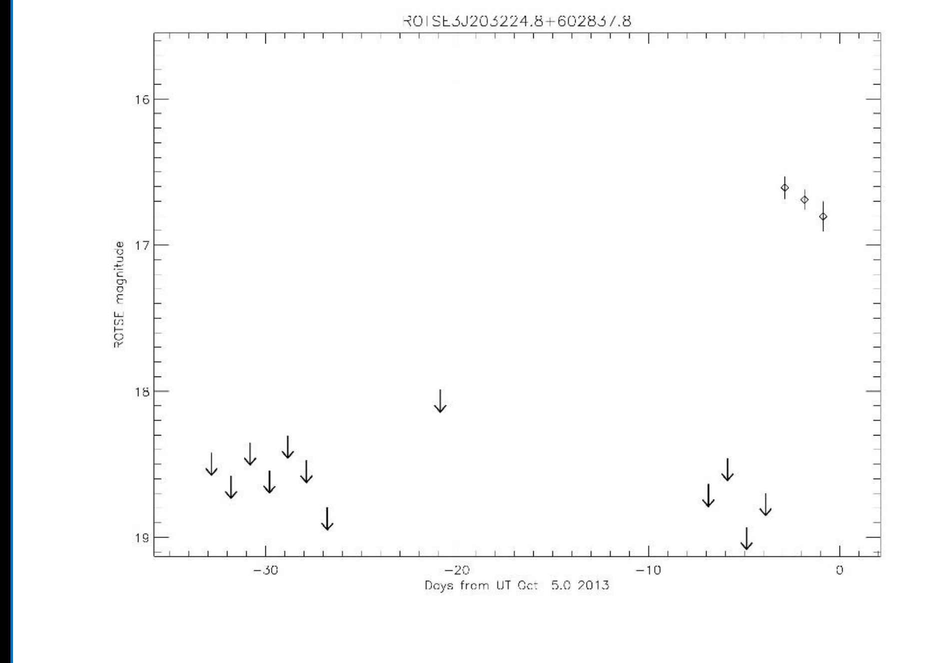


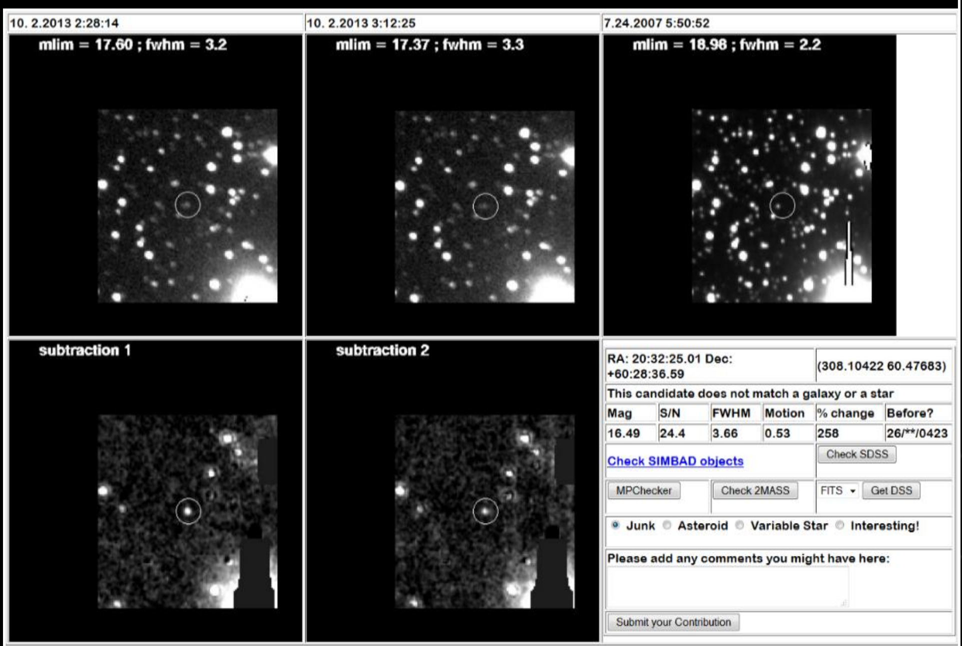
An artist's concept of the accretion disk around the binary star WZ Sge. Using data from Kitt Peak National Observatory and N Spitzer Space Telescope, a new picture of this system has emer which includes an asymmetric outer disk of dark matter.

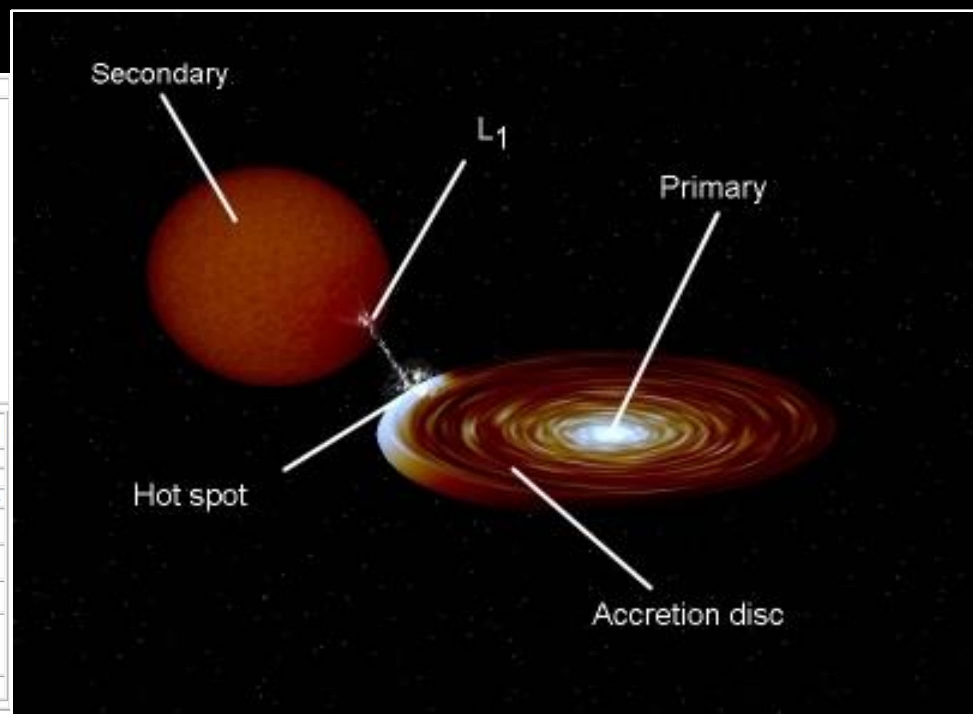
ROTSE3 J203224.8+602837.8

- 1st detection (110706):
  - ROTSE-IIIb & ROTSE-IIId
  - ATel #2126
- Outburst (131002 – 131004):
  - ROTSE-IIIb
  - ATel #5449
- Magnitude (max): 16.6
- (RA, Dec) = (20:32:25.01, +60:28:36.59)
- UG Dwarf Nova
  - Close binary system consisting of a red dwarf, a white dwarf, & an accretion disk surrounding the white dwarf
  - Brightening by 2 - 6 magnitudes caused by instability in the disk
  - Disk material infalls onto white dwarf

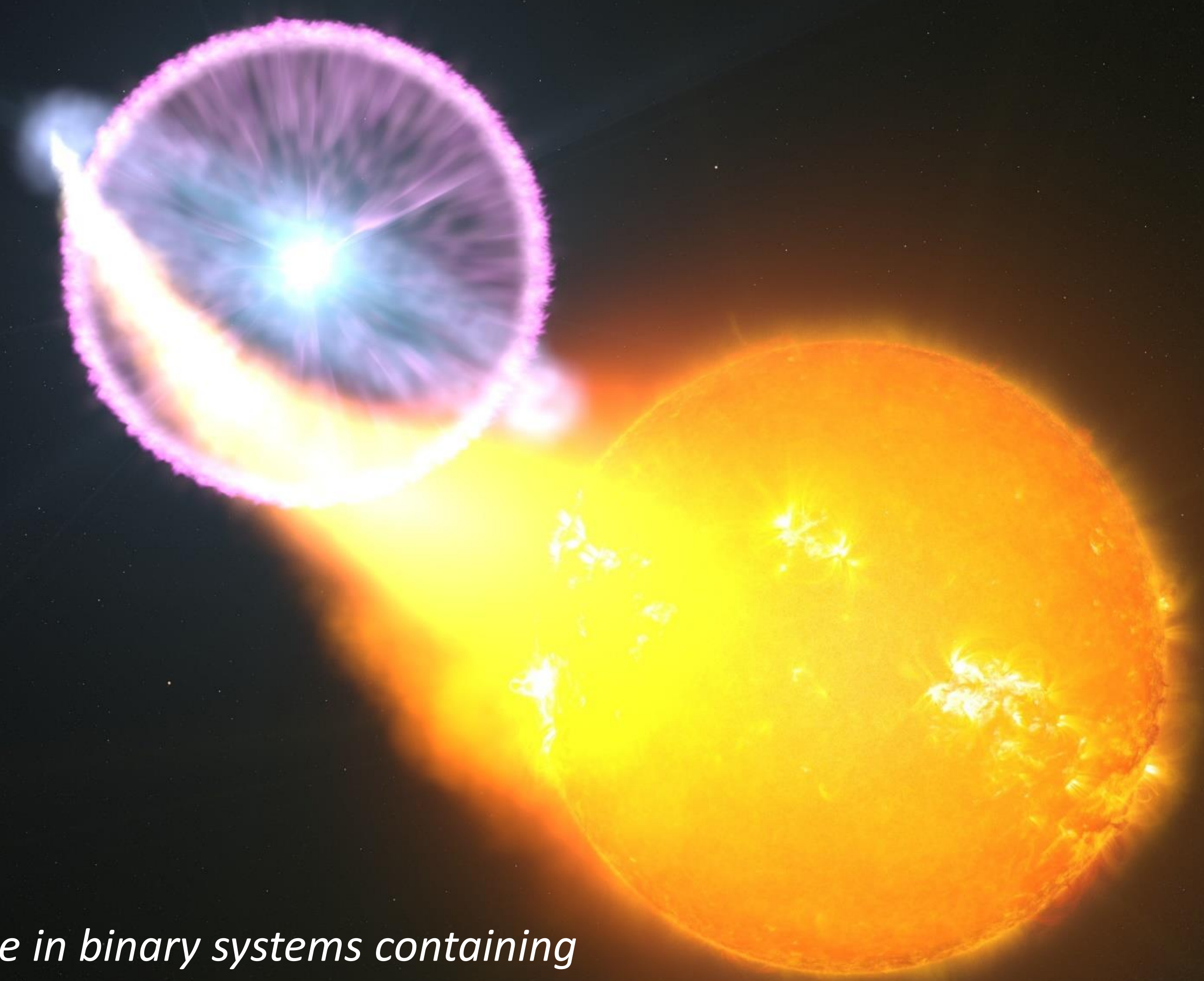
"Damien"







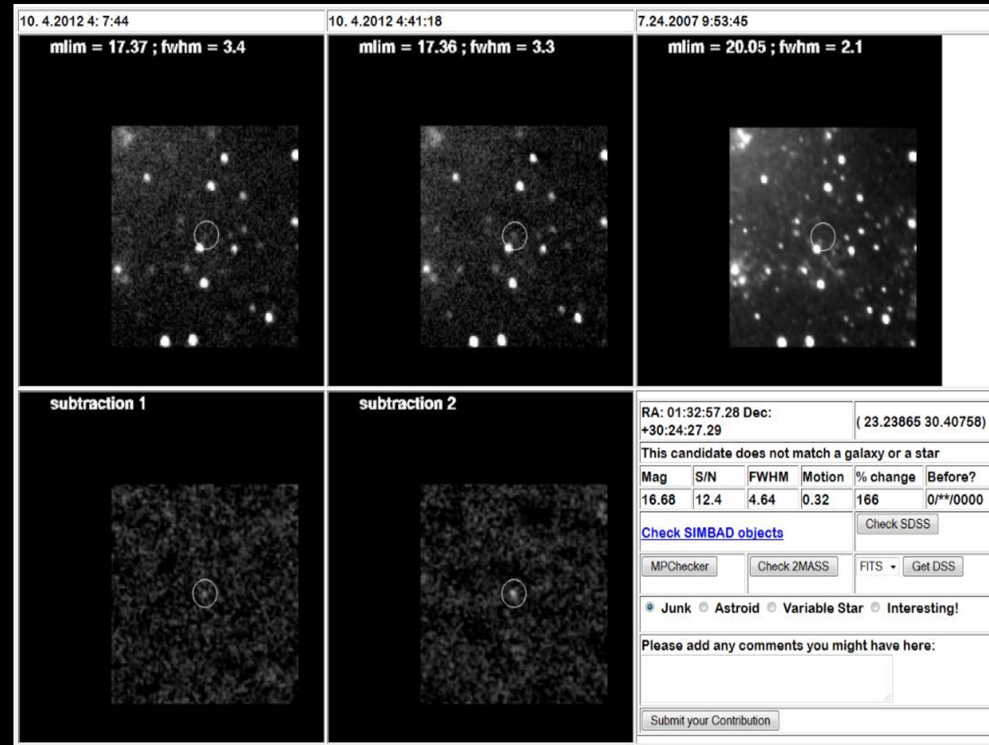
Novae (classical)



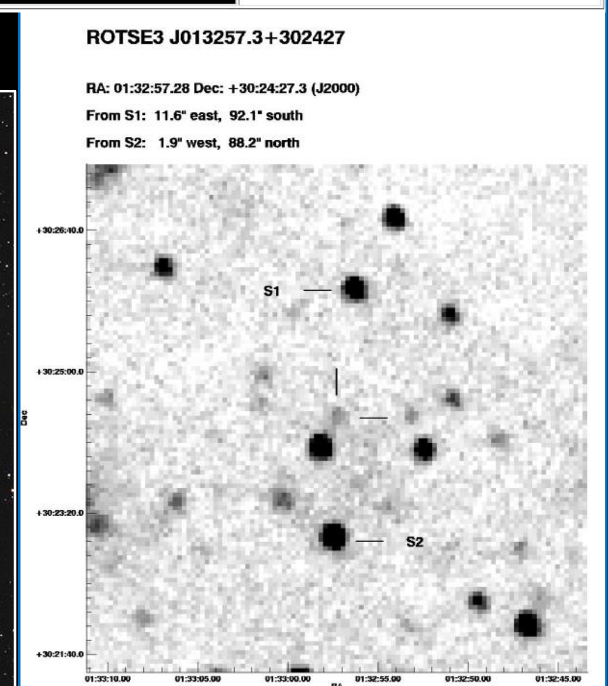
Novae typically originate in binary systems containing sun-like stars, as shown in this artist's rendering.

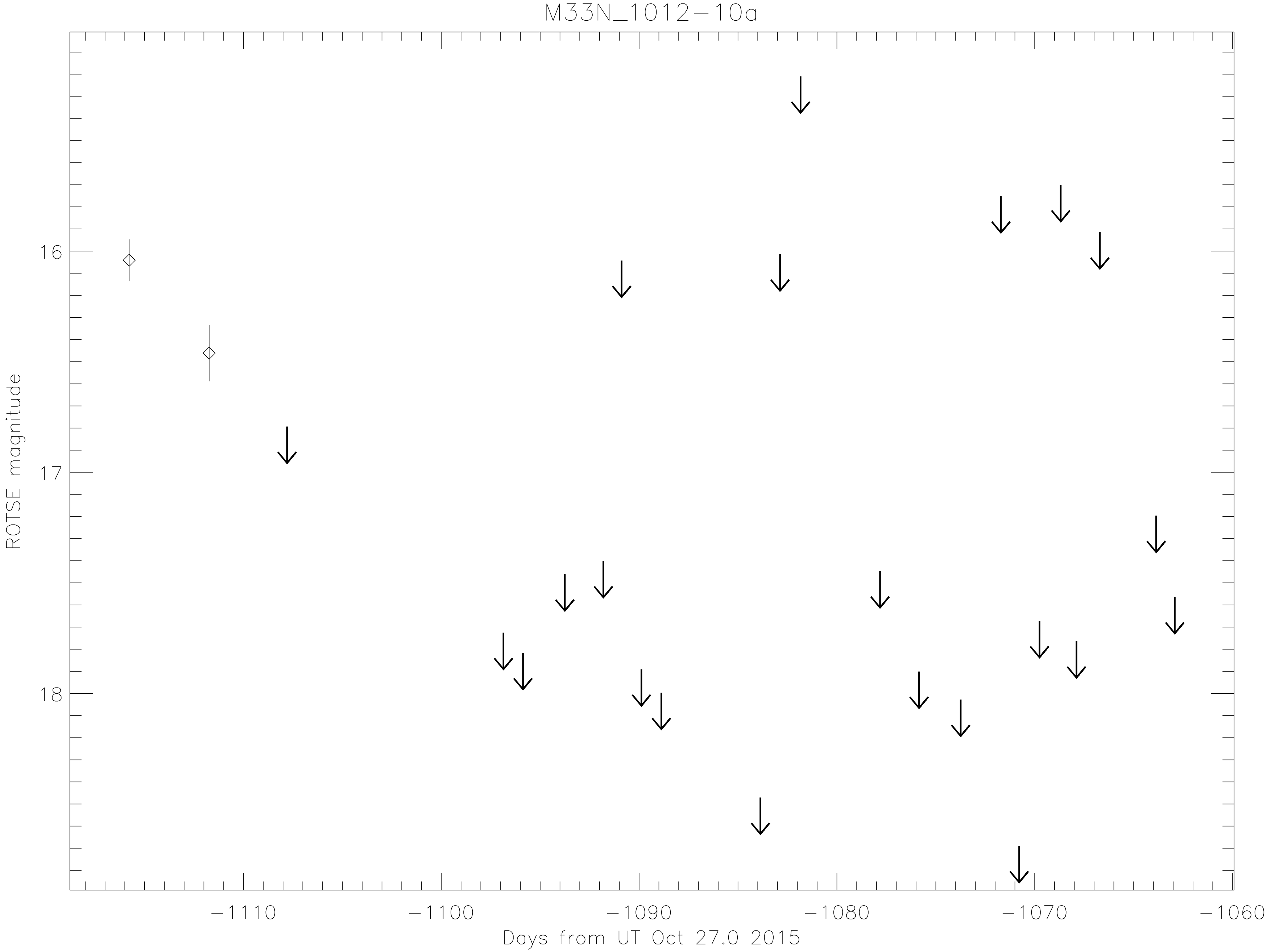
M33N 2012-10a

- 1st detection: 121004 (ROTSE-IIIb)
- (RA, Dec) = (01:32:57.3, +30:24:27)
- Constellation: Triangulum
- Host galaxy: M33
- Magnitude (max): 16.6
- z = 0.0002 (~0.85 Mpc, ~2.7 Mly)
- Classical nova
  - Explosive nuclear burning of white dwarf surface from accumulated material from the secondary
  - Causes binary system to brighten 7 - 16 magnitudes in a matter of 1 to 100s days
  - After outburst, star fades slowly to initial brightness over years or decades
- CBET 3250




M33 Triangulum Galaxy

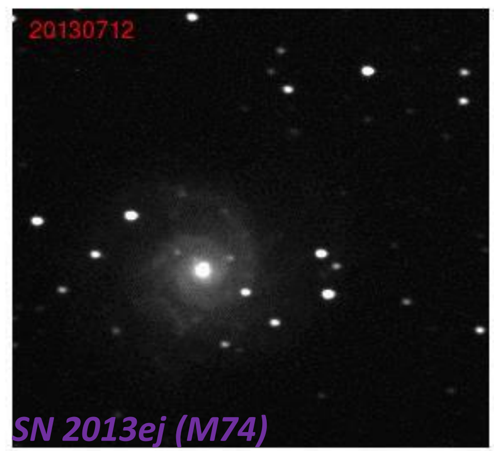




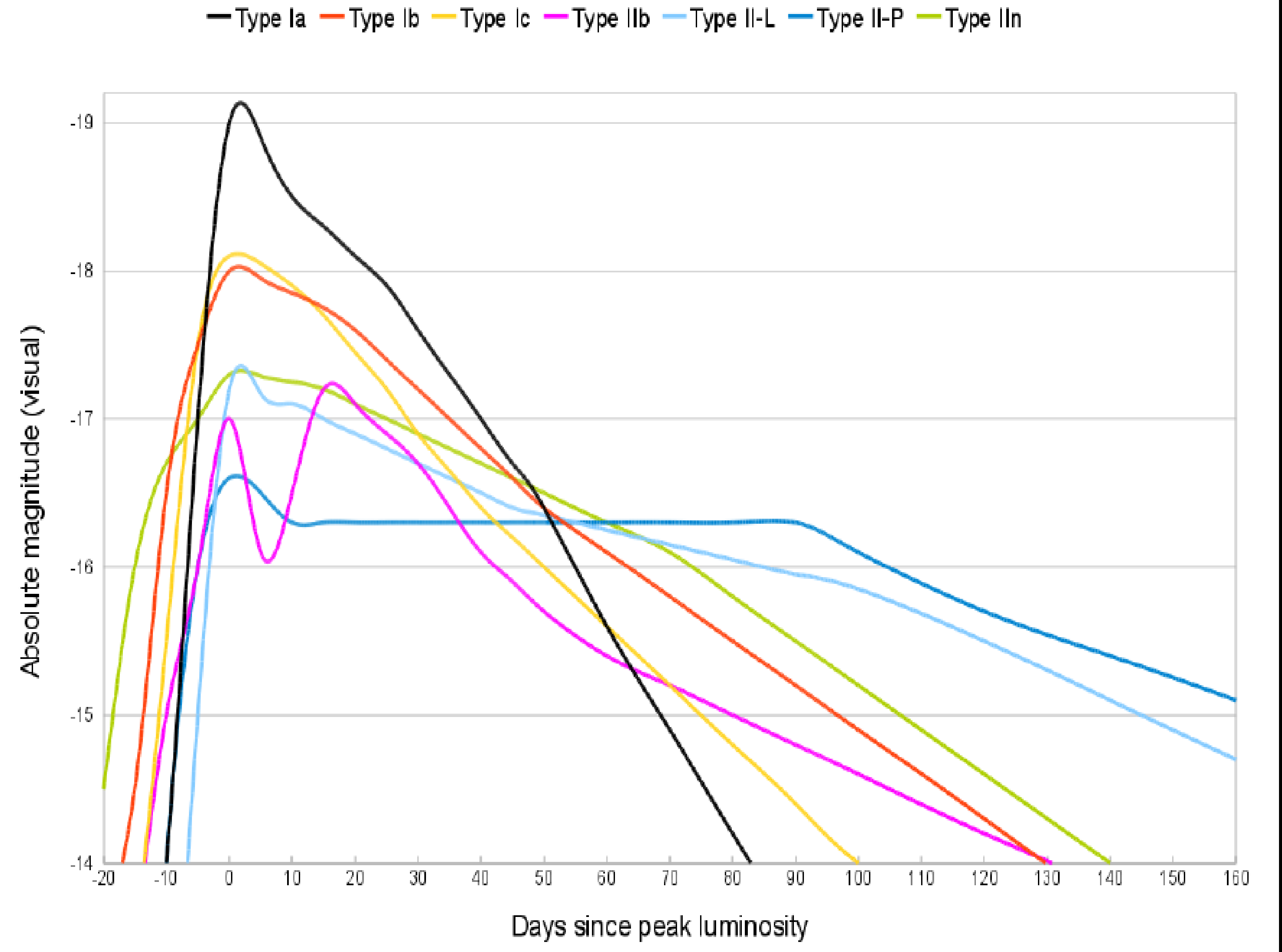
M33N_1012-10a
ROTSE magnitude
16
17
18
−1110
−1100
−1090
−1080
−1070
−1060
Days from UT Oct 27.0 2015

Supernovae

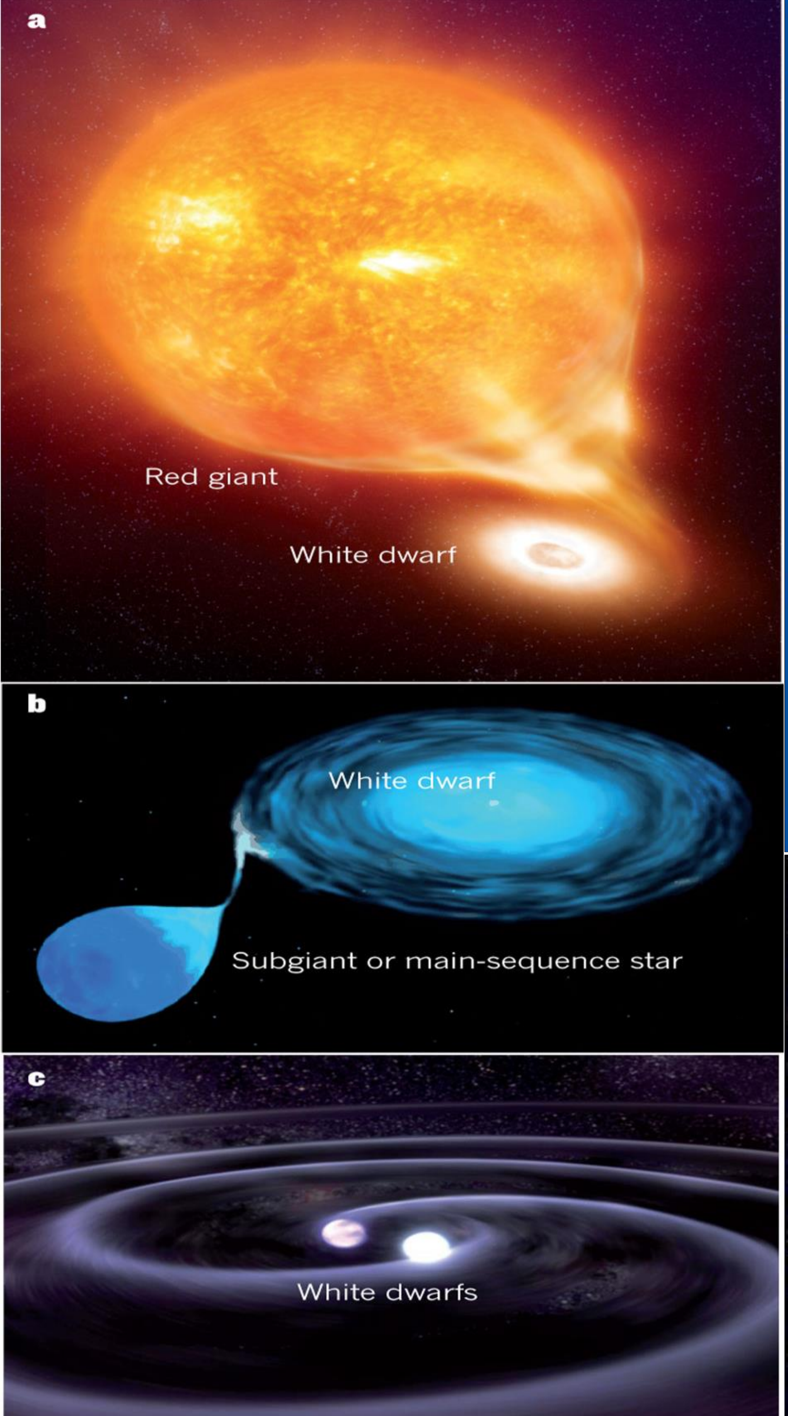
SN 1994D (NGC 4526)

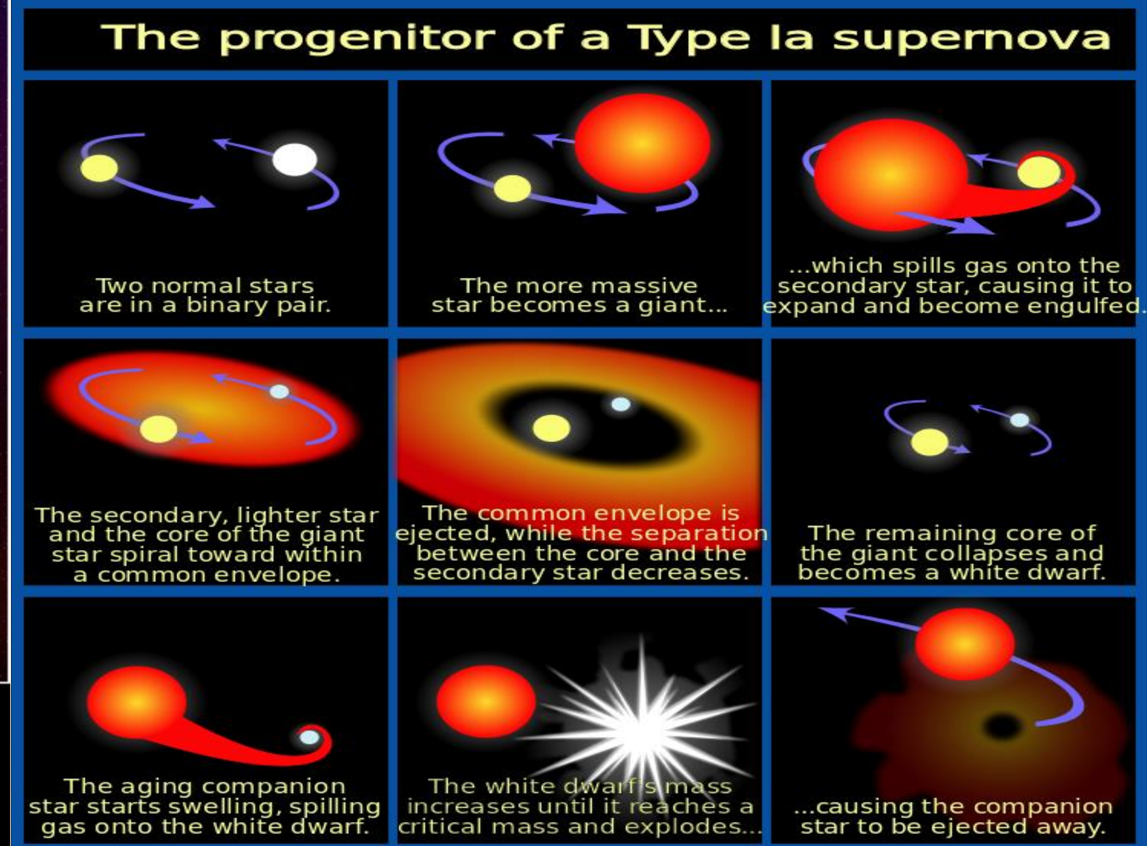


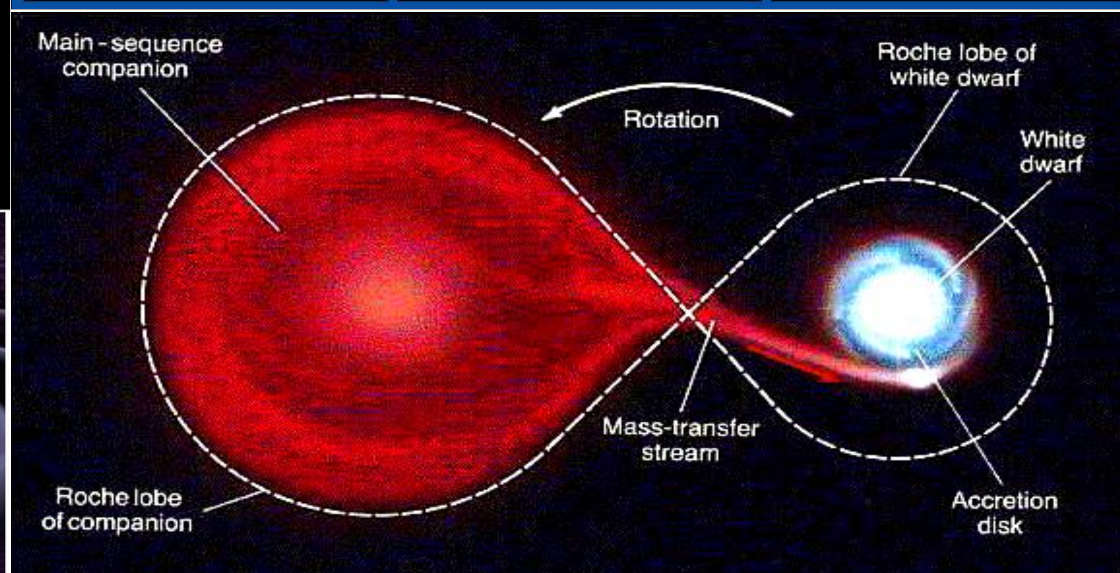
SN 2013ej (M74)



Type Ia
Type Ib
Type Ic
Type IIb
Type II-L
Type II-P
Type IIn
Absolute magnitude (visual)
-19
-18
-17
-16
-15
-14
-20
-10
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
Days since peak luminosity


a
Red giant
White dwarf
b
White dwarf
Subgiant or main-sequence star
c
White dwarfs

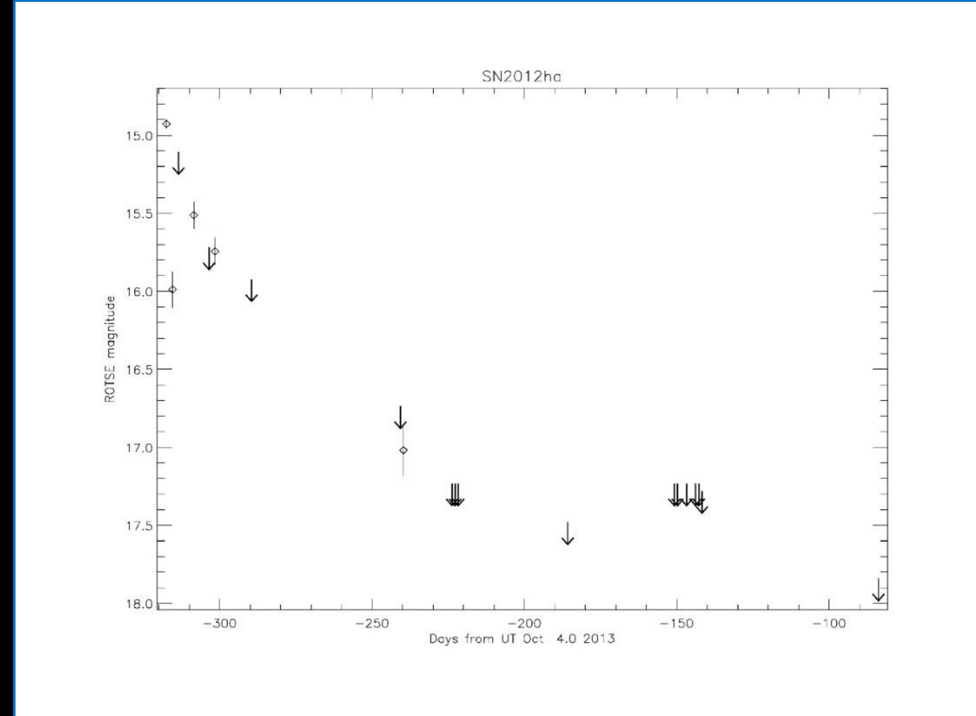

The progenitor of a Type Ia supernova
Two normal stars are in a binary pair.
The more massive star becomes a giant...
...which spills gas onto the secondary star, causing it to expand and become engulfed.
The secondary, lighter star and the core of the giant star spiral toward within a common envelope.
The common envelope is ejected, while the separation between the core and the secondary star decreases.
The remaining core of the giant collapses and becomes a white dwarf.
The aging companion star starts swelling, spilling gas onto the white dwarf.
The white dwarf's mass increases until it reaches a critical mass and explodes...
...causing the companion star to be ejected away.

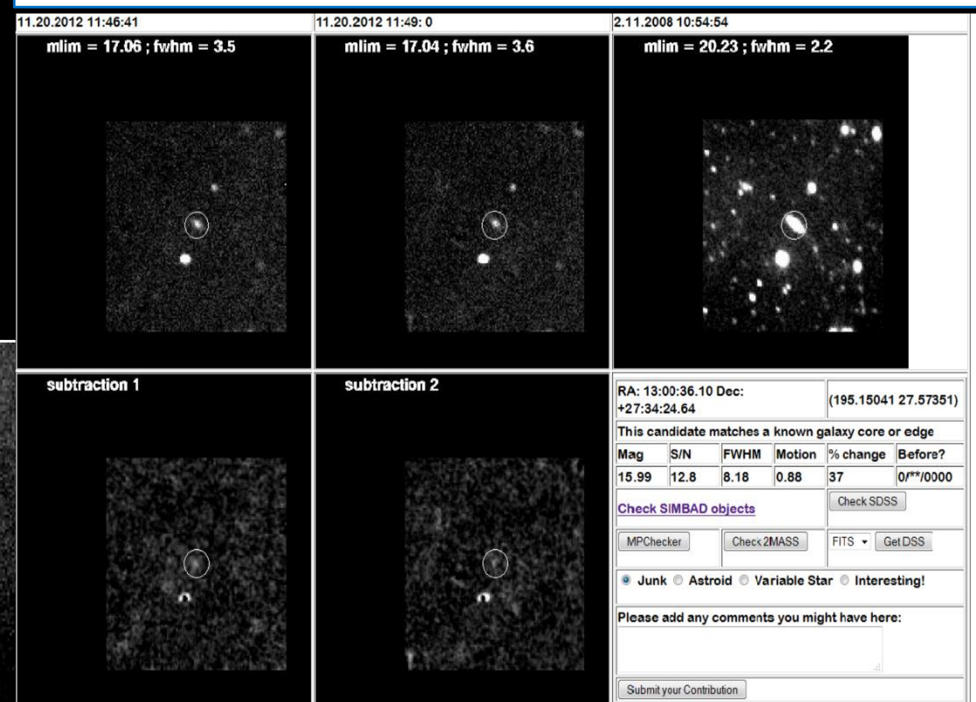

Main-sequence companion
Rotation
Roche lobe of white dwarf
White dwarf
Mass-transfer stream
Roche lobe of companion
Accretion disk

[SN 2012cg](#) (*NGC 4424)*

SN 2012ha *("Sherpa")*

- 1st detection: 121120 (ROTSE-IIIb)
- Type: Ia-normal
  - Electron degeneracy prevents collapse to neutron star
  - Single degenerate progenitor: C-O white dwarf in binary system accretes mass from companion (main sequence star)
  - Mass → Chandrasekhar limit (1.44 $M_{\odot}$)
  - Thermonuclear runaway
  - Deflagration or detonation?
  - Standardizable candles
    - acceleration of expansion
    - dark energy
- Magnitude (max): 15.0
- Observed 1 month past peak brightness
- (RA, Dec) = (13:00:36.10, +27:34:24.64)
- Constellation: Coma Berenices
- Host galaxy: PGC 44785
- *z* = 0.0170 (~75 Mpc; ~240 Mly)
- CBET 3319

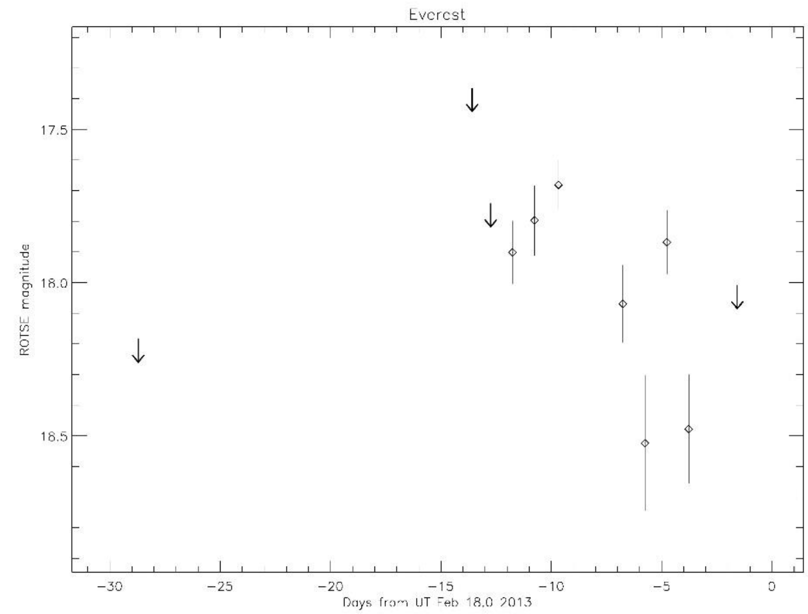


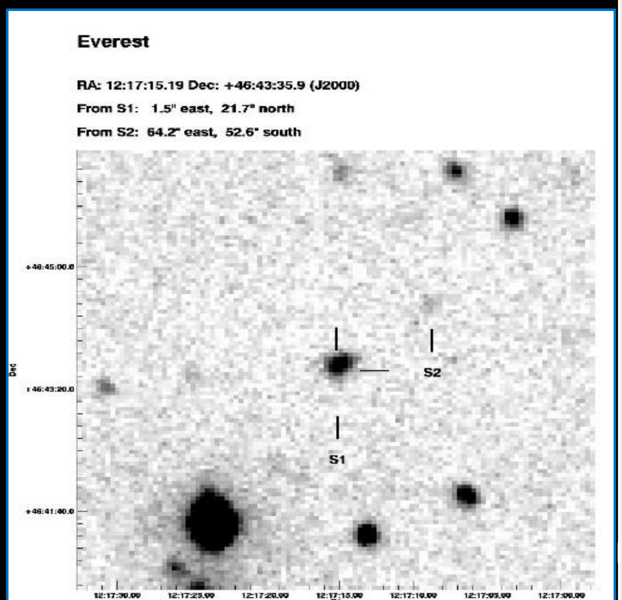


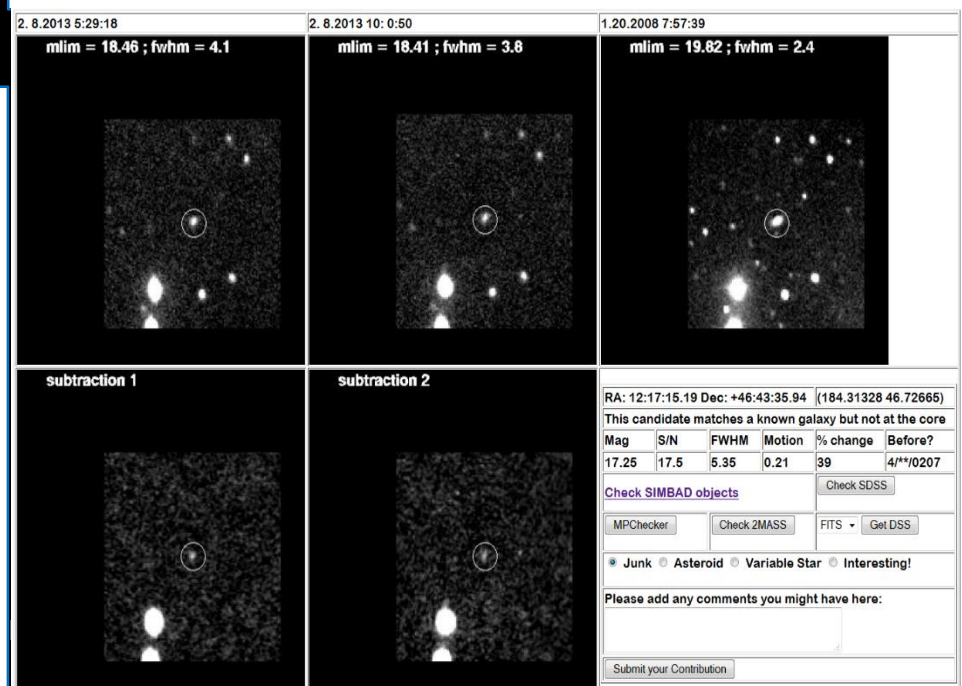

SN 2012ha: HET finder scope

SN 2013X *("Everest")*

- Discovered 130206 (ROTSE-IIIb)
- Type Ia 91T-like
  - Overluminous
  - White dwarf merger?
    - Double degenerate progenitor?
- Magnitude (max): 17.7
- Observed 10 days past maximum brightness
- (RA, Dec) = (12:17:15.19, +46:43:35.94)
- Constellation: Ursa Major
- Host galaxy: PGC 2286144
- z = 0.03260 (~140 Mpc; ~450 Mly)
- CBET 3413





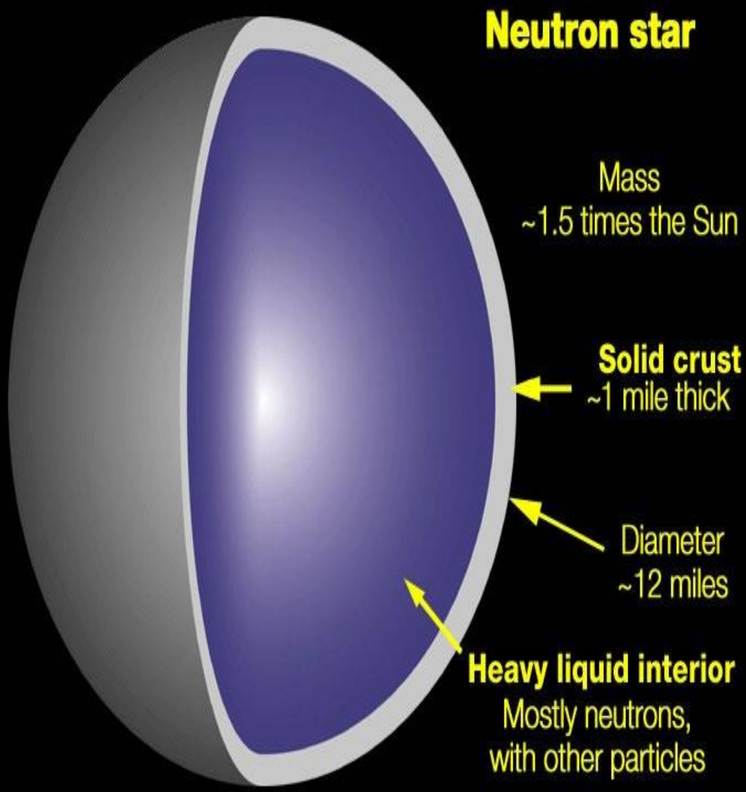


What happens to a star more massive than 1.4 solar masses?

1. There aren't any
2. They shrink to zero size
3. They explode
4. They become something else

Neutron Stars

- Very compact – about 10 km radius
- Very dense – one teaspoon of neutron star material weighs as much as all the buildings in Manhattan
- Spin rapidly – up to 2000 *rev/s*
- High magnetic fields – compressed from magnetic field of progenitor star



Neutron Stars

- Degenerate stars heavier than 1.4 solar masses collapse to become neutron stars
- Formed in supernovae explosions
- Electrons are not separate
  - Combine with nuclei to form neutrons
- Neutron stars are degenerate gas of neutrons

Near the center of the Crab Nebula is a neutron star that rotates 30 times per second. Photo Courtesy of NASA.

Neutron Stars

Similar to white dwarfs - basic physics is degenerate fermion gas. However, we have neutrons, not electrons. Replace m_e with m_p.

$$r_{\rm ns} \approx 2.3 \times 10^9 \text{ cm} \frac{m_e}{m_n} \left(\frac{\mathcal{Z}}{A}\right)^{5/3} \left(\frac{M}{M_\odot}\right)^{-1/3} \approx 14 \text{ km} \left(\frac{M}{1.4 M_\odot}\right)^{-1/3}$$

Note: the Z/A factor is one, since almost all nucleons are neutrons.

Important Effects (we neglected):

1. Nuclear interactions play an important role in the EOS. The EOS is poorly known due to our poor understanding of details of the strong interaction.
2. The star is so compact that the effects of GR must be taken into account.

Compare gravitational and rest mass energies of a test particle of mass m.

$$E_{gr} = \frac{GMm}{2r} \qquad \text{and} \qquad E = mc^2$$

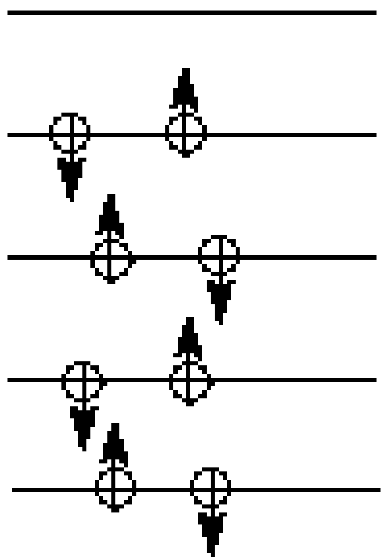
$$\frac{E_{\rm gr}}{mc^2} = \frac{GM}{rc^2} \approx \frac{6.7 \times 10^{-8}\ \text{cgs} \times 1.4 \times 2 \times 10^{33}\ \text{g}}{10 \times 10^5\ \text{cm}\ (3 \times 10^{10}\ \text{cm s}^{-1})^2} \approx 20\%$$

Matter falling onto a neutron star loses 20% of its rest mass and the mass of the star as measured via Kepler's law is 20% smaller than the total mass that composed it!

Detailed calculations that take into account GR and nuclear interactions give a radius of 10 km for a neutron star of $1.4M_{\rm sun}$.

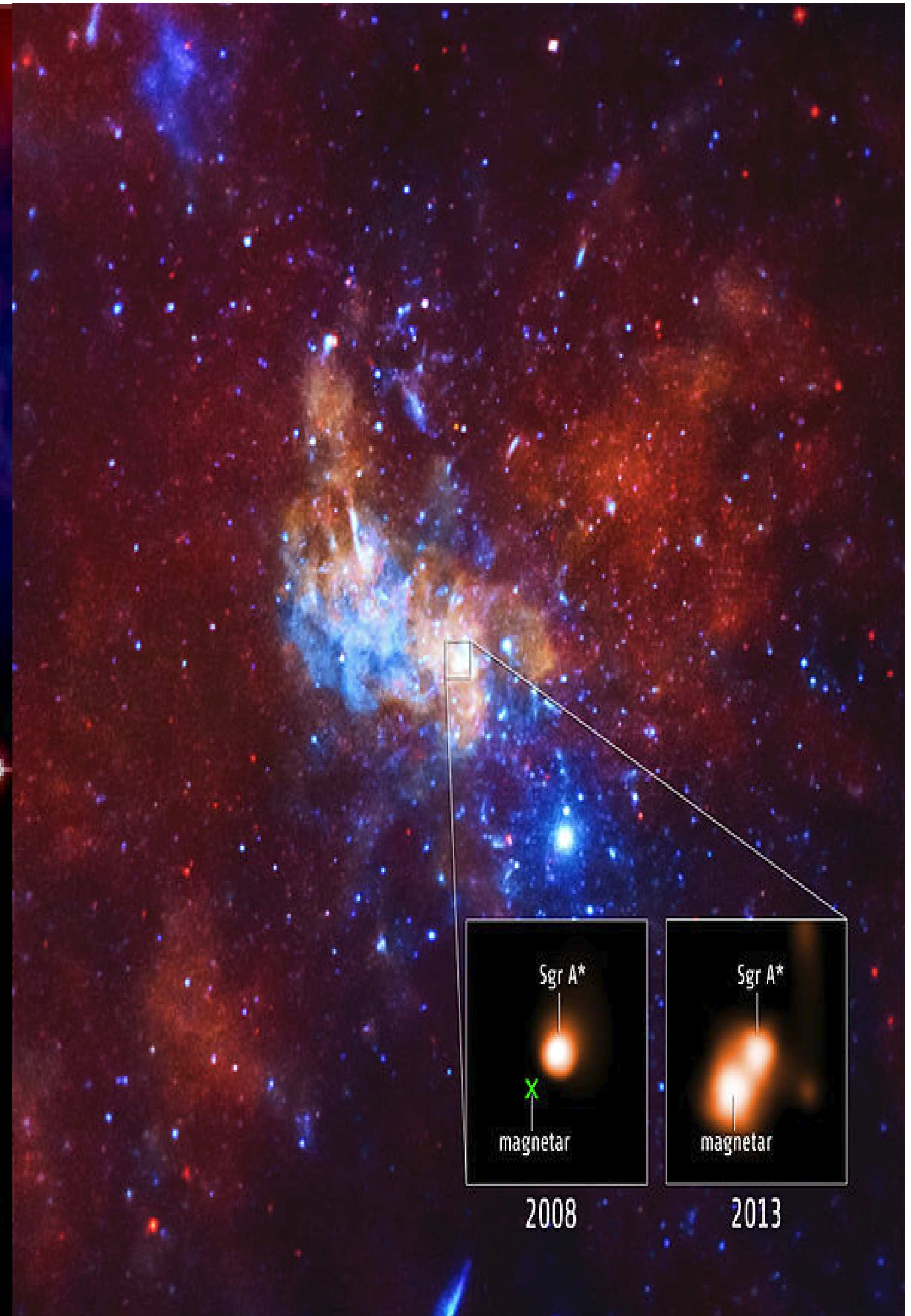
Limiting mass of a neutron star is not accurately known. The value is between $2M_{\rm sun}$ and $3.2M_{\rm sun}$.

Neutron energy levels



Degenerate gas: all lower energy levels filled with two particles each (opposite spins). Particles **locked** in place.

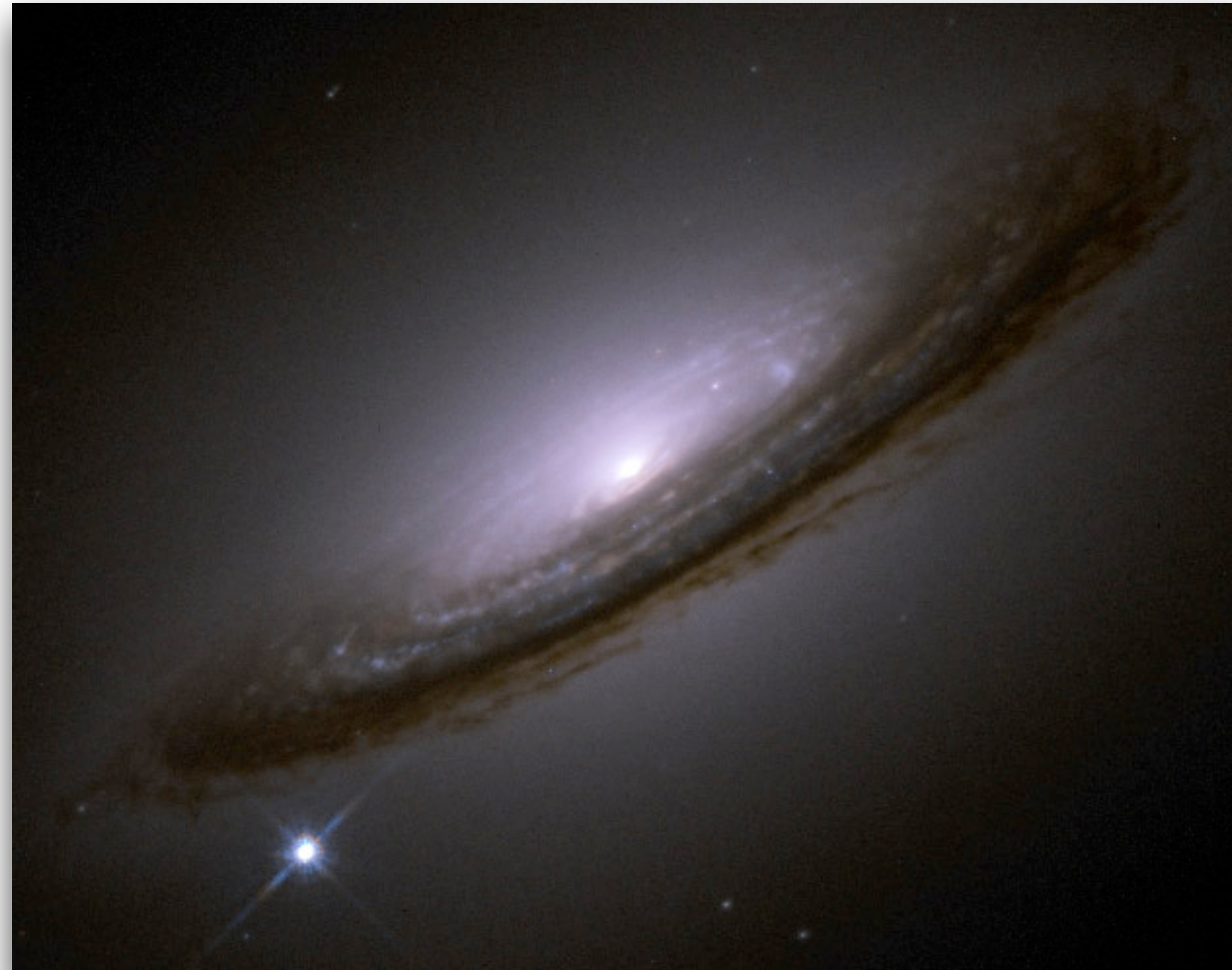
- Only two neutrons (one up, one down) can go into each energy level
- In a degenerate gas, all low energy levels are filled
- Neutrons have energy, and therefore are in motion and exert pressure even if temperature is zero
- Neutron star are supported by neutron degeneracy



Sgr A*
magnetar
2008
Sgr A*
magnetar
2013


BBC TWO

Supernova Explosions



SN1994D

Properties

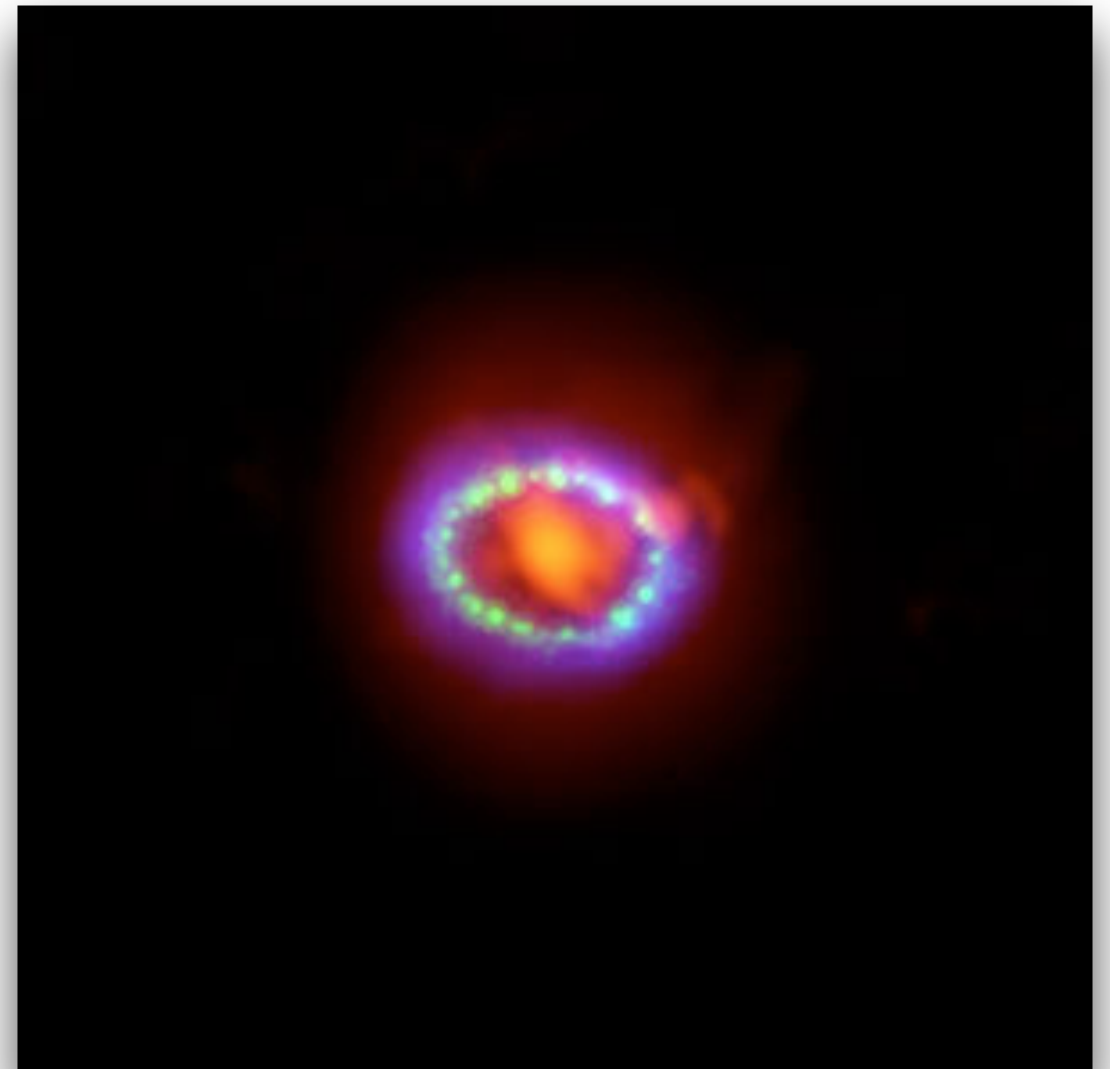
- The energy imparted by material flying out is about 3×10^{51} erg.
- Luminous energy of ~ 3×10^{49} erg can be observed for ~ 1 month after the explosion. This is driven by the decay of radioactive elements synthesized just before, during collapse & during explosion.
- The mean luminosity is

$$L_{SN} \sim 10^{43}\, erg\, s^{-1} = 3 \times 10^9 L_{sun}$$

- The bulk of the energy released in SN is carried away by neutrino-antineutrino pairs.
- The density is so high, that photons can not emerge from the star. (Too many photon-photon collisions).

$$\gamma + \gamma \rightarrow e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e, \nu_\mu + \bar{\nu}_\mu, \nu_\tau + \bar{\nu}_\tau$$

Supernova 1987A



SN 1987A in the Large Magellanic Cloud.
Distance 50 kpc from Earth.
Nearest SN since 1604.

- 20 antineutrinos were discovered in a span of a few seconds by two different underground experiments (Kamiokande II and IMB).
- First time neutrinos were detected. The neutrinos were detector prior to the emission of visible light.
- The experiments were originally designed to detect proton decay.
- Neutrinos are detected via the process

$$\bar{\nu}_e + p \rightarrow n + e^+$$

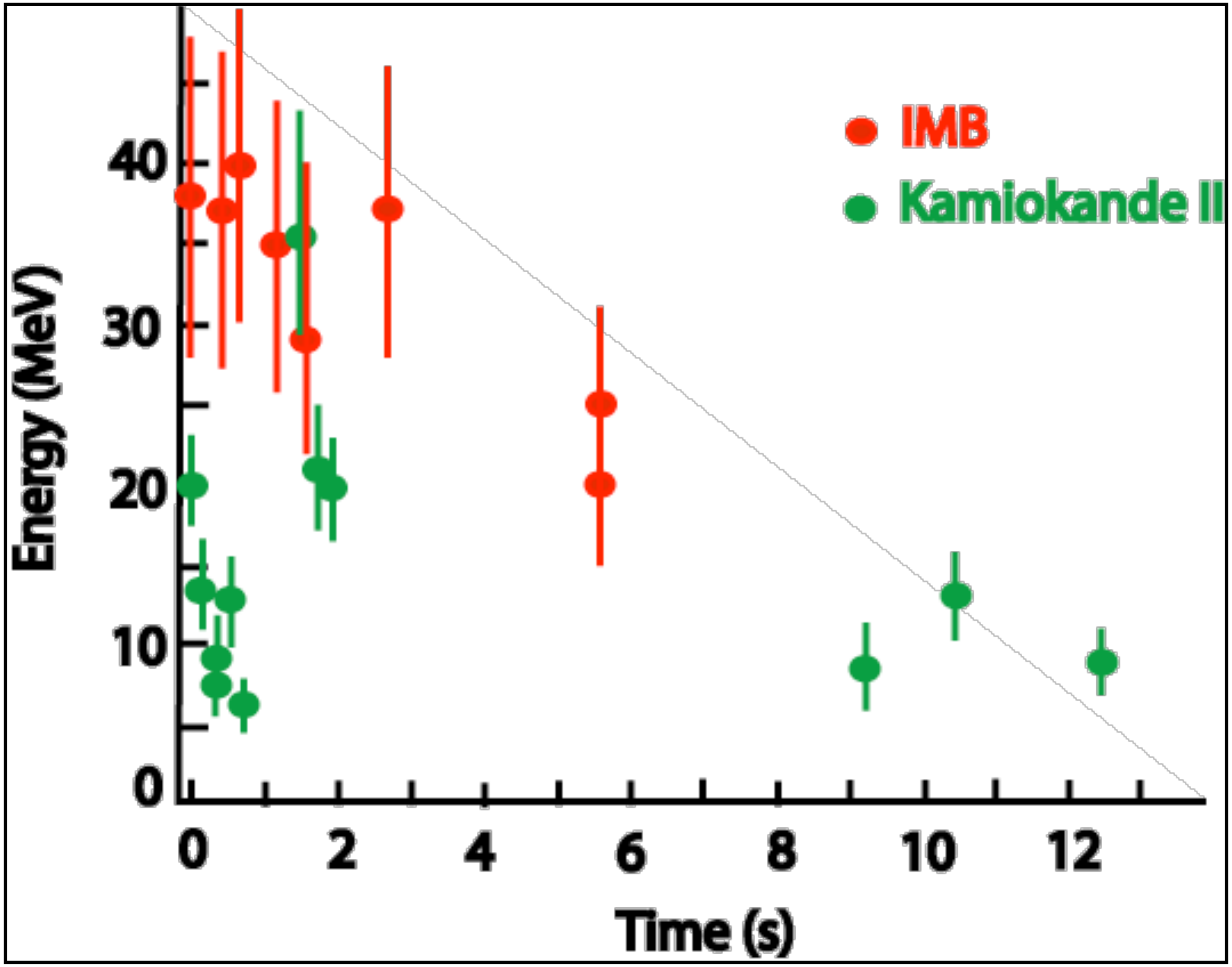
Nuclear Physics B (Proc. Suppl.) 3 (1988) 441–452
North-Holland, Amsterdam

2002 Nobel Prize in Physics for the first real time observation of supernova neutrinos. (This prize was shared with Ray Davis.)

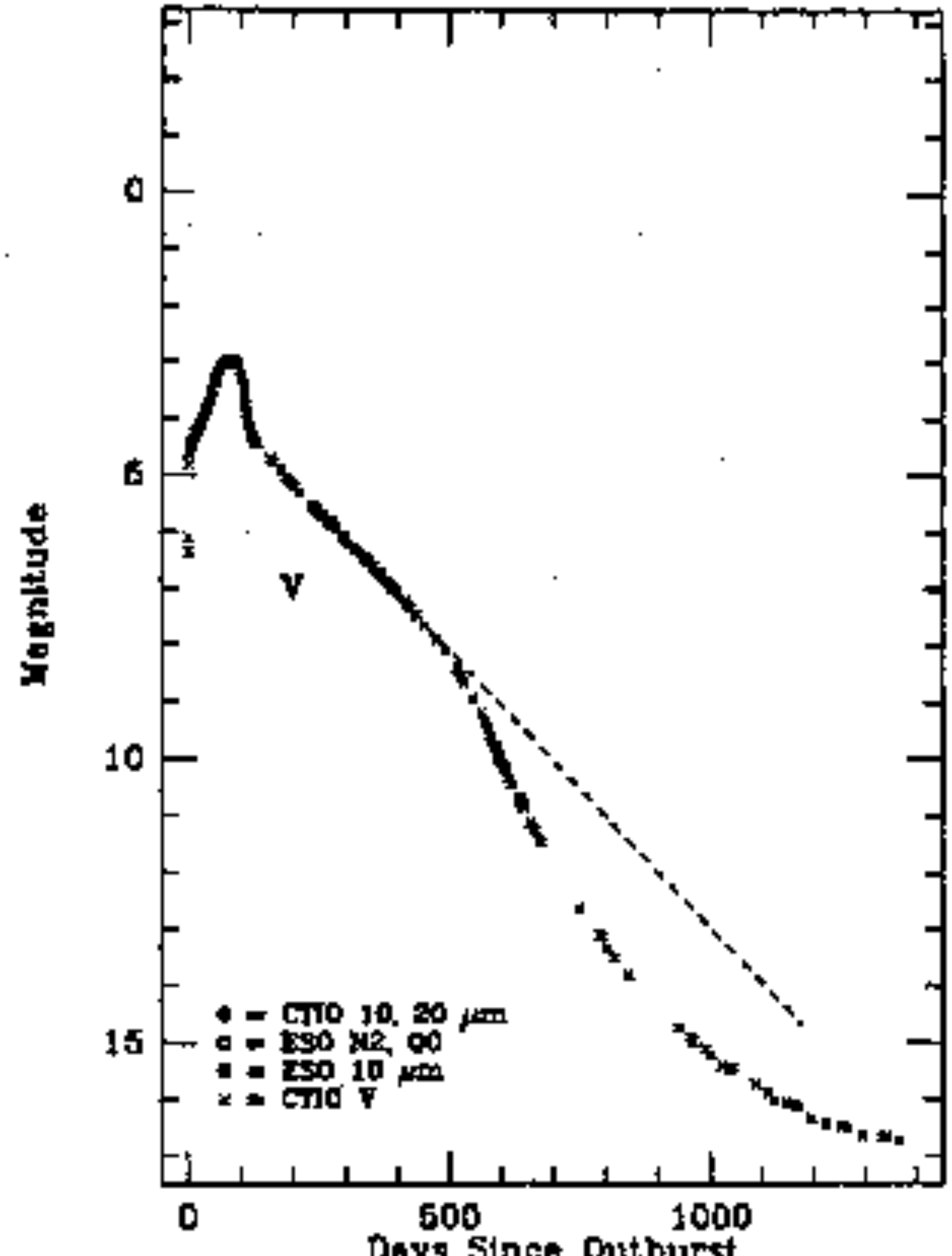
SUPERNOVA NEUTRINO SIGNAL AND KAMIOKANDE-II

M. Koshiba
DESY and University of Tokyo

The collaborators of this experiment are:
K.Hirata, T.Kajita, M.Koshiba, M.Nakahata, Y.Ohyama, N.Sato, A.Suzuki, M.Takita and Y.Totsuka (Dept. of Physics and ICEPP, Univ. of Tokyo); T.Kifune and T.Suda (Inst. Cosmic Ray Research, Univ. of Tokyo); K.Takahashi and T.Tanimori (Nat. Lab. High Energy Physics (KEK)); K. Miyano and M.Yamada (Dept. of Physics, Univ. of Niigata); E.W.Beier, L.R.Feldsher, S.B.Kim, A.K.Mann, F.M.Newcomer, R.Van Berg and W.Zhang (Dept. of Physics, Univ. of Pennsylvania); B.G. Cortez (AT&T Bell Laboratories, Holmdel).

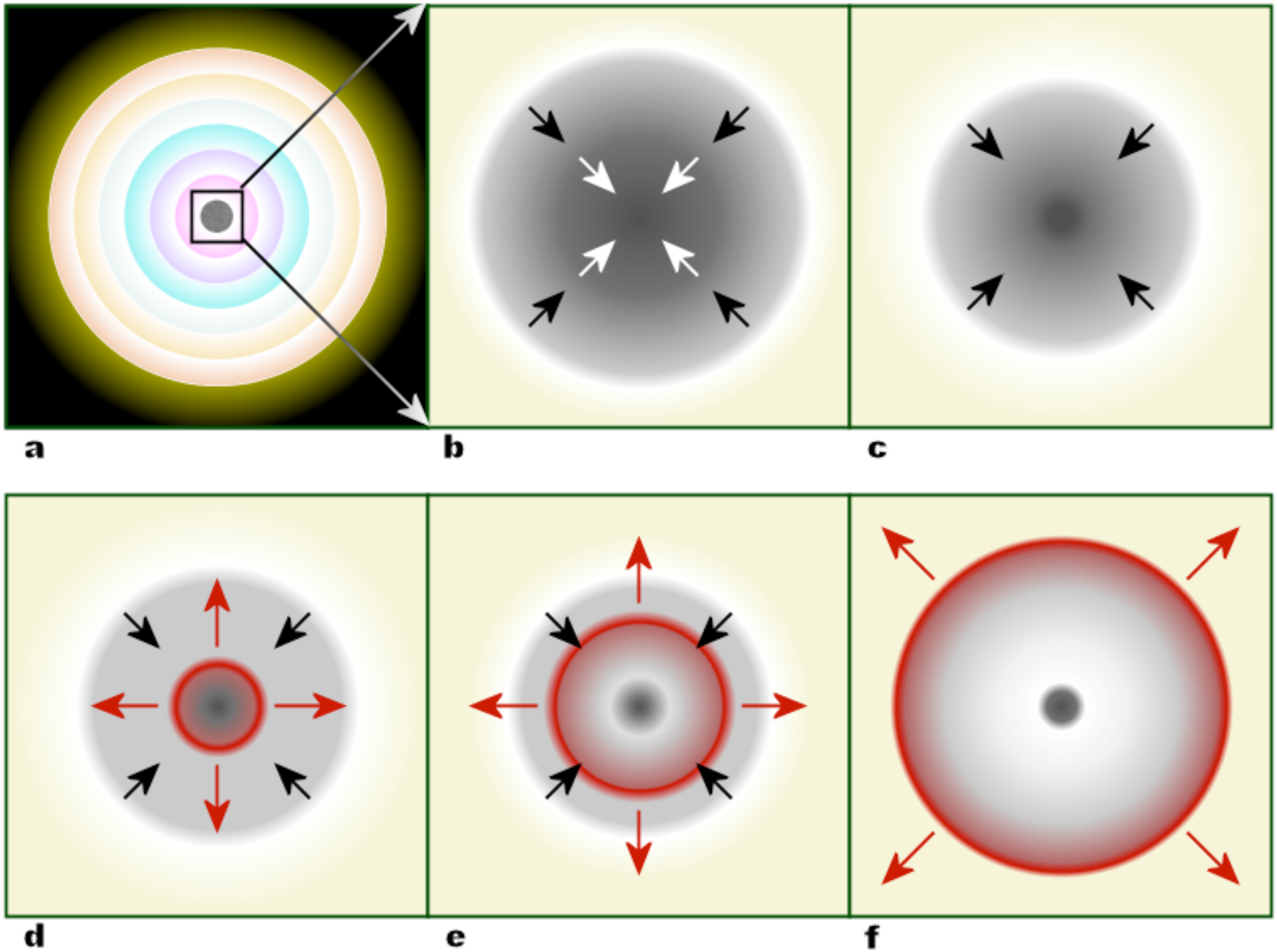


Neutrinos from SN1987A. The 8 neutrinos by the IBM experiment have greater energy than the 12 detected by the Kamiokande experiment because the IBM detector was not sensitive to low energy neutrinos.

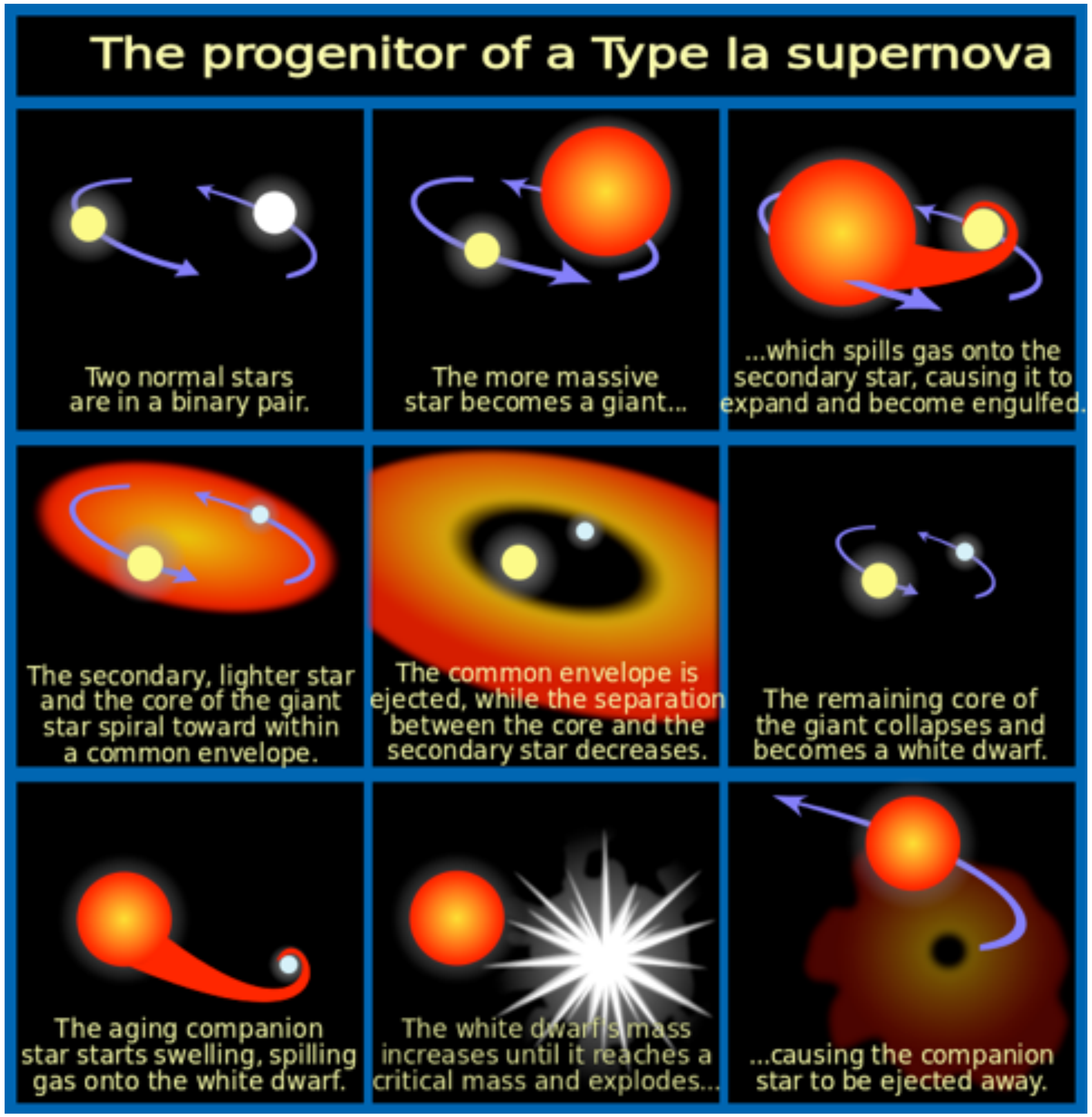


Optical light curve of SN1987A. Light faded at almost the same observed light at ^{66}Co (77 days).

Type II Supernova: Summary



Type Ia SN



- Mass transfer from companion onto WD happens until reach M_{ch}.
- At (or before) that stage, carbon core ignites. Temp rises, pressure increases. Classically, star should expand.
- Degenerate conditions prevent star from expanding, causes nuclear reaction rate to increase.
- Ends in a thermonuclear runaway — star explodes.

Supernova Summary

Type II SN

- Produced by core collapse of a massive star.
- Leaves behind a neutron star or black hole.

Type Ia SN

- Produced by the feeding of a white dwarf from a companion star.
- Leaves behind no stellar remnant.
- The kind of star the companion is unknown.
    - Merger of two WD?

Material expelled by both types of SN is essentially the only source of heavy elements in the universe. Lighter elements were formed in the early universe (more details later in the course).

Gamma Ray Bursts

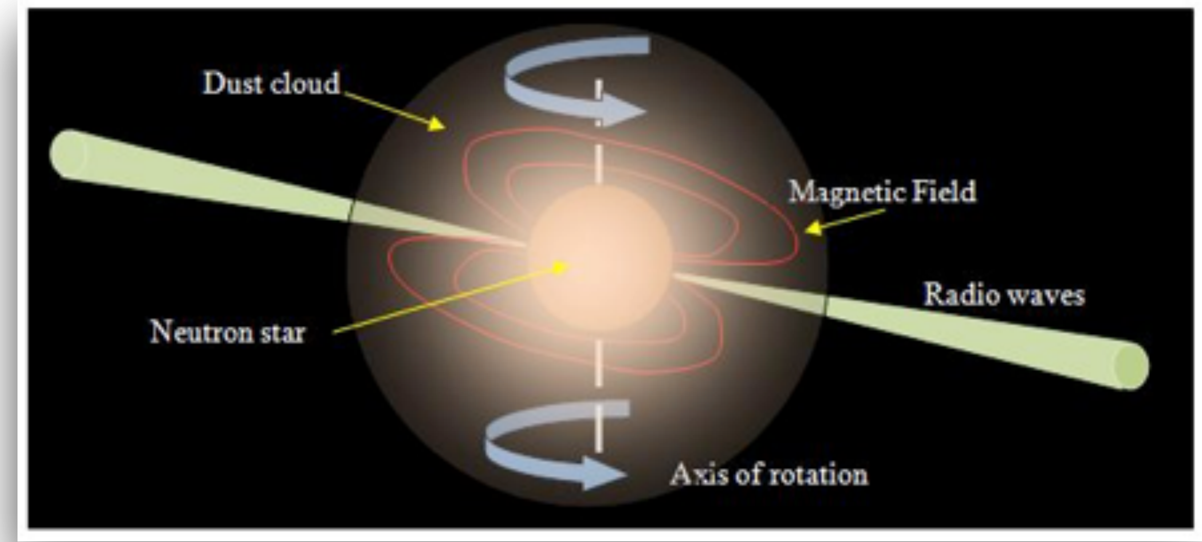
- Even more luminous than a SN explosion.
- Release 10^{51} erg over just a few seconds.
- Initially energy released at gamma frequencies, fading afterglows can be in x-rays (minutes), optical (days), and radio frequencies (weeks).
- Occurrence: Observe approximately 1 per day.
- Half of the explosions are in star forming galaxies.
- Nature and mechanism for GRBs is still widely debated.
    - Involved in formation of black holes.
    - Links to SN explosions (Type Ic)
    - Result from core collapse of massive stars (long-duration GRB).

Pulsars and Supernova Remnants

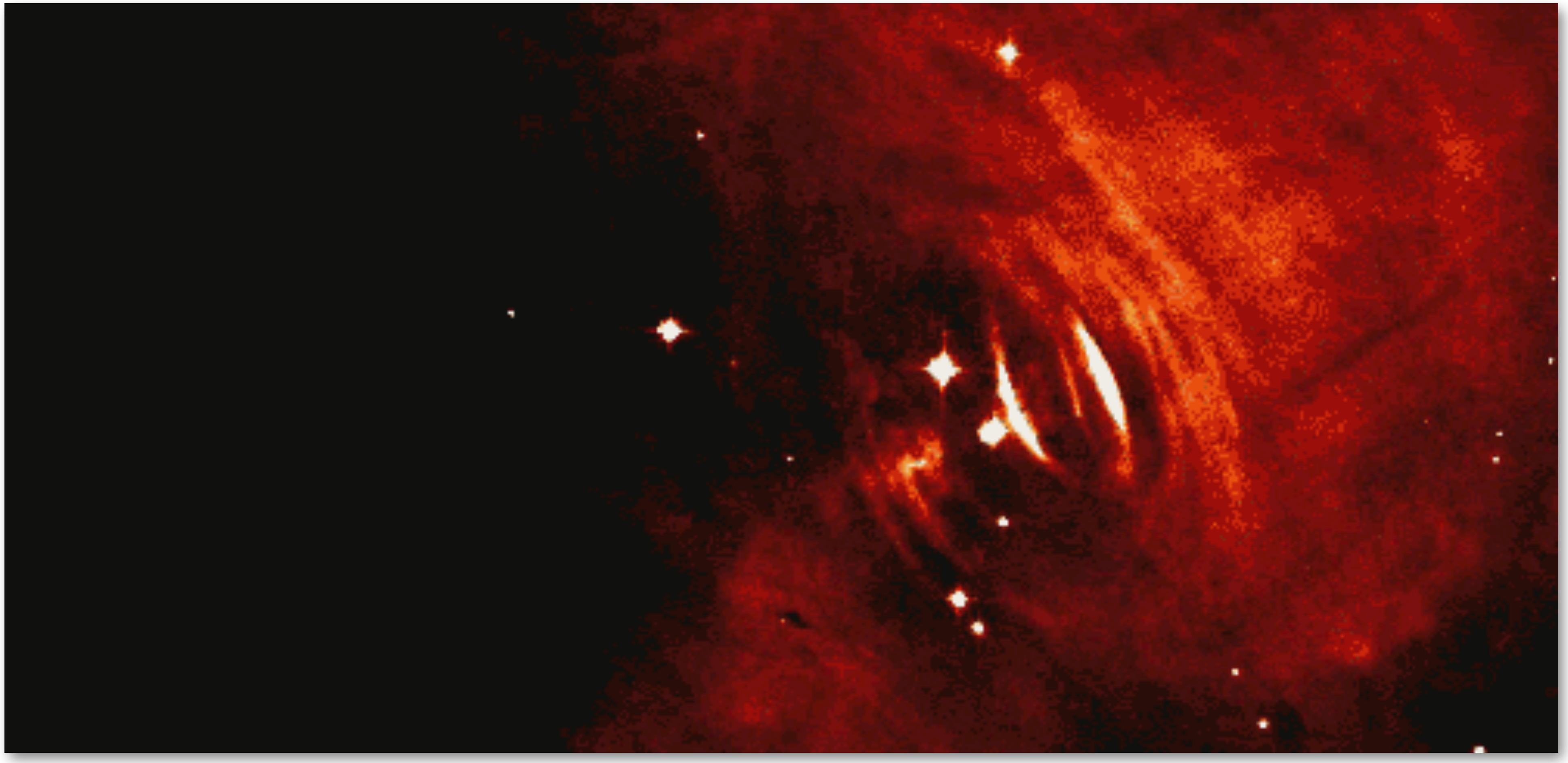
- The first pulsar was discovered in 1967. It had a pulse period of 1.33 s.
- It was named “LGM-1”. Any ideas what this stands for?

Little Green Men

- Today over 1000 pulsars are known. Some have periods as short as 0.03 s.



The Crab Pulsar



- First observed in 1054 by Chinese, Japanese and Korean Astronomers.
- Period τ = 33 ms
- $L_{tot} \sim 5 \times 10^{38}$ erg s^{-1}
- $\omega = \frac{2\pi}{\tau} = 190\ \mathrm{s}^{-1}$

Pulsars as Neutron Stars

What are possible mechanisms for producing the periodicity of the observed magnitude and regularity in these stars?

1. binaries
2. stellar pulsations
3. stellar rotation

Binaries:

Angular frequency, mass and separation are related by Kepler's law.

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$

$$a = \frac{[G(M_1 + M_2)]^{1/3}}{\omega^{2/3}} \quad = 2 \times 10^7 \text{ cm} = 200 \text{ km}$$

where a is the separation and we assume object are of solar mass.

How does the separation distance compare to the radius of a normal star?

It is much smaller than a normal star or even a white dwarf. Only neutron stars could exist in such a binary.

BUT GR predicts orbiting masses as such a separation will lose energy (via gravitational waves), separation will shrink and orbital frequency will grow. Observed pulsar frequencies decrease with time.

Stellar Pulsations:

Stars are observed to pulsate regularly in various modes.

$$\tau \propto \rho^{-1/2}$$

Normal stars oscillate with periods between hours and months. WD oscillate with periods of 100 to 1000 s.

Neutron stars (10^8 x denser) should, therefore, pulsate with periods of 0.1s.

Pulsars commonly have a period of ~0.8 s. There is no class of stars that produce this pulsation period.

Stellar Rotation:

Assume anisotropic emission from a rotating star. What is the fastest a star can spin?

Angular frequency at which centrifugal forces do not break it apart.

$$\frac{GMm}{r^2} > m\omega^2 r$$

$$\frac{M}{r^3} > \frac{\omega^2}{G}$$

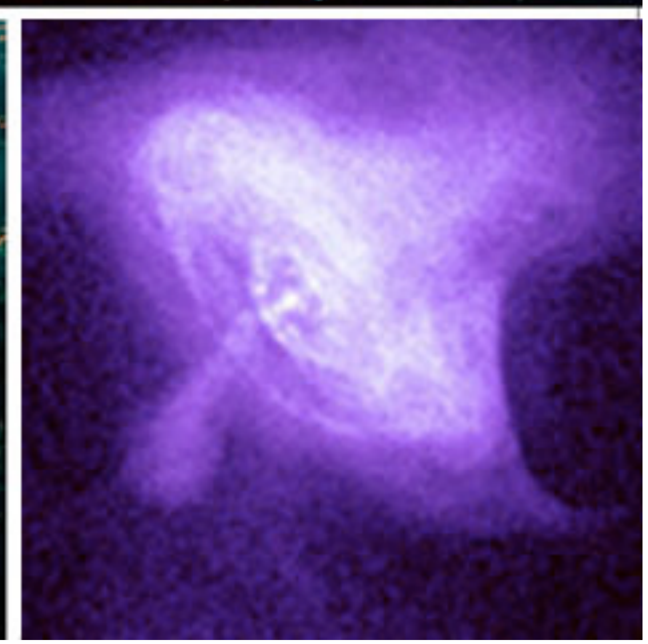
$$\bar{\rho} = \frac{3M}{4\pi r^3} > \frac{3\omega^2}{4\pi G} = 1.3 \times 10^{11} \text{ g cm}^{-3}$$

If the Crab is a spinning star and not flying apart, it's mean density must be 5x WD, but consistent with neutron star.

The Crab Nebula

Optical image - scale 4 pc per side.

Zoom in of marked area in optical.



Zoom in of marked area in x-rays.

Last Time:

What are possible mechanisms for producing the periodicity of the observed magnitude and regularity in these stars?

1. ~~binaries~~
2. ~~stellar pulsations~~
3. stellar rotation

1. The separation distance required between 2 binaries is 200 km. Normal stars and WD are too large, neutrons stars are okay. However, GR requires that stars in tight binary lose energy, spiral inward and orbital velocity increases. Observations indicate that pulsars slow over time.

2. No known class of stars produces a pulsation period of ~0.8 s. Normal stars and WD pulsate at 100 - 1000 s. Neutrons stars pulsate at 0.1 s.

Stellar Rotation:

Assume anisotropic emission from a rotating star. What is the fastest a star can spin?

Angular frequency at which centrifugal forces do not break it apart.

$$\frac{GMm}{r^2} > m\omega^2 r$$

$$\frac{M}{r^3} > \frac{\omega^2}{G}$$

$$\bar{\rho} = \frac{3M}{4\pi r^3} > \frac{3\omega^2}{4\pi G} = 1.3 \times 10^{11} \text{ g cm}^{-3}$$

If the Crab is a spinning star and not flying apart, it's mean density must be 5x WD, but consistent with neutron star.

Assume that the luminosity of the Crab nebula is powered by the pulsar's rotational energy loss as it spins down.

What is the formula for rotational energy?

$$E_{\mathrm{rot}} = \frac{1}{2} I \omega^2$$

How would I get the total luminosity due to rotational energy?

$$L_{\mathrm{tot}} = -\frac{dE_{\mathrm{rot}}}{dt} = -I\omega\frac{d\omega}{dt}$$

What is the moment of inertia of a sphere?

$$I = \tfrac{2}{5} M r^2$$

$$L_{tot} = -I\omega\frac{d\omega}{dt} \qquad I = \tfrac{2}{5}Mr^2$$

Substitute and solve for Mr^2

$$L_{tot} = -\frac{2}{5}Mr^2\omega\frac{d\omega}{dt}$$

$$Mr^2 = -\frac{5}{2}\frac{L_{tot}}{\omega\frac{d\omega}{dt}} = -\frac{5 \times 5 \times 10^{38}\ \mathrm{erg\ s^{-1}}}{2 \times 190\ \mathrm{s^{-1}}(-2.4 \times 10^{-9}\ \mathrm{s^{-2}})} = 3 \times 10^{45}\ \mathrm{g\ cm^2}$$

Compare this to a 1.4 M_{sun} neutron star of radius 10 km.

$$Mr^2 = 1.4 \times 2 \times 10^{33}\ \mathrm{g} \times (10^6\ \mathrm{cm})^2 = 2.8 \times 10^{45}\ \mathrm{g\ cm^2}$$

How does this compare to our sun?

$$Mr^2 = (2 \times 10^{33}\ g) \times (7 \times 10^{10}\ cm)^2 = 9.8 \times 10^{54}\ g\ cm^2$$

How fast would the sun spin if it were to collapse to a neutron star of radius 10 km? The rotation period of the sun is 25 days and the sun's radius is 7 x 10^{10} cm.

Use conservation of momentum to solve.

$$I_i \omega_i = I_f \omega_f$$

$$\frac{2}{5} M R_i^2 \omega_i = \frac{2}{5} M R_f^2 \omega_f$$

$$\omega_f = \omega_i \left(\frac{R_i}{R_f}\right)^2 = \underline{3 \times 10^{-6}\ s^{-1}} \times \left(\frac{7 \times 10^{10}\ cm}{10 \times 10^5\ cm}\right)^2 = \underline{1.5 \times 10^4\ s^{-1}}$$

Spin up rates on order of 10^9. Collapse of main sequence stars are expected to produce objects with a spin on the order of ms.

Is the nail in the coffin?

- The spin rate of pulsars is that expected from the collapse of the cores of main sequence stars.
- The mean densities are those of neutron stars
- Their rotational energy accounts for the luminosity of supernova ejecta in which the stars are embedded.
- Location of pulsars at the sites of historical SN is expected to accompany the formation of a neutron star.

Magnetic Fields

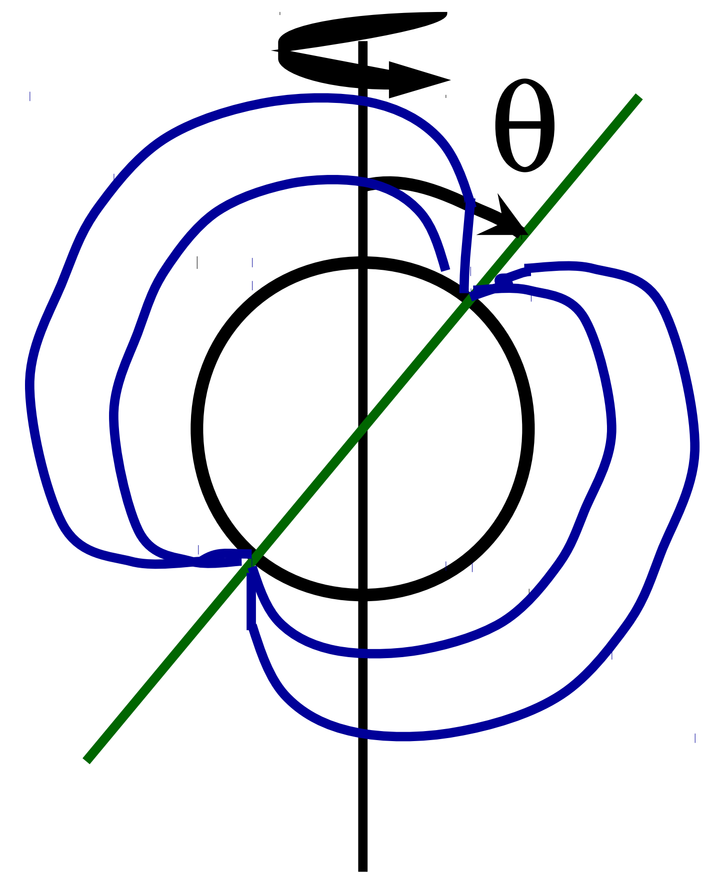
If a solar type star collapses to form a neutron star, while conserving magnetic flux, we would expect

$$R_{sun}^2 B_{sun} = R_{NS}^2 B_{NS}$$

$$\frac{B_{NS}}{B_{sun}} = (\frac{7 \times 10^{10}\ cm}{10^6\ cm})^2 \quad \sim 5 \times 10^9$$

For the sun, B ~ 100 G, so we would expect a NS to have a field of magnitude ~10^{12} G.

Consider a NS having a magnetic field axis misaligned w.r.t. the stars rotation axis by some angle θ.



A spinning magnetic dipole radiates an EM luminosity of

$$L = \frac{1}{6c^3} B^2 r^6 \omega^4 \sin^2\theta \quad \propto \omega^4$$

Solving for B, and substituting in observed values of the Crab gives B ~ 8 x 10^{12} Gauss.

If the EM radiation is leading to the pulsar's rotation energy, then

$$\frac{dE_{\mathrm{rot}}}{dt} = I\omega \frac{d\omega}{dt} \propto \omega^4 \longrightarrow \frac{d\omega}{dt} = C\omega^3$$

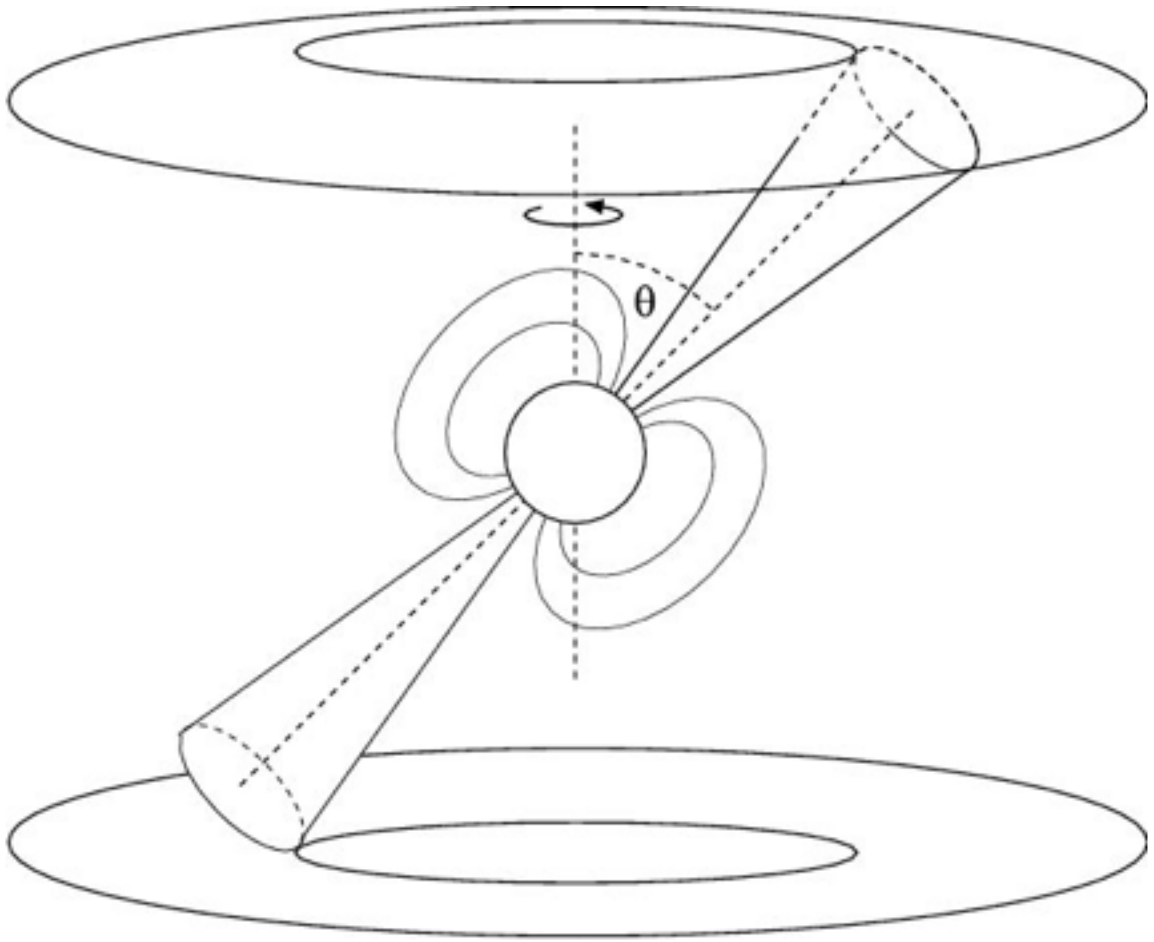
Separating variables and solving yields the age of a pulsar :

$$t_{\mathrm{pulsar}} = \frac{\omega_0^3}{2\dot{\omega}_0}\left(\frac{1}{\omega^2} - \frac{1}{\omega_i^2}\right)$$

For the Crab, pulsar = 1260 years. This is consistent with the historical age of ~960 years.

Neutron Star Cooling

- Only a small fraction of neutron stars are observable from Earth.
- As NS slow down and lose rotational energy, they become undetectable as pulsars.
- Detailed calculations of NS cooling are much less certain than those for WD.
- Poorly constrained EOS for nuclear matter leads to uncertainty in the structure and composition of a neutron star.



Black Holes

- In the case of a stellar remnant with mass > the allowed mass of a neutron star, no known mechanism can prevent complete gravitational collapse.
- GR predicts that even if a new form of pressure kicks in at high densities, it will not be strong enough to overcome gravity.
- The star will collapse to a black hole from where no radiation or matter can escape.

Let's find the "radius" of a black hole.

$$\frac{GMm}{r} = U > E_k = \frac{1}{2}mv_e^2$$

$$\frac{GM}{r} > \frac{1}{2}v_{\mathrm{e}}^2 = \frac{1}{2}c^2$$

Rearranging terms

$$\boxed{r_s = \frac{2GM}{c^2} = 3\ \mathrm{km}\frac{M}{M_\odot}}$$

This is the Schwarzschild radius

Note that the equation on the previous page is incorrect for two reasons.

1. The KE of a photon is not $mc^2/2$
2. The gravitational PE is not described by Newton's limit.

We will outline the correct derivation.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein's tensor energy-momentum tensor

energy-momentum tensor:

Represented a 4x4 matrix, each of the indices runs over the 4 space-time coordinates. This term in the equations includes mass-energy density and pressure.

Einstein's tensor:

Consists of combinations of 1st & 2nd PDEs wrt spacetime coordinates of the metric $g_{\mu\nu}$.

The metric tells us how to calculate the interval ds is the interval between two spacetime events.

$$(ds)^2 = \sum_{\mu,\nu} g_{\mu\nu} dx_\mu dx_\nu$$

normally 1 time and 3 space elements.

In the absence of matter, spacetime is flat. In that case, we can use the **Minkowski metric**.

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

The 4 x 4 matrix describing $g_{\mu\nu}$ then look like

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$(ds)^2 = (cdt)^2 - (dx)^2 - (dy)^2 - (dz)^2$$

What happens if $dt = 0$?

$$(ds)^2 = -[(dx)^2 + (dy)^2 + (dz)^2] \;= |ds|$$

This is just the distance between 2 points.

What happens if $dx = dy = dz = 0$?

$$(ds)^2 = (cdt)^2$$

ds/c = time between two points. Proper time $\tau = ds/c$ is the time elapsed on a clock moved between two points.

Light travels along "null geodesics" for which $ds = 0$.

What if we use spherical coordinates?

$$(ds)^2 = (cdt)^2 - (dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

The 4 x 4 matrix describing $g_{\mu\nu}$ then look like

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}$$

In the case of spacetime in a vacuum surrounding a static, spherically symmetric, mass distribution we get the **Schwarzchild metric:**

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

$$(ds)^2 = \left(1 - \frac{r_s}{r}\right)(cdt)^2 - \left(1 - \frac{r_s}{r}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2$$

where r_s is the Schwarzschild radius

$$r_s = \frac{2GM}{c^2}$$

$$(ds)^2 = \left(1 - \frac{2GM}{rc^2}\right)(cdt)^2 - \left(1 - \frac{2GM}{rc^2}\right)^{-1}(dr)^2 - (rd\theta)^2 - (r\sin\theta d\phi)^2 \qquad r_s = \frac{2GM}{c^2}$$

For a clock at rest, what is the proper time?

$$d\tau \equiv \frac{ds}{c} = \left(1 - \frac{2GM}{rc^2}\right)^{1/2} dt = \left(1 - \frac{r_s}{r}\right)^{1/2} dt$$

Consider a stellar remnant so compact that its radius fits within r_s.

- As $r \rightarrow r_s$, $d\tau \rightarrow 0$. Gravitational time dilation becomes infinite.
- EM and magnetic fields will appear to oscillate more slowly, leading to a gravitational redshift

$$\frac{\lambda}{\lambda_0} = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} = \left(1 - \frac{r_s}{r}\right)^{-1/2}$$

where λ_0 = emitted wavelength, λ = observed (at infinity).

Note, the redshift becomes infinite as $r \rightarrow r_s$.

Event Horizon

Recall that light moves along null geodesics. If we set *ds = 0* the coordinate speed of a light beam moving radially becomes

$$\frac{dr}{dt} = \pm c\left(1 - \frac{2GM}{rc^2}\right) = \pm c\left(1 - \frac{r_s}{r}\right)$$

- What happens if $r >> r_s$?

The speed is c, as expected.

- What happens if $r_s >> r$?

The speed appears to approach 0.

No information can emerge from a radius smaller than r_s, which constitutes an **event horizon** around the black hole.

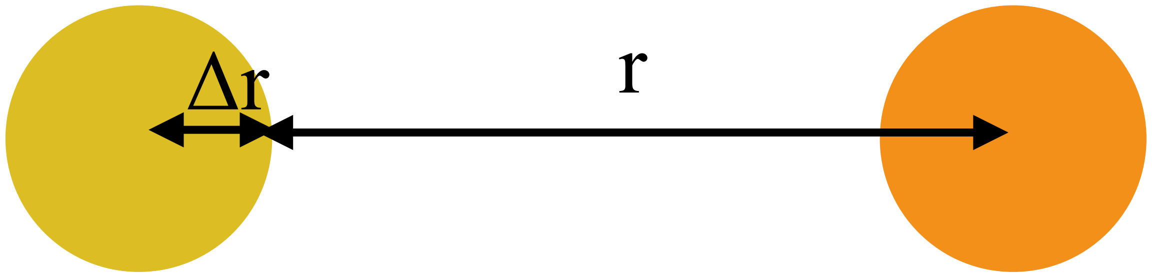
The collapse of matter to r_s takes an infinite amount of time for an observer at infinity (but finite amount of time for someone falling in). As such, the matter is "frozen" in time as it falls in. However, there is no observable differences in frozen stars and truly collapsed black holes. More details can be found on pages 97-98 of your textbook.

Interacting Binaries

- Many objects (including stars) are powered not by nuclear reactions, but by accretion of matter onto gravitational wells.
- We will focus on stars in binaries which will exert forces on each other.
  - Force on center of mass maintains binary orbit.
  - Force is stronger for parts of the star facing towards companion and weaker for parts facing away from companion. This is a result of **tidal forces** that stars exert at small distances.

Tidal Forces:

Forces that cause distortions of equipotential surfaces.



Consider mass element *m* in star 1 at a distance Δr from the center. What is the force on *m* due to star 1?

$$\frac{F_{\text{grav}}}{m} = \frac{GM_1}{(\Delta r)^2}$$

What is the tidal force felt by *m* due to star 2 at distance r (assume Δr << r)

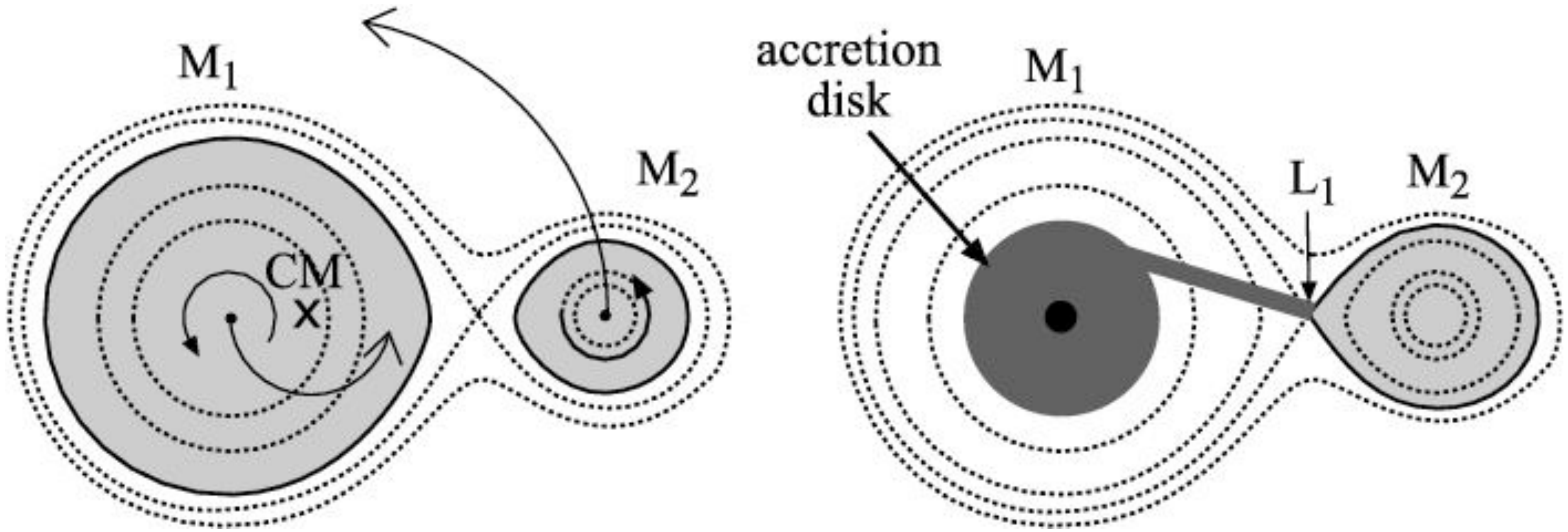
$$\frac{F_{\text{tide}}}{m} = GM_2 \left(\frac{1}{r^2} - \frac{1}{(r+\Delta r)^2} \right) \approx \frac{2GM_2 \Delta r}{r^3}$$

Taking the ratio of forces yields:

$$\frac{F_{\text{tide}}}{F_{\text{grav}}} = \frac{2M_2}{M_1} \left(\frac{\Delta r}{r} \right)^3$$

Tidal forces are largest when Δr/r is biggest.

Once the stars achieve synchronized, circularized orbits **tidal locking** is achieved. Everything will appear stationary in a frame rotating at binary frequency.



Roche lobes are the deepest non-disjoint equipotential surface in the rotating frame.

Binary systems can be:

- detached: neither star fills its Roche lobe
- semi-detached: one star fills its Roche lobe
- contact: both stars fill their Roche lobes.

If a star fills its Roche lobe, matter transfers via the first Lagrangian point. Matter will have angular momentum and form an accretion disk around the other star.

Semi-detached Binary

In the case of a semi-detached binary, there is always mass transfer from the Roche-lobe-filling star to its companion.

Binary Type	Receiving Star
Algol-type	main sequence
cataclysmic variables	white dwarf
type Ia supernova	white dwarf
x-ray binary	neutron star or black hole

Accretion Disks

To model accretion disks we will assume:

- particles move on an approximate circular orbit
- they lose energy and angular momentum due to viscous interactions with particles on nearby orbits
- frictional heat is radiated away with each disk annulus acting as a blackbody of a given temperature.

Note: The nature of viscosity is still not well known

How does energy change when a mass dM in an accretion disk around a star of mass M change when it's orbit goes from radius $r+dr$ to radius r?

$$dE_g = GMdM\left(\frac{1}{r} - \frac{1}{r+dr}\right) \sim \frac{GMdMdr}{r^2}$$

This is only the gravitational potential energy.

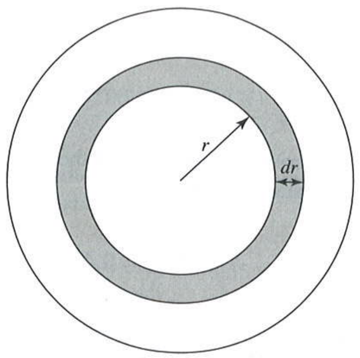
Half of the total energy is the potential energy. We must also consider the thermal energy.

Recall that the viral theorem gives

$$E_{total} = E_{th} + E_{gr} = \frac{E_{gr}}{2}$$

Thus,

$$dE_{\text{th}} = \frac{1}{2}\left(\frac{GMdM}{r} - \frac{GMdM}{r+dr}\right) \sim \frac{GMdMdr}{r^2}$$



What is the luminosity?

accretion rate of particular annulus

surface area annulus

$$2(2\pi r)dr\sigma T^4 = dL = \frac{dE}{dt} = \frac{1}{2}GM\frac{dM}{dt}\left(\frac{1}{r} - \frac{1}{r+dr}\right) = \frac{1}{2}GM\dot{M}\frac{dr}{r^2}$$

surface area annulus includes top and bottom

Solve for T

$$2(2\pi r)dr\sigma T^4 = \frac{1}{2}GM\dot{M}\frac{dr}{r^2}$$

$$\boxed{T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma}\right)^{1/4} r^{-3/4}}$$

Notice, $T \propto r^{-3/4}$. This means that the inner regions of the disk are hottest and thus most luminous.

To find the total luminosity of the disk, we integrate over all annuli.

$$L = \int_{r_{\rm in}}^{r_{\rm out}} 2(2\pi r)\sigma T^4(r)dr = \frac{1}{2}GM\dot{M}\left(\frac{1}{r_{\rm in}} - \frac{1}{r_{\rm out}}\right)$$

In the case the $r_{out} >> r_{in}$

$$L = \frac{1}{2}\frac{GM\dot{M}}{r_{\rm in}}$$

Radiative efficiency:

The fraction of rest mass energy of accreted material that is radiated.

$$\eta = \frac{L}{\dot{M}c^2} = \frac{1}{2}\frac{GM}{c^2 r_{\rm in}}$$

Accreting Object	Inner Radius of Disk	Radiative Efficiency
neutron star 1.4 M_{sun}	10 km	0.10
non-rotating black hole	$3r_s$	0.057
maximally rotating black hole	$0.5r_s$	0.42

The radiative efficiency of nuclear burning is 0.007 or less in main sequence stars.

Example: White Dwarf

Calculate the typical luminosity of an accretion discs where the accretor is a white dwarf with a mass of M_{sun} and radius 10^4 km. The typical accretion rate is 10^{-9} M_{sun} yr^{-1}.

$$L = \frac{1}{2}\frac{GM\dot{M}}{r_{\rm in}} = \frac{6.7\times10^{-8}\ \text{cgs}\times2\times10^{33}\ \text{g}\times10^{-9}\times2\times10^{33}\ \text{g}}{2\times3.15\times10^{7}\ \text{s}\times10^{9}\ \text{cm}}$$

$$\boxed{L = 4\times10^{33}\ \text{erg s}^{-1}\approx L_\odot.}$$

Calculate the temperature at the inner radius (which dominates the disk).

$$T(r) = \left(\frac{GM\dot{M}}{8\pi\sigma}\right)^{1/4} r^{-3/4} = \left(\frac{6.7\times10^{-8}\ \text{cgs}\times2\times10^{33}\ \text{g}\times10^{-9}\times2\times10^{33}\ \text{g}}{3.15\times10^{7}\ \text{s}\times8\pi\times5.7\times10^{-5}\ \text{cgs}}\right)^{1/4}(10^{9}\ \text{cm})^{-3/4}$$

$$\boxed{T(r) = 5\times10^{4}\text{K}}$$

$$T(r) = 5 \times 10^4 \mathrm{K}$$

In which part of the EM spectrum does this star peak?

$$\lambda_{max} = \frac{0.29\, cm\, K}{T} = \frac{0.29\, cm\, K}{5 \times 10^4\, K} = 5.8 \times 10^{-6}\, cm = 58\, nm$$

This is well into the UV part of the EM spectrum.

Compare this to a neutron star accretor. For a typical neutron star of 1.4 M_{sun} with radius 10 km we have

- L ~ 10^{37} ergs (vs ~ 10^{33})
- T ~ 10^7 K (vs ~ 10^4)
- λ_{max} ~ 0.58 nm (x-ray specturm)

Accreting White Dwarfs

Novae are a class of cataclysmic variable binary stars. Mass transfers though disk, builds up on WD surface and eventually undergoes nuclear fusion.

- Typical energy ~ 10^{46} erg.
- Duration ~ 1 month, typical luminosity ~ 4 x 10^{39}erg s^{-1}.

Type Ia supernovae are the runaway version of the nova eruption. Mass builds up on the WD until mass exceeds the Chandrasekhar limit. WD fuses to iron-group elements and explodes.

- Typical energy ~ 10^{51-52} erg.
- Duration ~ 1 month, typical luminosity ~ 10^{43-44} erg s^{-1} ~$10L_{sun}$.
- 99% of energy is carried away by neutrinos (thus, core-collapse SN are far more energetic)
- have a narrow range of observed optical luminosities.
- useful as "standard candles" for measuring distances.

Eddington Limit

Consider radiation pressure from an object of luminosity L acting on ionized inflowing gas. The dominant interaction will be Thomson scatter.

The rate at which an electron scatters photons depends on the # photons per unit area is the energy flux at that frequency dived by the energy of an individual photon.

$$\Sigma = \frac{f_\nu}{h\nu} = \frac{L_\nu}{4\pi r^2 h\nu}$$

The electron will scatter via Thomson scatter at a rate

$$R_{scat} = \sigma_T \frac{L_\nu}{4\pi r^2 h\nu}$$

Each scattering event transfers, on average a momentum to the electron given by

$$p = \frac{h\nu}{c}$$

The force exerted on the electron is then

$$F_\nu = \frac{dp}{dt} = R_{\rm scat}\frac{h\nu}{c} = \frac{L_\nu \sigma_T}{4\pi r^2 c}$$

The total force is found by integrating over all frequencies, ν.

$$F_{\rm rad} = \frac{L\sigma_T}{4\pi r^2 c}$$

Gravitational attraction prevents the electron from being repelled by the accreting source of luminosity. The gravitational force will be felt more strongly by protons, but electrons are attracted to the protons by the coulomb attraction. Thus,

$$F_{\rm grav} = \frac{GMm_p}{r^2}$$

$$F_{\text{rad}} = \frac{L\sigma_T}{4\pi r^2 c} \qquad\qquad F_{\text{grav}} = \frac{GMm_p}{r^2}$$

The accretion flow will stop if $F_{\text{rad}} > F_{\text{grav}}$ since the net force on matter in the flow would then be outward. The maximum accretion rate and maximum luminosity occurs when the radiation pressure exactly balances gravity. This is the **Eddington Luminosity**.

$$\frac{L_E\sigma_T}{4\pi r^2 c} = \frac{GMm_p}{r^2}$$

$$L_E = \frac{4\pi cGMm_p}{\sigma_T}$$

Calculate this limit in terms of the M/M_{sun}.

$$L_E = \frac{4\pi \times 3\times 10^{10} \times 6.7\times 10^{-8}\ \text{cgs} \times 2\times 10^{33}\ \text{g} \times 1.7\times 10^{-24}\ \text{g}}{6.7\times 10^{-25}\ \text{cm}^2}\frac{M}{M_\odot}$$

$$= 1.3\times 10^{38}\ \text{erg s}^{-1}\frac{M}{M_\odot} = 6.5\times 10^4\ L_\odot\frac{M}{M_\odot}$$

Limiting luminosity is called the Eddington Luminosity

Notes:

- Our calculations of luminosities for accretion onto neutron stars implies we would get luminosities higher than the Eddington limit. This is not really true. We made an assumptions/ simplifications of spherical accretion and an isotropically radiating source.
- Matter is taken in along an equatorial plan and radiates preferentially in directions perpendicular to the plane..
- Detailed calculations show that accretion disks become unstable when radiating near L_E.
- L_E applies to systems undergoing steady-state accretion.

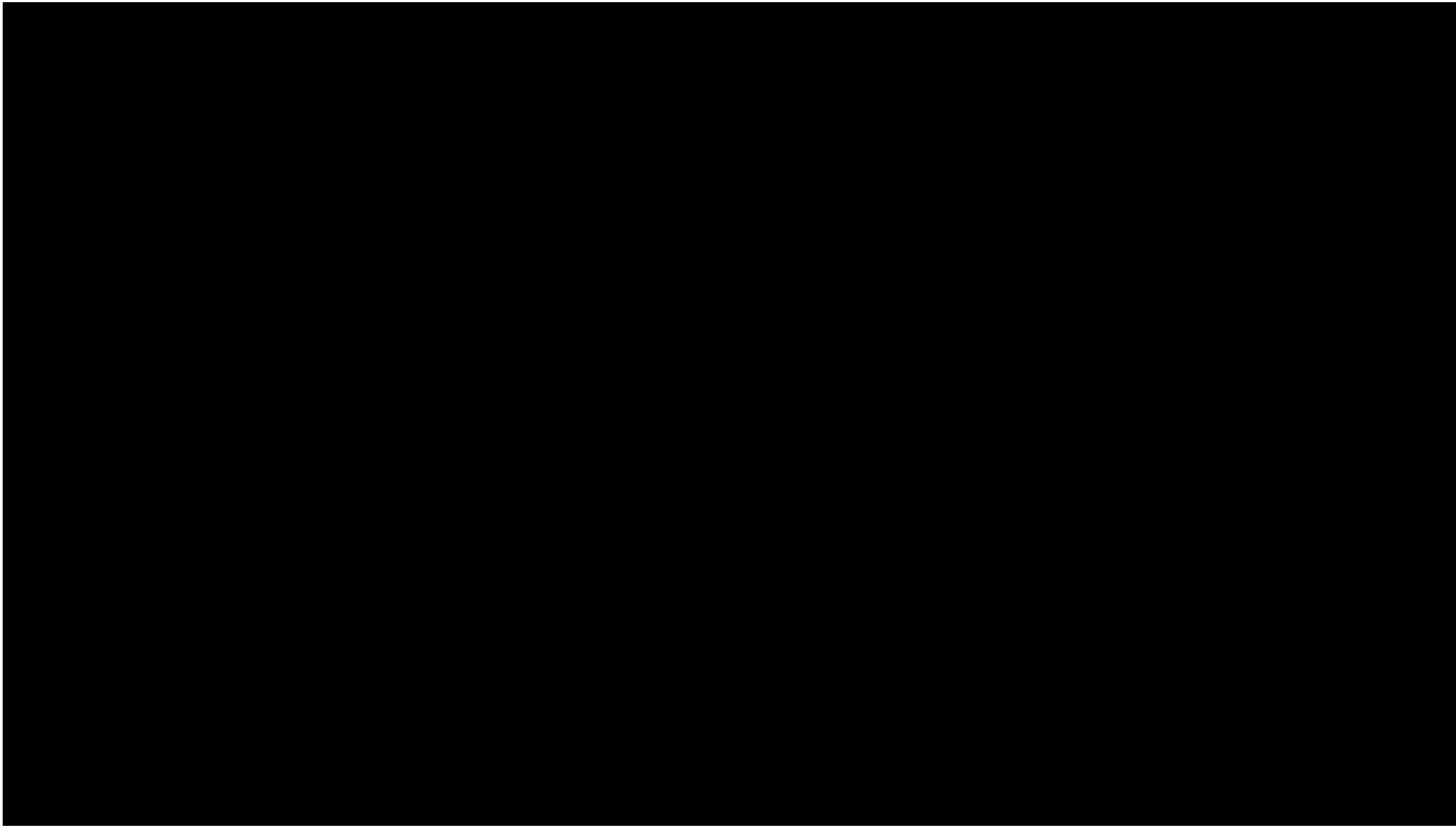
Evolution of Interacting Binary Systems

Recall that isolated neutron stars power their pulsar emission and their surrounding SN remnant emission at the expense of their rotational energy.

Neutron stars in binary systems that are accreting matter from a companion can GAIN angular momentum.

The jets and beams present in pulsars can hit one side of the donor star, heat it, ablate it or completely destroy it. These pulsars are known as *black-widow pulsars*. Example: a millisecond pulsars with no companion.

http://www.nasa.gov/content/goddard/with-a-deadly-embrace-spidery-pulsars-consume-their-mates/#.VSKgpEaRqC8



http://www.nasa.gov/content/goddard/with-a-deadly-embrace-spidery-pulsars-consume-their-mates/#.VSKgpEaRqC8

Let's examine the changes in evolution to of the parameters in a binary system.

The orbital angular momentum of a circular binary composed of M_1 and M_2 with separation distance a.

$$J = I\omega = \mu a^2 \omega$$

where I is the moment of inertia and μ is the reduced mass,

$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

Recall Kepler's law (from chapter 2).

$$\omega^2 = \frac{G(M_1 + M_2)}{a^3}$$

Substituting yields

$$J = \mu a^2 \frac{\sqrt{G(M_1 + M_2)}}{a^{3/2}} = \mu \sqrt{G(M_1 + M_2)a}$$

$$J = \mu\sqrt{G(M_1 + M_2)a}$$

Conservation of total mass and angular momentum require

$$\frac{dJ}{dt} = 0$$

Which term(s) are chaining with time? Need to invoke the chain rule.

$$\frac{dJ}{dt} = \sqrt{G(M_1 + M_2)}\left(\frac{d\mu}{dt}\sqrt{a} + \frac{\mu}{2\sqrt{a}}\frac{da}{dt}\right) = 0$$

$$-\sqrt{a}\frac{d\mu}{dt} = \frac{\mu}{2\sqrt{a}}\frac{da}{dt}$$

$$-\frac{2}{\mu}\frac{d\mu}{dt} = \frac{1}{a}\frac{da}{dt}$$

$$\mu = \frac{M_1 M_2}{M_1 + M_2} \qquad\qquad -\frac{2}{\mu}\frac{d\mu}{dt} = \frac{1}{a}\frac{da}{dt}$$

Examine dμ/dt

$$\frac{d\mu}{dt} = \frac{1}{M_1 + M_2}\left(\frac{dM_1}{dt}M_2 + M_1\frac{dM_2}{dt}\right)$$

However, conservation mass requires that

$$\dot{M}_1 = -\dot{M}_2$$

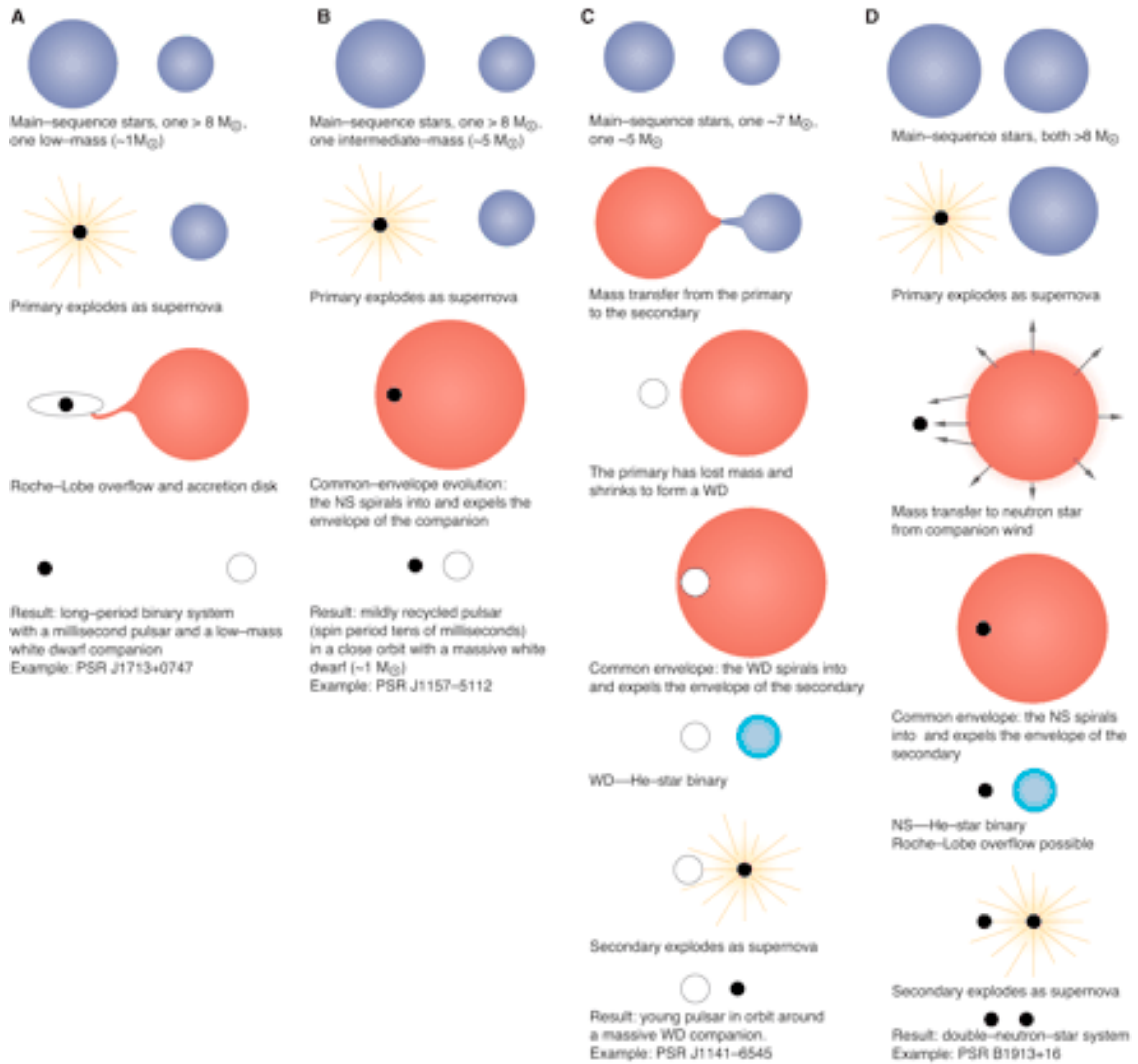
Thus, we can write

$$\frac{d\mu}{dt} = \frac{\dot{M}_1}{M_1 + M_2}(M_2 - M_1)$$

Substituting yields

$$\boxed{2\dot{M}_1\frac{M_1 - M_2}{M_1 M_2} = \frac{1}{a}\frac{da}{dt}}$$

This equation determines how period and separation evolve.



http://www.sciencemag.org/content/304/5670/547/F1.expansion