At the Brillouin zone boundaries

$k = \pm \frac{\pi}{a}$,

$\omega_{optical} = \sqrt{\frac{2C}{m_2}}$ $\quad m_1 \geq m_2$

$\omega_{acoustic} = \sqrt{\frac{2C}{m_1}}$

and $\left.\frac{d\omega}{dk}\right|_{k=\pm\frac{\pi}{a}} = 0 = v_{group}$

What does the motion look like?

$\omega_{optical}$ is independent of m_1

$\Rightarrow$ m_1 frozen

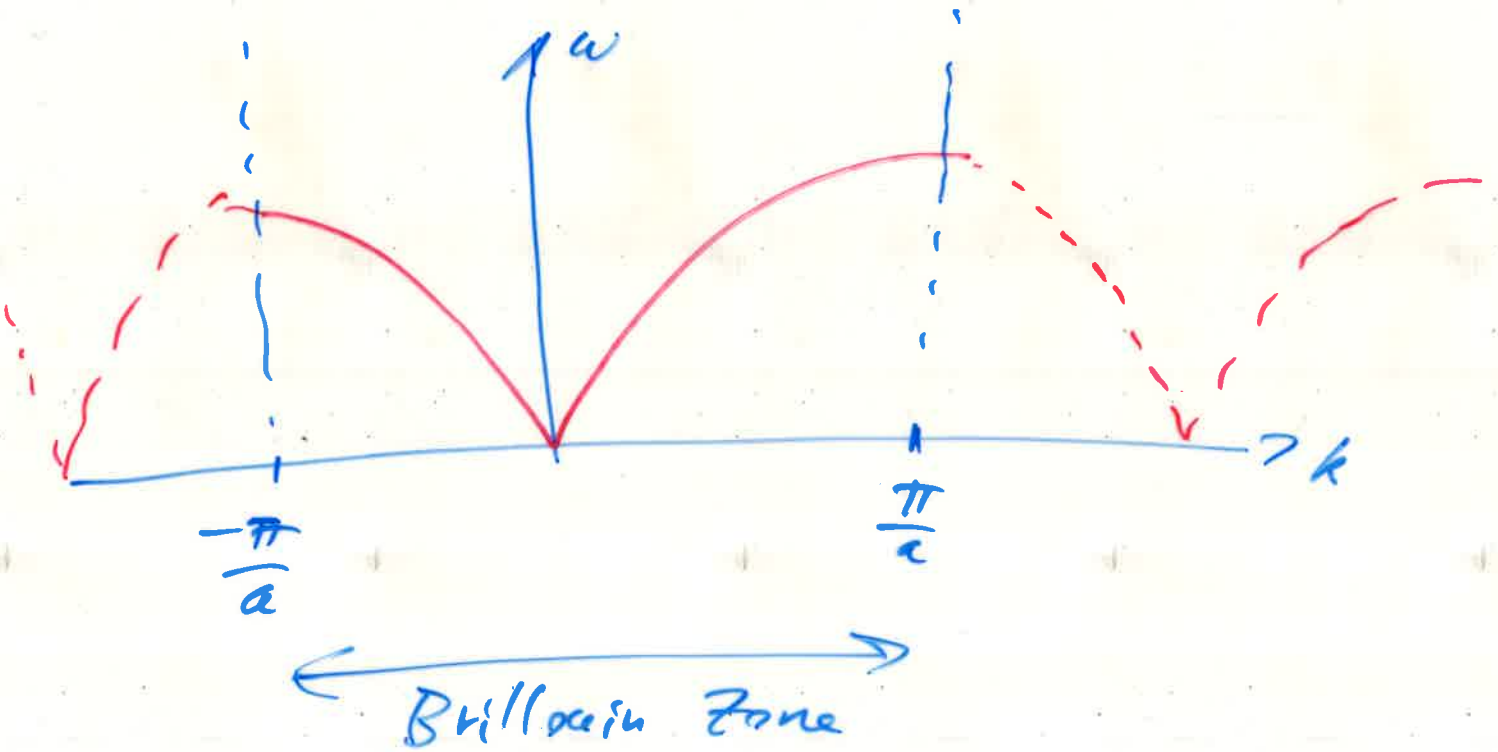
standing wave, no energy propagation ✓

m_2 attached to two springs $\Rightarrow$ force constant = $2C$ ✓

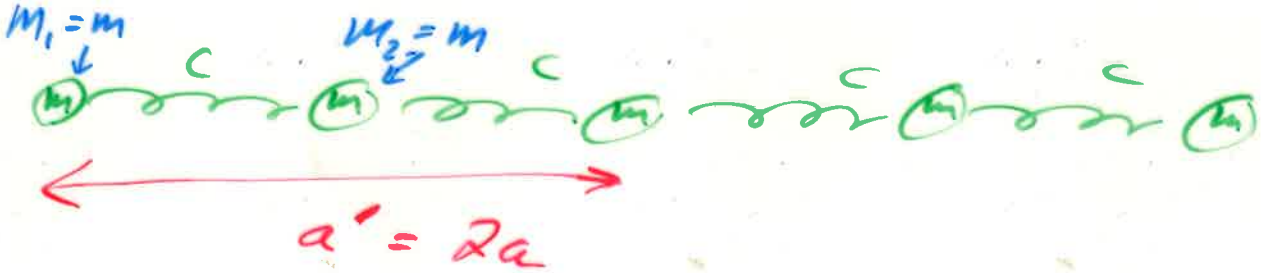
$\omega_{acoustic}$

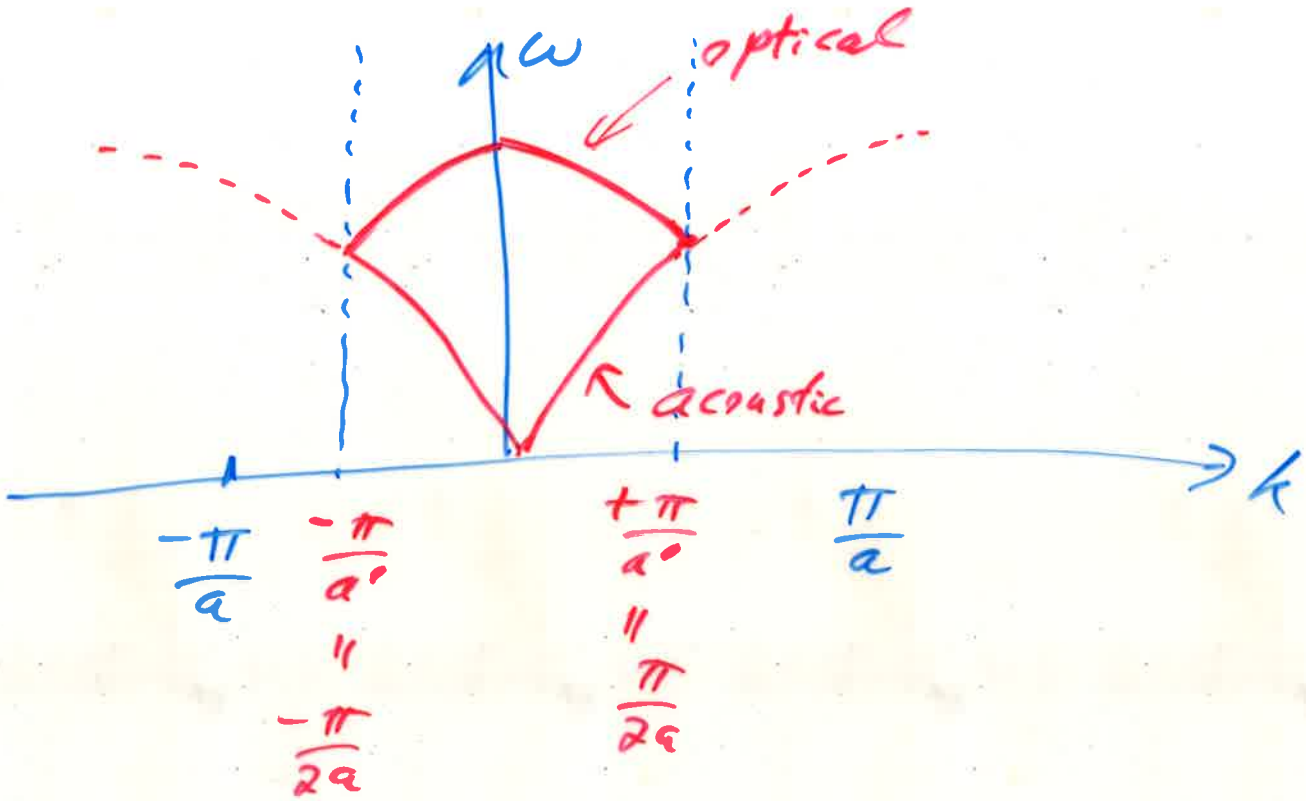
Dispersion Relation

$$\omega = 2\sqrt{\frac{c}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$$

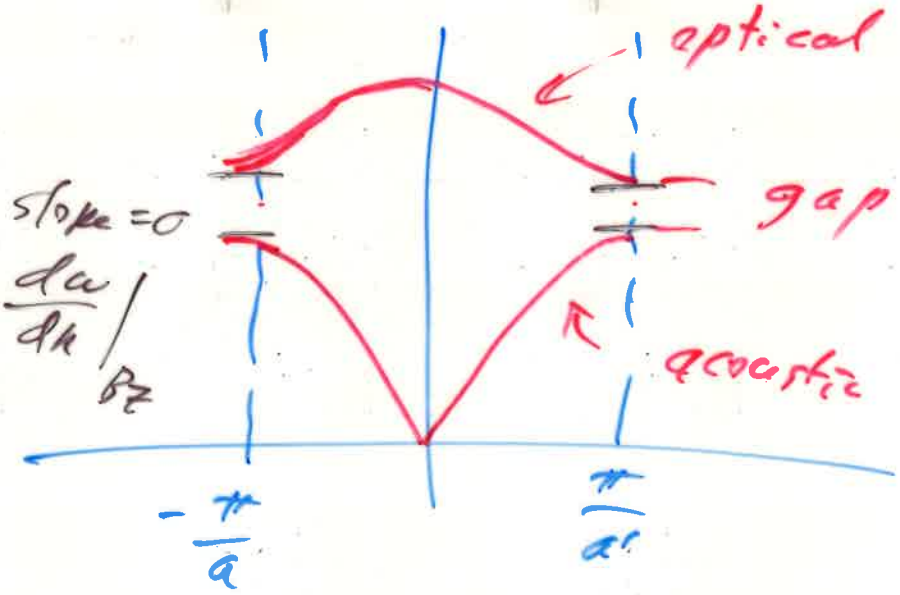


Same Xtal, lattice spacing 2a and two atom basis




ω
optical
acoustic
k
$-\frac{\pi}{a}$
$-\frac{\pi}{a'}$
$+\frac{\pi}{a'}$
$\frac{\pi}{a}$
$-\frac{\pi}{2a}$
$\frac{\pi}{2a}$

Now $m_1 \neq m_2$


optical
gap
acoustic
slope = 0
$\frac{d\omega}{dk}\Big|_{BZ}$
$-\frac{\pi}{a}$
$\frac{\pi}{a'}$